индекс 3624

ЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

ЕФИ-753(66)-84

ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

R.G.BADALYAN, H.R.GULKANYAN

MULTIPARTON RECOMBINATION MODEL

ЕРЕВАН-1984

1. Introduction

The recent rise of interest in the study of hadron processes with small transverse momenta is related to the attempts to explain the features of these processes basing on the quark-parton structure of hadrons. A number of recombination-type models [1-8] are proposed where connection is established between the incident hadron quark-parton distribution functions and inclusive cross sections of hadron production (hadrons with small P_T are meant further) in the fragmentation region at values of the Feynman variable $X=P_{\parallel}/P \gtrsim 0.2$, where P is the incident hadron momentum, $P_{\parallel}$ is the longitudinal component of the secondary hadron momentum in the c.m.s. As a basis for this served the experimental fact of the approximate similarity of distribution function of the proton valence quarks, defined from deep inelastic lepton processes, and inclusive spectra of proton fragmentation into $\pi^{\pm}$-mesons. The latter, according to the models, are produced as a result of recombination of a fast valence quark and low-energy antiquark from the composition of the incident proton. The recombination probability is defined by the recombination function that is chosen phenomenologically [1-5] or is expressed via distribution functions of compound valence quarks (valons) in the final hadron [6-7].

One of the basic assumptions made in recombination models consists in the fact that in "soft" collisions the valence part of the structure function of incident hadrons does not undergo essential changes* (soft gluons participate in the interaction act [1-2]). As for distributions of sea partons, they may undergo changes (e.g. some part of the gluon sea may change into the quark-antiquark sea). However, in the framework of recombination models these changes practically do not affect the form of inclusive spectra of final hadrons in the fragmentation region but influence their absolute yield [4-7].

The characteristic feature of the above recombination models is the assumption that a limited number of partons (two in the case of meson production, three in the case of (anti) baryon production) whose total longitudinal momentum defines the longitudinal momentum of the final hadron, "transfer" from the parent hadron into the detected hadron. An opposite from this viewpoint is developed in the framework of additive quark model [9-10], according to which whole clusters of partons, constituent quarks (valence quark-parton with its "cloud" of sea partons) take part in hadroproduction. The portion of the incident hadron sea, contributing the longitudinal momentum

* The approximate account of annihilation effects of valence quarks of colliding hadrons is done in [8].

into the final hadron, is approximately $W \simeq m'/m$ where m is the number of valence quarks in the incident hadron, m' is the number of valence quarks common for incident and final hadrons.

The question what role the parton sea of the incident hadron plays in the production of final particles in fragmentation region, apparently cannot be solved theoretically as yet. In this paper we propose a phenomenological approach which would allow, comparing predictions to experiment, to make certain conclusions on the fact what portions of the final hadron longitudinal momentum are brought by valence and sea partons of the incident hadron. It is shown (after the example of proton fragmentation) that in the dependence on the type of the final hadron h, for definite classes of processes $p \to h$ a mechanism is realized similar to two (three)-parton recombination [1-8], and for other processes a situation occurs when an unlimited number of partons participate in the hadron h production (as in the constituent quark model [9-10]).

In section 2 we present a brief description of the Kuti-Weisskopf model [11] that is used, following [4,5,8] to describe the multiparton configuration of the incident hadron. Parameters of the Kuti-Weisskopf multiparton distribution function are fixed [8] in accord with the data of deep inelastic lepton-nucleon scattering. The procedure of separation of multiparton substates is described, that contain a valence quark and an arbitrary number of sea partons, and the x -distribution of such a subsystem in the nucleon is shown to correspond to the distribution for valons.

In a similar way in section 3 we separate the multiparton substate with a definite quark composition corresponding to that of the final hadron. The inclusive spectrum of the final hadron is shown to have at $x \to 1$ the dependence $(1-x)^{\beta}$ and is defined by the x-distribution of the separated subsystem and is practically independent of the function of its recombination into a final hadron. It is shown that the exponent β is defined by the number of common valence quarks m' and parameter W, that define the probability for the incident hadron sea parton to belong to the final hadron sea. Values of W for various processes $p \to h$ are estimated basing on the comparison with expeimental data. Calculation results of inclusive spectra for the processes $p \to \pi^{\pm}$ are presented in a wide interval by the variable x.

