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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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THE PROTON LIFETIME IN SO(10) GRAND UNIFICATION MODEL

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ВРЕМЯ ЖИЗНИ ПРОТОНА В МОДЕЛИ ВЕЛИКОГО ОБЪЕДИНЕНИЯ SO (IO)

Рассматривается модель великого объединения, основанная на ортогональной группе SO (IO). Симметрия нарушается до стандартной группы $SU(3)_c \times SU(2)_L \times U(1)_y$ в два этапа через вакуумные средние полей Хиггса 45 и 126. С помощью уравнений ренормгруппы в двухпетлевом приближении вычислены масштаб нарушения SO (IO) - симметрии M_X и масштаб нарушения лево-правой симметрии M_R. Показано, что время жизни протона в модели SO (IO) в рамках современных экспериментальных ограничений на значение угла Вайнберга может быть достаточно большим и согласуется с последними экспериментальными данными по распаду протона. Рассмотрены также возможные ограничения на массы нейтрино, которые возникают в модели.

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THE PROTON LIFETIME IN SO(10) GRAND UNIFICATION MODEL

A model of grand unification based on SO(10) orthogonal group is considered. The symmetry is violated to the standard $SU(3)_C \times SU(2)_L \times U(I)_Y$ in two stages via the vacuum averages of the 45 and 126 Higgs fields. Using the renorm-group equations, the scale M_X of the SO(10) symmetry breaking and the scale M_R of the left-right symmetry breaking were calculated in two-loop approximation. It has been shown that the proton lifetime in the SO(10) model can be rather large in the framework of recent experimental restrictions on the value of Weinberg angle and agreed with available data on the proton decay measurements. Possible limitations on the neutrino mass arising in the model were also considered.

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Recent experiments on the proton lifetime measurement allowed to considerably rise the lower limit for τ_p to the value $\tau_p^{e^+\pi^\circ} > 6.5 \cdot 10^{31}$ yr [1]. This result is at variance with predictions of the minimal model of grand unification based on SU(5) group which gives $\tau_p = 10^{29\pm2}$ yr [2,3]. This problem is difficult to solve in the framework of SU(5) model, although such attempts were made [4].

In this connection it seems necessary to consider other grand unification models which will possibly give more acceptable predictions for the proton lifetime.

The simplest after SU(5) is the model of grand unification based on the orthogonal group SO(10). Unlike SU(5) this model allows to unite all the fermions of one family into one irreducible representation. On the other hand, SO(10) contains higher symmetry than SU(5). In particular, unlike SU(5) it is left-right symmetrical. For that reason, the breaking of SO(10) to the standard group $G_1 = SU(3)_C \times SU(2)_L \times U(1)_Y$ may proceed in several stages.

As was noted in [5], the breaking of SO(10) can go through one of the following maximal subgroups of SO(10):

(A) $SU(5) \times U(1)$; (B) $SU(4) \times SU(2)_L \times SU(2)_R$.

In the first case the predicted value of τ_p doesn't exceed that for the conventional SU(5) model [5]. Of special interest is the second case when the breaking goes through the Pati-Salam group. In this case the subsequent violation could proceed by different ways:

$$SO(10) \xrightarrow{M_X} SU(4)\times SU(2)_L \times SU(2)_R$$
$$\xrightarrow{M_C} SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$$
$$\xrightarrow{M_R} SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R$$
$$\xrightarrow{M'} SU(3)_c \times SU(2)_L \times U(1)_y \qquad (1)$$

$$SO(10) \xrightarrow{M_X} SU(4)\times SU(2)_L \times SU(2)_R$$
$$\xrightarrow{M_R} SU(4)\times SU(2)_L \times U(1)_R$$
$$\xrightarrow{M_C} SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R$$
$$\xrightarrow{M'} SU(3)_c \times SU(2)_L \times U(1)_y \qquad (2)$$

In the violation path (1) the quark-lepton symmetry is violated sooner than the left-right symmetry (i.e., $SU(2)_R$).

