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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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STOCHASTIC QUANTIZATION OF GAUGE-INVARIANT AND SUPERSYMMETRIC THEORIES

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1. Introduction

The problem of stochastic quantization of the Yang-Mills theory in the scheme of Parisi-Wu [1] has been discussed in [2-5]. Results of [2-4] may be formulated in the following sequence of operations:

1. Let us solve and find the Yang-Mills field $A^a_\mu(x,t)$ as a functional of the source $J^a_\mu(x,t)$ by the following equation:

$$\partial_t A^a_\mu(x,t) + \frac{\delta S_{cl}}{\delta A^a_\mu(x,t)} + D^{ab}_\mu U^b(A) = J^a_\mu(x,t) \qquad (1.1)$$

where x is the four-dimensional coordinate, t is the additional time, S_{cl} is the Euclidean classical Yang-Mills action, U is the arbitrary functional of A^a_μ.

2. Let us substitute the solution of eq.(1.1) A^J in the gauge invariant functional $F_{GI}(A)$ and average it over J assuming that J is distributed as white noise:

$$\langle F_{GI}(t)\rangle_J \equiv \int F_{GI}(A^J)\exp - \int J^2 dt\, \mathcal{D}J \qquad (1.2)$$

3. In the limit $t \to \infty$ eq.(2) is independent of the

functional U and yields the standard quantum average of the functional F with regard to the Faddeev-Popov ghosts:

$$\lim_{t\to\infty} \langle F_{GI}(t) \rangle_{\mathfrak{I}} = \langle F_{GI} \rangle_{a} \tag{1.3}$$

In eq.(1.1) the term $D_\mu U$ actually fixes the gauge. However, ghosts in this scheme do not emerge.

Our aim is to free eq.(1.1) from the term $D_\mu U$, i.e. to begin with a naive Langevin equation:

$$\partial_t A^a_\mu(x,t) + \frac{\delta S_{cl}}{\delta A^a_\mu(x,t)} = \mathfrak{I}^a_\mu(x,t) \tag{1.4}$$

We will prove that solving eq.(1.4) with the condition

$$A^a_\mu(x,0) = 0 \tag{1.5}$$

and later on, acting in accord with points 2 and 3, we obtain the standard quantum averages for gauge invariant functionals, i.e. eq.(1.3) occurs. A similar problem has been discussed in 5 , where it is proved by means of explicit calculations that using the above method in g^2 approximation for the operator $F^a_{\mu\nu} F^a_{\rho\sigma}$ one obtains the standard quantum theoretical-perturbation average.

Note that eq.(1.5) actually fixes the gauge under -dependent gauge transformations but does not complicate the Langevin equation with an additional term $D_\mu U$

The stochastic quantization of N = 1 abelian supersymmetric theories has been discussed in [6,7].

In [7] the quantization is performed in the superfield components and in [6] - in the superfield formalism. For the N = 1 abelian supergauge theory the following result is obtained in [6]; let us write the Langevin equation for the gauge

superfield $\Phi(x,t,\alpha,\bar\alpha)$ that depends on the additional time t

$$\partial_t \Phi + \frac{\delta S}{\delta \Phi} + \alpha^{-1}(\bar D D)^2 \Phi = \mathfrak{I}(x,t,\alpha,\bar\alpha) \tag{1.6}$$

where $D, \bar D$ are the supercovariant derivatives, S is the superaction continued to the Euclidean space by the rule Further, acting in accord with points 2 and 3, we obtain usual quantum averages in the α gauge both for gauge invariant functionals and Green functions. In this paper we discuss also the N = 1 nonabelian supergauge theories. It appears that the result obtained for pure gauge theories (eq.(1.5)) is generalized for the supergauge case. As a result, the naive Langevin equation

$$\partial_t \Phi^a + \frac{\delta S}{\delta \Phi^a} = \mathfrak{I}^a \tag{1.7}$$

with a zero initial condition

$$\Phi^a\big|_{t=0} = 0 \tag{1.8}$$

yields standard quantum averages for gauge invariant functionals. Thus, Faddeev-Popov ghosts are automatically taken into account.