2. Modified Kuti-Weisskopf Model

The multiparton distribution function for protons is described in the Kuti-Weisskopf model [4,5,8,11] by the expression

$$dP = Z(P) \sum_n dP_n \tag{1}$$

where

$$dP_n = \frac{1}{2!} \frac{V_u(x_1)\,dx_1}{\sqrt{x_1^2 + x_T^2}} \frac{V_u(x_2)\,dx_2}{\sqrt{x_2^2 + x_T^2}} \frac{V_d(x_3)\,dx_3}{\sqrt{x_3^2 + x_T^2}} \times$$

$$\times \prod_a \frac{1}{(n_a)!} \prod_{j=1}^{n_a} \frac{S_a(x_{aj})\,dx_{aj}}{\sqrt{x_{aj}^2 + x_T^2}} \delta\left(1 - \sum_{j=1}^{3} x_j - \sum_a \sum_{j=1}^{n_a} x_{aj}\right) \tag{2}$$

Here $a = u, \bar{u}, d, \bar{d}, s, \bar{s}, G$ denotes the sort of the sea

parton, n_a is the number of partons of the sort a, $n = \sum_a n_a$. $x_T = \mu/P$ where μ is the effective transverse mass of the parton, P is the proton momentum $(P \gg \mu)$. $V_u(x)$, $V_d(x)$ and $S_a(x)$ are the so-called primitive distribution functions of valence and sea partons, respectively. Their form is presented in Appendix A. $Z(P)$ is the normalization factor defined from the normalization condition $\int dP = 1$.

The distribution density of the valence quark-parton or sea parton of the type a is obtained by summation of the expression (1) by the quantity of remaining partons and integration over their x-variables. Distributions for valence u, d-quarks and sea quarks and gluons, respectively, have the form (see Appendix A):

$$u(x) = \frac{2}{\sqrt{x}} (1-x)^{3,6} \quad B_2^{ud}(1; 1-x) \tag{3a}$$

$$d(x) = \frac{1}{\sqrt{x}} (1-x)^{4,6} \quad B_2^{uu}(1; 1-x) \tag{3b}$$

$$a_s(x) = \frac{g_a^2}{x} (1-x)^{4,1+K_a} \quad B_3(1; 1-x) \tag{3c}$$

The form of functions $B_N(\omega, x)$ at various values of the parameter ω is given in fig.1. Parameters g_a^2 and K_a from the expression (3c) are given in Appendix A. Distribution densities for partons of various sorts are presented in fig.2.

Let us now find the distribution density of the multiparton substate that contains a valence quark and an arbitrary number $n' = \sum_a n'_a$ of sea partons. To do so. one should sum the expression (1) over the number $n-n'=\sum_a (n_a - n'_a)$ of partons that are not included in the separated substate, and integrate over their x -variables. Let us require that this substate contain on the average $W = 1/3$ of the total number of sea partons (which occurs in the case of the constituent quark-valon). In this case an additional factor $\prod_a C_{n_a}^{n'_a} W^{n'_a}(1-W)^{n_a-n'_a}$, where $C_{n_a}^{n'_a}$ are the binominal coefficients, is introduced in the sum (1) on summation over $n-n'$. The thus obtained distribution densities of U - and D -valons in a parton have the form:

$$U(x) = x^{0,7}(1-x)^{2,4} B_1^u\left(\tfrac{1}{3},\tfrac{2}{3};x,1-x\right) \tag{4a}$$

$$D(x) = x^{0,7}(1-x)^{2,4} B_1^d\left(\tfrac{1}{3},\tfrac{2}{3};x,1-x\right) \tag{4b}$$

where functions $B_1^{u(d)}(\omega_1,\omega_2;x_1,x_2)$ are defined in Appendix B. Functions $U(x)$ and $D(x)$ satisfy the normalization condition:

$$\int_0^1 U(x)\,dx = 1\ , \qquad \int_0^1 D(x)\,dx = 1 \tag{5}$$

The form of the functions $U(x)$ and $D(x)$ as well as distribution densities of U and D -valons, presented in [6-7] are given in fig.3.

3. Multiparton Recombination Model

Consider now hadron production in the proton fragmentation region. We suppose that the inclusive cross section of the hadron production of the given sort h is proportional to the probability of finding in the proton a substate with the valence composition of the hadron h and having the momentum equal to that of this hadron. This probability depends both on the number m' of valence quark-partons, that are common for proton and hadron h ($m' = 0,1,2,3$), and the probability W for the sea parton of the proton (besides the sea partons that occur in the final hadron as valence quark-partons) being a part of the hadron h . Note that in the above models [1-10] the value of W is actually given a priori: in recombination models [1-8] it is always $W = 0$, whereas in the constituent quark model [9-10] $W \simeq m'/3$. In our calculations the value of W appears as a free parameter depending on m' , and is defined from the comparison with the experiment.