In the latter case the process is inverse. As was shown in [5], if the left-right symmetry is breaked sooner than the quarks separate from leptons, then in such a model the proton lifetime strongly depends on the Higgs content of the theory and may be both larger and smaller than that in SU(5) model. The predictions are more definite if the quark-lepton symmetry is violated sooner,- in this case τ_p can exceed the experimental limit. However, in the complete violation path (1) for SO(10) with $M_X \gg M_C \gg M_R \gg M'$ the renormalization-group equations fail to permit M_X to be expressed through observable quantities and to obtain any predictions for τ_p . Besides, such an important parameter of the theory of M' violation of $U(1)_{B-L}$ symmetry, as that by means of which the masses of right neutrino are determined (and with their help also the masses of conventional, left neutrino) is not present in renormalization-group equations at all and could not be determined

In the present work we consider the "shortened" path of type (1) violation. The Higgs fields in our model are 45, 126, and 10. Their decomposition into the representations of $SU(3)_C \times SU(2)_L \times SU(2)_R$ group has the form

$$\underline{45} = (8,1,1) + (1,3,1) + (1,1, \ldots$$
$$(3,2,2) + (\bar{3},2,2) + (3,1 \ldots$$
$$(\bar{3},1, \ldots$$
$$\underline{126} = (6, \ldots) + \ldots (1,3) + (1,3, \ldots$$
$$(1,1, \ldots) + \ldots + (\bar{?}, \ldots$$
$$(3,2,2) + \ldots + (\bar{3},1 \ldots$$
$$(3,1, \ldots$$
$$\underline{10} = (3,1,1) + \ldots + (1,2 \ldots$$

Then the breaking of SO(10) proceeds on as follows: the vacuum average (VA) of the 45 Higgs field component (1,1,1) violates SO(10) to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_Y$, then VA of the 126 Higgs field component (1,1,3) violates this group to G_1, i.e., in our case in violation path (1)

$$M_X = M_1 = \langle 45 \rangle, \quad M_R = M' = \langle 126 \rangle .$$

The further violation of G_1 to the exact symmetry $G =$ $SU(3)_C \times U(1)$, may go through the component (1,2,2) of the 10-plet (or 126-plet). As we shall see in what follows, such a violation scheme will ensure unequivocal determination of M_X and M_R from renormalization-group equations in terms of observed quantities. The knowledge of M_X allows one to determine the proton lifetime, τ_p, while the value of M_R defines the scale of left-right and simultaneously of $U(1)_{B-L}$ symmetry violation. As we shall see, the proton lifetime (depending on the value of Weinberg angle) may be rather large already in this simple scheme of SO(10) violation.

To allow for the contribution of Higgs fields to coefficients of β -functions entering the renormalization-group equations, it is necessary to know the masses of these fields. They are determined by VA of Higgs fields and an unknown coupling constant defining the self-action of these fields, i.e., they are actually the unknown parameters of the theory. If we assume, however, that no conditions on the Higgs fields potential except for the conditions of hierarchy of $M_X \gg M_R \gg M_W$ type vacuum averages are imposed, then one can obtain the most natural order of the magnitude of Higgs fields masses [6].

Let a multistage breaking of grand unification symmetry G to G_0 takes place:

$$G \xrightarrow{M_n} G_{n-1} \xrightarrow{M_{n-1}} \cdots \xrightarrow{M_{k+1}} G_{k+1} \cdots \xrightarrow{M_0} G_0 \qquad (4)$$
$$M_n \gg \cdots \gg M_{k+1} \gg M_k \gg \cdots \gg M_0 = M_W$$

and the Higgs field Φ is a representation of G, the vacuum average of which $\sim M_K$ violates G_{K+1} to G_K. Let us decompose Φ into the representations of G_{K+1}

$$\Phi = \Phi_0 + \sum_i \Phi_i \qquad (5)$$

where Φ_0 is that part of Φ, VA component of which violates G_{K+1}. Then, the remaining Higgs fields from Φ_0 acquire the mass $\sim M_K$. The masses of the rest of the Higgs fields from Φ are determined in the following way: every set of Φ_i fields acquires the mass M_r $(r > K)$, if it is a representation of G_r group and is not a representation of G_{r+1} group [6].

One can call these rules the survival hypothesis for the

Higgs fields,- the Higgs fields acquire the maximum possible mass.

It is easy to determine from the aforesaid the masses of Higgs fields in our model. One $SU(2)_L$ doublet of Higgs fields acquires the mass $\sim M_W$, another $SU(2)_L$ doublet - the mass $\sim M_R$. The representation (1,1,3) of $SU(3)_C \times SU(2)_L \times SU(2)_R$ group from the expansion of the 126-plet (3) acqures the mass $\sim M_R$. All the rest Higgs fields acquire the mass $\sim M_X$.