This result is valid in the abelian case as well. In this case, due to (1.8), we get rid of the third term in the left hand side of eq.(1.6) and obtain standard quantum averages for gauge invariant functionals (from (1.6) we have obtained also standard Green Functions in the α gauge).

2. Gauge theories

Equation (1.4) with the initial condition (1.5) may be solved by the perturbation theory [5]. The solution will read:

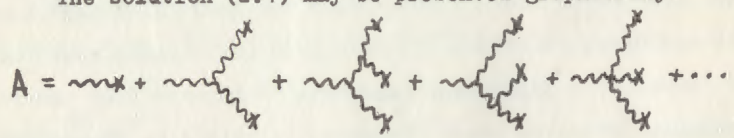
$$A = G\mathcal{J} + gGV(G\mathcal{J})(G\mathcal{J}) + g^2 GV\{GV(G\mathcal{J})(G\mathcal{J})\}(G\mathcal{J}) + g^2 GV(G\mathcal{J})\{GV(G\mathcal{J})(G\mathcal{J})\} + g^2 W(G\mathcal{J})(G\mathcal{J})(G\mathcal{J}) + \cdots \qquad (2.1)$$

where G is the zero order Green function of eq.(1.4):

$$G(k,t-t') = \{(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})e^{-k^2|t-t'|} + \frac{k_\mu k_\nu}{k^2}\}\theta(t-t') \qquad (2.2)$$

and V, W are the three- and four-point vertices, respectively (see [5]).

The solution (2.1) may be presented graphically:



where

$\sim \equiv G$

$x \equiv \mathcal{J}$

It is proved in ref.[5] that making use of the stochastic method (1.2), (1.3) in g^2 approximation for $F^a_{\mu\nu}(x)\, F^a_{\rho\sigma}(x)$ we will obtain a standard theoretical-perturbation average with account of Faddeev-Popov ghosts.

In the present paper we will prove a more general statement formulated in the introduction (eqs.(1.4),(1.5),(1.2), (1.3)).

Turn now to the proof of our statement. The latent supersymmetry, inherent in the stochastic quantization scheme, plays an important role in our proof. To make it explicit, we will pass to the superfield formalism of the stochastic quantization [8,9,10]. Let us first cut-off the additional time (denote the cut-off time via T) and assume that the current $\mathcal{J}$ on T passes into zero:

$$\mathcal{J}^a_\mu(x,T) = 0 \qquad (2.3)$$

We then take the limit $T \to \infty$

As is done in [9], the stochastic average of the gauge invariant functional at the moment T , calculated by eqs.(1.4), (1.5),(1.2), may be represented in the superfield form:

$$\langle F_{GI}(T)\rangle_{\mathcal{J}} = \int F_{GI}(A_\mu(T)) \exp - S(\Phi)\mathcal{D}\Phi_\mu \qquad (2.4)$$

where

$$\Phi^a_\mu = A^a_\mu + \bar\theta\psi^a_\mu + \bar\psi^a_\mu\theta + \bar\theta\theta C^a_\mu$$

$$S = \int_0^T \mathcal{L}(\Phi)\,dt\,d\theta\,d\bar\theta$$

$$\mathcal{L}(\Phi) = \mathcal{L}_{cl}(\Phi) - \Phi\left(\frac{\partial^2}{\partial\theta\partial\bar\theta} + \theta\frac{\partial}{\partial\theta}\partial_t\right)\Phi \qquad (2.5)$$

$\mathcal{L}_{cl}$ is the classical Euclidean Yang-Mills Lagrangian. In the action a four-dimensional integration is also implied that is omitted for briefness. Later on we omit also isotopic indices of fields. The follwoing boundary conditions that follow from eqs.(1.5) and (1.6) and the reality of the term $\int\bar\psi\partial_t\psi\,dt$ from the action (2.4), are imposed in the integral (2.4):

$$A_\mu(0) = 0$$

$$\psi_\mu(0) = \psi_\mu(T) = \bar\psi_\mu(0) = \bar\psi_\mu(T) = 0 \qquad (2.6)$$

$$C_\mu(T) = 0$$

The integrand in the functional integral (2.4) is invariant with respect to gauge transformations independent of t , θ , $\bar\theta$:

$$\Phi_\mu \to \Omega^{-1}\Phi_\mu\Omega + \Omega^{-1}\partial_\mu\Omega \qquad (2.7)$$

This implies that one may fix the gauge only in one point in . Let us impose this condition at $t = T$:

$$F(A_\mu(T)) = 0$$

This is achieved by means of the Faddeev-Popov trick. As a result, eq.(2.4) may be rewritten as

$$\langle F_{GI}(T)\rangle_J = \int F_{GI}(A_\mu(T))\delta(F(A_\mu(T)))\Delta_{FP}(A_\mu(T))e^{-S(\Phi)}\mathcal{D}\Phi_\mu \tag{2.8}$$

where Δ_{FP} is the Faddeev-Popov determinant.