Consider the process of the proton fragmentation into π^+-meson. Let us separate in the proton the substate that contains a valence u -quark of the proton, a $\bar{d}$ -antiquark and an arbitrary number of sea partons that "belong" to this substate with the probability W . Consider now the recombination of the separated multiparton state into a π^+-meson. It is assumed, as in [6-7], that recombination stages are preceded by the formation of constituent objects - valons that recombinate into a final hadron. However, unlike [6-7], where the formation of valons proceeds due to quantum-chromodynamic evolution of the separated quark-parton, whose momentum is equal to that of the valon, in the present report it (in this case formation of U and $\bar{D}$ -valons) is assumed to proceed by means of statistical regrouping of partons within the separated multiparton state. Probabilities of the sea parton belonging to U- or $\bar{D}$-valons are assumed then identical and equal to $W/2$. Then the probability density of U- and $\bar{D}$-valons carrying, respectively, portions x_1/x and x_2/x of the total momentum of the separated multiparton substate, is defined by the expression (see Appendix B)

$$F_{u\bar{D}}(x_1,x_2) = 2(1-x)^{3,6(1-W)} x_1^{-0,5+1,8W} x_2^{-1+1,8W} \times$$

$$\times C_2^{ud}(1-W;1-x)\, C_1^{u}\left(\frac{W}{2};x_1\right) C_0^{\bar{d}}\left(\frac{W}{2};x_2\right) \Big/ C_3(1;1) \qquad (6)$$

Here $x = x_1 + x_2$, functions $C_N(\omega;x)$ are defined in Appendix A.

The function $R(y_1, y_2)$ of the U- and $\bar{D}$-valons $\left(y_j = \frac{x_j}{x}, j=1,2\right)$ recombination into a π^+-meson is defined, as in [6-7], by means of the distribution density of valons in a pion $G^{\pi}(y_1, y_2)$

$$R(y_1, y_2) = y_1 y_2\, G^{\pi}(y_1, y_2) \qquad (7)$$

The inclusive cross section of the proton fragmentation into a π^+-meson for the case, when the latter has a common with the proton valence u-quark, equals

$$\frac{x}{\sigma}\frac{d\sigma}{dx}(P\to\pi^+) = 2(1-x)^{3,6(1-W)} C_2^{ud}(1-W;1-x) \times$$

$$\int x_1^{-0,5+1,8W} x_2^{-1+1,8W} C_1^{u}\left(\frac{W}{2};x_1\right) C_0^{\bar{d}}\left(\frac{W}{2};x_2\right) \times$$

$$\times R\left(\frac{x_1}{x},\frac{x_2}{x}\right) \delta\left(1-\frac{x_1}{x}-\frac{x_2}{x}\right) dx_1\, dx_2 \qquad (8)$$

It is seen from the expressions (7) and (8) that at any finite function $G^{\pi}(y_1, y_2)$ x-dependence of the inclusive cross section (8) at $x\to 1$ is defined by the x-dependence of the expression $F_{u\bar{D}}(x_1,x_2)$ (formula (6)) and behaves like $(1-x)^{\beta}$. Similarly one may ensure that the same occurs for other fragmentation processes $h\to h'$ as well.

Thus inclusive spectra of hadrons at $x\to 1$ are defined by properties of the separated multiparton state and are practically independent of the form of recombination function, i.e. of the final hadron structure. Note that one may come to a similar conclusion considering recombinations of few-parton (two- and three-parton) systems.

Consider now properties of separated multiparton states for the general case $P\to h$.

Let us separate in proton a substate containing m' common with proton valence quarks and n'' quarks (antiquarks) from the sea that define the valence composition of the final hadron, and an arbitrary number of sea partons belonging to this substate with the probability W that depends on m'. Grouping partons into valons, we obtain the distribution density of the configuration, when the latter (valons) carry portions $x_1/x \ldots x_{m'+n''}/x$ $\left(x = \sum_{j=1}^{m'+n''} x_j\right)$ of the momentum of the considered substate (see Appendix B).

In virtue of the abovestated, the inclusive cross section of the process $P\to h$ at $x\to 1$ behaves like $(1-x)^{\beta}$, where the exponent β is defined by the expression

$$\beta = -1 + (1-W)\sum_{a} g_a^2 + \sum_{j=m'+1}^{m} \beta_j \qquad (9)$$

Values of g_a^2 and β_j are presented in Appendix A.