The renormalization-group equations determining the scales M_X and M_R of SO(10) violation have the form:

$$\bar{\alpha}^{-1} = \alpha_3^{-1}(\mu) + \frac{b_3}{2\pi}\ln\frac{M_R}{\mu} + \frac{\bar{b}_3}{2\pi}\ln\frac{M_X}{M_R}$$

$$\bar{\alpha}^{-1} = \frac{\sin^2\theta_W}{\alpha(\mu)} + \frac{b_2}{2\pi}\ln\frac{M_R}{\mu} + \frac{\bar{b}_2}{2\pi}\ln\frac{M_X}{M_R} \qquad (6)$$

$$\bar{\alpha}^{-1} = \frac{3}{5}\frac{\cos^2\theta_W}{\alpha(\mu)} + \frac{b_1}{2\pi}\ln\frac{M_R}{\mu} + \frac{1}{2\pi}\left(\frac{2}{5}b_1' + \frac{3}{5}b_2'\right)\ln\frac{M_X}{M_R}$$

$$b_3 = \bar{b}_3 = 7, \quad b_2 = 3.167, \quad \bar{b}_2 = 3$$

$$b_1 = -4.1, \quad b_1' = -5.5, \quad b_2' = 2.333,$$

where $\bar{\alpha}$ is the grand unification constant (in the M_X point). On the analogy of [7] we choose $\mu = M_W$. The fine structure constant in this point is $\alpha(\mu) = (127.8 \pm 0.5)^{-1}$. The parameter Λ corresponding to modified path of minimal subtractions is considered to be between 0.1 and 0.2 GeV [2,3]. The strong interaction constant in the μ point (in two-loop approximation with due regard for the thresholds) is $\alpha_3(\mu) = (9.884)^{-1}$ for $\Lambda = 0.1$ GeV and is $\alpha_3(\mu) = (6.901)^{-1}$ for $\Lambda = 0.2$ GeV.

One can determine M_X and M_R in terms of Λ and Weinberg angle $\sin^2\theta_W$ from equations (6). However, to have more exact estimates. It is necessary to take into account the two-loop corrections which can change the result by 2.5 - 3 times[8]. In our case these corrections must allow for the presence of two violation scales M_X and M_R:

$$\bar{\alpha}^{-1} = \alpha_i^{-1}(\mu) + \frac{b_i}{2\pi}\ln\frac{M_R}{\mu} + \frac{\bar{b}_i}{2\pi}\ln\frac{M_X}{M_R} - \frac{1}{4\pi}\sum_j \frac{b_{ij}}{b_j}\ln\left(1 + \frac{b_j}{2\pi}\alpha_j(\mu)\ln\frac{M_R}{\mu}\right) - \frac{1}{4\pi}\sum_j \frac{\bar{b}_{ij}}{\bar{b}_j}\ln\left(1 + \frac{\bar{b}_j}{2\pi}\alpha_j(M_R)\ln\frac{M_X}{M_R}\right) \tag{7}$$

where b_i, $\bar{b}_i$, b_{ij}, $\bar{b}_{ij}$ are coefficients of β-functions in the first and the second loops (with due regard for the contribution of Higgs fields), $\alpha_i(\mu)$, $\alpha_i(M_R)$ - are gauge coupling constants of $SU(3)_C$, $SU(2)_L$, $U(1)_Y$, $SU(2)_R$, $U(1)_{B-L}$ groups[3].

The allowance for threshold effects connected with the thresholds of heavy particles is also important. Weinberg has shown [9] that if the gauge group G is broken into a product of subgroups $H_1 \times H_2 \times \ldots$, then efficient gauge coupling constants g_1, g_2, ... of these groups are related to the gauge constant g of the G group in the vicinity of violation point by means of the following relation:

$$g_i(M) = g(M) + \frac{g^3(M)}{96\pi^2}\left\{ Tr\left(t_{iS}^2 \ln M_S/M\right) + Tr\, t_{iV}^2 + 8\, Tr\left(t_{iF}^2 \ln\sqrt{2}M_F/M\right) - 21\, Tr\left(t_{iV}^2 \ln M_V/M\right)\right\} \tag{8}$$

where M_S, M_F, M_V are masses of heavy scalars, fermions and gauge bosons arising at the breaking of G, t_{iS}, t_{iF}, t_{iV} are the corresponding representations of H_i groups. As it was the case with SU(5), the threshold effect connected with gau-

..e particles is also small here [..]. However, as was noted in [10], the uncertainties connected with heavy scalar partic- les may be essential.