The boundary condition for A_μ in the integral (2.8) changes as compared to (2.4). Now $A_\mu(0)$ can take gauge values:

$$A_\mu(0) = \Omega^{-1}\partial_\mu \Omega(x) \tag{2.9}$$

where $\Omega(x)$ is the arbitrary matrix of gauge transformations. The boundary conditions for the remaining components of Φ_μ are the same.

Let us introduce a production functional for the calculation of averages of the type of (2.8):

$$Z(h_\mu(x)) = \int \delta(F(A_\mu(T)))\Delta_{FP}(A_\mu(T))\exp - \int_0^T dt\, d\theta\, d\bar\theta(\mathcal{L}(\Phi) + H_\mu \Phi_\mu)\mathcal{D}\Phi_\mu$$

$$H_\mu = \bar\theta\theta\,\delta(t-T)\,h_\mu(x) \tag{2.10}$$

The boundary conditions of integral (2.10) coincide with those of (2.8), i.e.

$$A_\mu(0) = \Omega^{-1}\partial_\mu \Omega$$

$$\psi(0) = \psi(T) = \bar\psi(0) = \bar\psi(T) = 0$$

$$C_\mu(T) = 0 \tag{2.11}$$

Let us intriduce, just as in refs.[11,12], one-parameter production functionals Z^λ with the same boundary conditions (2.11):

$$Z^\lambda(h_\mu) = \int \delta(F(A_\mu(T)))\Delta_{FP}(A_\mu(T))\exp - \int_0^T dt\, d\theta\, d\bar\theta(\mathcal{L}^\lambda(\Phi) + H_\mu \Phi_\mu)\mathcal{D}\Phi_\mu$$

$$\mathcal{L}^\lambda(\Phi) = [\lambda + (1-\lambda)\bar\theta\theta\delta(t-T)]\mathcal{L}(\Phi) \tag{2.12}$$

At $\lambda = 1$ we have

$$Z^1(h_\mu) = Z(h_\mu) \tag{2.13}$$

At $\lambda = 0$ we have

$$Z^0 = \int \delta(F(A_\mu(T)))\Delta_{FP}(A_\mu(T))\exp - (\mathcal{L}_{cl}(A_\mu(T)) + h_\mu A_\mu(T))\mathcal{D}A_\mu(T)$$

$$Z^0(h_\mu) = Z^{FP}(h_\mu) \tag{2.14}$$

i.e. Z^0 coincides with usual Faddeev-Popov production functional. We will prove that Z^λ in the limit $T\to\infty$ will be independent of λ , and hence, from eqs.(2.13), (2.14) the desired equation will follow:

$$\lim_{T\to\infty} Z(h_\mu) = Z^{FP}(h_\mu) \tag{2.15}$$

Let us differentiate $\ln Z^\lambda$ with respect to λ :

$$\frac{\partial \ln Z^\lambda}{\partial\lambda} = -\int_0^T dt\, d\theta\, d\bar\theta\,(1 - \bar\theta\theta\delta(t-T))\langle \mathcal{L}(\Phi)\rangle \tag{2.16}$$

The production functional Z^λ with the boundary condition (2.11) is invariant with respect to supershifts:

$$t \to t - \bar\theta\epsilon$$
$$\theta \to \theta + \epsilon \tag{2.17}$$

The only form that is invariant with respect to these transformations is

$$\tau = t + \bar\theta\theta$$

i.e., e.g. $\langle\mathcal{L}(\Phi)\rangle$ in (2.16) is a function of τ and, therefore, (2.16) may be transformed as follows:

$$\frac{\partial \ln Z^\lambda}{\partial\lambda} = -\langle \mathcal{L}(\Phi)\rangle_{t=\theta=\bar\theta=0} \tag{2.18}$$