Exponent β for various processes $P \to h$ at various values of W as well as experimental values of exponents of inclusive spectra are shown in the table. It should be noted that experimental values of β are defined (except for the diffraction process $P \to P$) in the region of the variable X , which is not very close to 1. In this case, as we shall see below, particularly for the processes $P \to \pi^{\pm}$, inclusive spectra do not obey the dependence $(1-X)^{\beta}$ but have a more complicated X -dependence, therefore, one may speak of some efficient exponent β_{eff} only, which somewhat differs from the exponent at $X \to 1$.

Nevertheless, using the data plotted in the table, one can make the following qualitative conclusions. Values of W are almost the same for similar quantities m' of valence quarks that are common for incident and final hadrons, and are different for various values of m' . In processes of diffraction type $P \to P$ $(m'=3)$ W is close to 1, i.e. the parton sea is almost completely transfered to the final hadron; in processes involving diquarks ($m'=2$) of the type $P \to n$, $P \to \Lambda$ etc. $W \simeq 2/3$, i.e. the diquark, as a strongly bounded system, "takes away" approximately 2/3 of the initial sea, whereas at $m'=1$ (in particular, processes of the type $P \to \pi^{\pm}$) the valence quark partially or almost completely loses its cloud ($0 \leq W < 1/3$): in processes not involving valence quarks ($m'=0$) practically no sea is transfered to the final hadron ($W \simeq 0$).

Consider now, after the example of the process $P \to \pi^{\pm}$,

inclusive spectra of hadrons in a wider region by X $(0.2 \leq X < 1)$. At $W \simeq 0$ inclusive spectra calculation results are reduced to results of two-parton recombination models. Using the distribution density of valons in a pion in the form [6-7]

$$G^{\pi}(y_1, y_2) = \left[\Gamma^2(\alpha+1)/\Gamma(2\alpha+2)\right](y_1 y_2)^{\alpha}$$

($\Gamma(X)$ are Euler's gamma functions) at $\alpha = 0$ [6-7], we obtain the following expressions for inclusive spectra of $\pi^{\pm}$ -mesons:

$$\frac{x}{\sigma}\frac{d\sigma}{dx}(P \to \pi^+) = \frac{4}{3} g_{\bar{d}}^2 x^{0,5}(1-x)^{3,6}\left(1-\frac{2}{5}x\right) B_2^{ud}(1; 1-x) + g_u^2 g_{\bar{d}}^2 (1-x)^{4,1}(1-x+x^2/6)\, B_3(1; 1-x) \quad (10a)$$

$$\frac{x}{\sigma}\frac{d\sigma}{dx}(P \to \pi^-) = \frac{2}{3} g_{\bar{u}}^2 x^{0,5}(1-x)^{3,6}(1-x+6x^2/35) B_2^{uu}(1; 1-x) + g_d^2 g_{\bar{u}}^2 (1-x)^{4,1}(1-x+x^2/6)\, B_3(1; 1-x) \quad (10b)$$

The first summand in (10a), (10b) describes the contribution made in the inclusive cross section by the process of recombination of U (D) valon that contains a valence u (d) quark of proton, with the $\bar{D}$ ($\bar{U}$) valon, whereas the second summand describes the contribution made in the inclusive cross section by the process of recombination of U (D) valon that contains no valence u (d) quark of proton, with $\bar{D}$ ($\bar{U}$) valon. Expressions (10a) and (10b) satisfactorily describe the X -dependences of inclusive spectra of $\pi^{\pm}$ -mesons in the region $X \gtrsim 0.4$, however, by their absolute value they (note that expressions (10a) and (10b) contain no additional normalization factor) are approximately twice lower that experimental ones. This is explained by the fact that after the incident proton interaction act, the contribution of sea quark-antiquark pairs is already not defined by values of g_a^2 , extracted from deep inelastic processes but increase due to gluon component (i.e. $G \to q\bar{q}$ transitions) [4-7]. The value of this increase is defined [4-7] phenomenologically - by comparing theoretical calculations with experimental spectra of pions. In expressions (10a) and (10b) we have used values of $g^2_{u(\bar{u})} = g^2_{\bar{d}(d)} = 0.265$ which are twice as large as those extracted from deep inelastic lepton-nucleon processes. Calculation results are compared to experimental inclusive spectra of $\pi^{\pm}$-mesons in fig.4.

In the region $x \geqslant 0.4$ the agreement with experimental data is satisfactory. In the region $x < 0.4$ a substantial contribution in the inclusive spectra of $\pi^{\pm}$-mesons may be made by decays of meson resonances (in particular, ρ -mesons [13]) as well as the pionization process in the central region.