One should also take note of an additional theoretical uncertainty in M_X arising due to higher order corrections, which can change M_X by a factor of nearly 1.5 [7].

Specific calculation of M_X and M_R shows that $\lg M_X$, $\lg M_R$ are nearly (to an accuracy of 1% in M_X and M_R) linear functions of $\sin^2\theta_W$. Plots of $\lg M_X$ and $\lg M_R$ versus $\sin^2\theta_W$ for Λ =0.1 GeV and Λ =0.2 GeV are shown in Fig....

M_X is seen to increase with $\sin^2\theta_W$, while M_R decreases. In the intersection point of these $\sin^2\theta_W$-dependences of and M_R we obtain the results of SU(5) [2] for the mass of grand unification and the Weinberg angle.

The values of $\sin^2\theta_W$ as obtained from the analysis of low-energy data and from masses of $W^\pm$, Z - bosons somewhat differ one from the other: in the former case $\sin^2\theta_W$=0.2[..] 0.014; in the latter case $\sin^2\theta_W$ =0.226±0.01 [11]. If we take the highest value of $\sin^2\theta_W$ even from the former estimate, we shall have for the proton lifetime a figure of 10^{35}, while in the latter case the upper value of τ_p is already about 10^{37} years. Both these estimates essentially exceed the experimental limit for τ_p. To get more definite predictions it is necessary to increase the accuracy of experimental determination of $\sin^2\theta_W$,- the variation of $\sin^2\theta_W$ by 0.01 changes τ_p almost by three orders of magnitude.

In Ref. [10], where, unlike our work, the violation path (2) has been considered, a conclusion was drawn that the proton lifetime in SO(10) not increased as compared with that in SU(5).

The choice of violation path (2) in Ref. [10] was due to the fact that the path (1) with intermediate $SU(2)_R \times SU(2)_L$ symmetry resulted in the equality of b and t quarks masses [12]. Concerning the last assertion we can say that it is valid only when in parallel with $SU(2)_R \times SU(2)_L$ there is an additional discrete L-R symmetry in the model. In our model, however, such a discrete symmetry is lacking,-it is connected with the mass spectrum of Higgs fields assumed by the survival hypothesis.

Thus, one can infer hence that in the SO(10) model the value of proton lifetime could be agreed with the experimental limit in the simplest violation path with intermediate left-right symmetry.

Using the obtained results for M_R we can make order-of-magnitude estimate of the neutrino mass. Indeed, VA of the 126-plet, breaking $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$, simultaneously imparts Mayorana mass $\sim \lambda_a M_R$ to the right neutrino, where λ_a is a constant coupling the 126-plet to a generation fermions (a =1,2,3). Then the conventional (left) neutrino acquires the Mayorana mass $m_{\nu_a} \sim \frac{m_{ua}^2}{\lambda_a M_R}$, where m_{ua} is the mass of a generation up quark (this mechanism was proposed by Gell-Mann et al.) . If we take that VA of $SU(2)_L$ doublets contained in 126 also contribute to the masses of fermions, then we can roughly estimate the constant λ_a by the violation of the relation of equality of symmetrical masses of a charged lepton and a lower quark, $\lambda_a \sim \frac{m_{da} - m_{ea}}{M_W}$ where m_{da} is the mass of -1/3 charge quark, m_{ea} is the mass of a generation lepton.

The value of M_R is estimated on the basis of experimental value of $\sin^2\theta_W$ and the lower bound on τ_p and turns to be 10^9 GeV $< M_R < 10^{13}$ GeV. Then we have the following estimates for neutrino masses: [1)]

$$
\begin{aligned}
10^{-4}\ \mathrm{eV} &< m_{\nu_e} < 1\ \mathrm{eV} \\
0.1\ \mathrm{eV} &< m_{\nu_\mu} < 100\ \mathrm{eV} \qquad (9) \\
10\ \mathrm{eV} &< m_{\nu_\tau} < 100\ \mathrm{keV}
\end{aligned}
$$

These estimates are certainly correct on the order of magnitude. Unlike Witten's estimates [13] which considered another SO(10) violation path (with no 126-plet), these estimates are not in contradiction with cosmological constraints.