The source from the averaging (2.18) acts at the moment T , therefore, at large T it does not affect the average (2.18) calculated at $t = 0$. Thus, at large T the right hand side of (2.18) may be calculated by the formula

$$N^{-1}\int \mathcal{L}(\varphi(0))\delta(F(A_\mu(T)))\Delta_{FP}(A_\mu(T))\exp - S^\lambda \mathcal{D}\varphi_\mu \mathcal{D}\Omega \qquad (2.19)$$

where

$$N = \int \delta(F(A_\mu(T)))\Delta_{FP}(A_\mu(T))e^{-S^\lambda}\mathcal{D}\varphi_\mu \mathcal{D}\Omega$$

In (2.19) we have written explicitly also the integral by the element of the group $\Omega(x)$ that yields the boundary value for A_μ at $t = 0$ (eq.(2.11)). Making an inverse with respect to (2.7) transformation

$$\varphi_\mu \to \Omega \varphi_\mu \Omega^{-1} + \Omega \partial_\mu \Omega^{-1}$$

we pass in (2.19) from the boundary conditions (2.11) to (2.6), and $\mathcal{L}(\varphi)\big|_{t=\theta=\bar\theta=0}$ there turns into zero. Hence, the right hand side of eq.(2.18) turns into zero, $\ln Z^\lambda$ is λ -independent, and the basic statement (2.15) is proved.

Thus, the naive Langevin equation (1.4) with the initial condition (1.5) yields correct quantum averages for gauge invariant functionals, which both aesthetically and technically are better than eq.(1.1). Besides, eqs.(1.4) and (1.5) unlike eq.(1.1) are easily generalized to the $N = 1$ nonabelian supergauge theory.

3. N = 1 nonabelian supergauge theory

The result of this section is already formulated in the introduction (eqs.(1.7) and (1.8)).

In this section a transition is assumed to the Euclidean space by the formula $x_0 \to i x_0$, and matrices and spinors remain in the Minkowski space.

Equation (1.7) with the condition (1.8), as in the case of the standard Yang-Mills theory (eqs.(2.1) and (2.2)), may be solved by means of the perturbation theory. The Green function of eq.(1.7) in the zero by g approximation reads

$$G = \theta(t-t')\{(1-\frac{1}{4p^2}(\bar D D)^2)e^{-tp^2} - \frac{1}{4p^2}(\bar D D)^2\} \qquad (3.1)$$

Unlike (2.1) the theoretical perturbation series for Φ is more complicated in this case, since there are vertices of arbitrary order and, therefore, it is rather of a principle than a calculational value. Thus, eq.(1.7) with the initial condition (1.8) at least by the perturbation theory is soluble. Further we will act as in the case of gauge theories (section 2).

Now instead of φ^a_μ from (2.5) the superfield Ψ^a should be introduced:

$$\Psi^a(x,t,\alpha,\bar\alpha,\theta,\bar\theta) = \Phi^a(x,t,\alpha,\bar\alpha) + \bar\theta\psi^a + \bar\psi^a\theta + \bar\theta\theta C^a \qquad (3.2)$$

where Φ^a, ψ^a, C^a are the ordinary four-dimensional superfields, are the four-dimensional anticommuting Majorana spinors. All the other formulae of section 2 are valid also for the supersymmetry case, only the formula (2.7) should be replaced by

$$e^{\Psi} \to \Omega_+ e^{\Psi} \Omega_-^{-1} \qquad (3.3)$$

the boundary condition (2.9) by

$$e^{\Phi(0)} = \Omega_+ \Omega_-^{-1}$$

and the Faddeev-Popov determinant by the superdeterminant.

Conclusion

Thus, in the case of gauge and N = 1 supergauge theories the stochastic quantization method with naive Langevin equa-

-ions (i.e. with a classical action without additional terms) with the zero initial condition at $t = 0$ yields standard theoretical-perturbation quantum averages for gauge invariant functionals. The contribution of ghosts is automatically taken into account.

In the stochastic quantization method, Gribov's problem related to the fixation of the Faddeev-Popov gauge, is lacking as well.

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