It is worthwhile to note in conclusion that the proposed model of SO(10) grand unification enables one to obtain constraints on the proton lifetime and the scale of left-right symmetry breaking. The predicted value of τ_p may exceed the experimental limit. Estimates on neutrino masses have been obtained as well.

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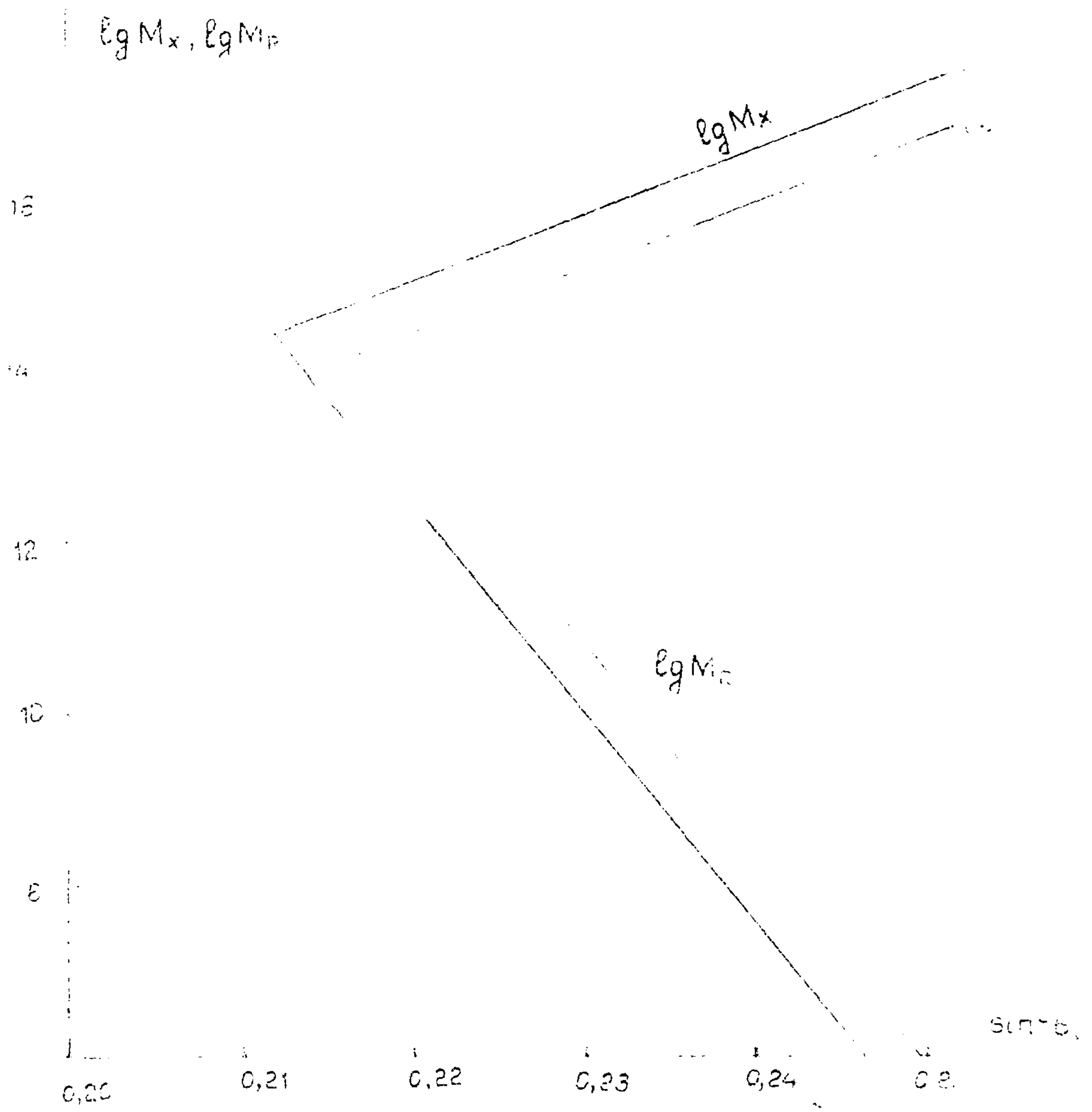
6) The vacuum averages of the doublets from 10- and 126-plets are taken to be of the same order of magnitude and to be $\sim M_W$.



Dependence of lg M_X, lg M_p on ... for Λ =0.2 GeV (curve (1)) and Λ = ... (curve (2)). M_X and M_p are measured ...

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