Experimental and calculated ratios of $\pi^{\pm}$-mesons in PP-interactions are presented in fig.5.

As has already been noted, the inclusive spectra (10a) and (10b) of $\pi^{\pm}$-mesons have the form $(1-x)^{b}$ at $x \to 1$ ($b = 3.6$) only. In the region of smaller values of x , where values of b are defined experimentally, the approximation of the expressions (10a) and (10b) by the dependence $(1-x)^{b_{eff}}$ yields the following efficient values for the exponents b_{eff} : in the region $x \simeq 0.9$ $b_{eff}(P\to\pi^+) \simeq 3.4$, $b_{eff}(P\to\pi^-) \simeq 3.7$,

in the region $x \simeq 0.6$ $b_{eff}(P\to\pi^+) \simeq 2.7$, $b_{eff}(P\to\pi^-) \simeq 3.3$. These values do not contradict the experimental estimates of the exponent b (see the table).

Inclusive spectra of baryon production ($P \to B$) will be presented in subsequent publications. Note only that at $W \simeq 0$ that corresponds to the three-parton recombination model, the calculated spectra for processes with two common valence quarks (ud or uu) strongly differ from experimental ones at values of the Feynman variable approximating 1. The best agreement of the model with the experiment is obtained at $W \simeq 2/3$, in other words, tnere occurs a mechanism like that proposed in the constituent quark model [9-10].

Note in conclusion that the proposed multiparton recombination model allows calculation of other one-particle and multiparticle spectra of the fragmentation process $h \to h_1 h_2 \cdots h_N$ if the structure functions of hadrons involved in this process are known.

Thanks are due to S.R.Gevorkyan, A.A.Grigoryan, A.Yu.Khojamirian and S.G.Matinyan for helpful discussions.

Appendix A

The Kuti-Weisskopf model [4,5,8,11] defines the multiparton distribution of a hadron composed of m valence quarks ($m = 2$ for mesons and $m = 3$ for baryons) and an arbitrary number of sea partons (quark-antiquark pairs and gluons). The latter are considered distributed independently and statistically. The probability density of ($m+n$)-parton configuration (n is the total number of sea partons in the configuration considered) is defined by the following expression [4,5,8,11] :

$$dP_n = \prod_{j=1}^{m} \frac{V_j(x_j)\,dx_j}{\sqrt{x_j^2 + x_T^2}} \prod_a \frac{1}{(n_a)!} \prod_{j=1}^{n_a} \frac{S_a(x_{aj})\,dx_{aj}}{\sqrt{x_{aj}^2 + x_T^2}} \times$$

$$\times\, \delta\Big(1 - \sum_{j=1}^{m} x_j - \sum_a \sum_{j=1}^{n_a} x_{aj}\Big) \qquad \text{(A.1)}$$

Here x is the Feynman variable; $a = u, \bar{u}, d, \bar{d}, s, \bar{s}, G$ denotes the sea parton sort (contributions of heavier quarks are not considered), n_a is the number of sort a partons, $n = \sum_a n_a$; $x_T = \mu/P$, where μ is the effective transverse mass of the parton, P is the hadron momentum ($P \gg \mu$) Functions $V_j(x)$ and $S_a(x)$ are the so-called primitive distribution functions (distribution functions without consideration of the energy-momentum conservation law) of valence and sea partons, respectively. These functions have the form [4, 5, 8] :

$$V_j(x) = x^{\beta_j} P_j(x); \quad P_j(0) = 1 , \quad \beta_j > 0 \qquad \text{(A.2a)}$$

$$S_a(x) = g_a^2 P_a(x) ; \quad P_a(0) = 1 , \quad g_a^2 > 0 \qquad \text{(A.2b)}$$

where $P_j(x)$, $P_a(x)$ are the polynomials ($j = 1,2 \ldots m$; $a = u, \bar{u}, d, \bar{d}, s, \bar{s}, G$). Hadron is considered as a sum of various ($m+n$)-parton configurations:

$$dP = Z(P) \sum_n dP_n \qquad \text{(A.3)}$$

where $Z(P)$ is the normalization factor (statistical weight) and is defined by the normalization condition $\int dP = 1$.

The Kuti-Weisskopf model allows to obtain one-parton and arbitrary ($m'+n'$)-parton distribution densities (m' is the number of valence quarks($0 \leq m' \leq m$)). Such a distribution is obtained after ($m+n-m'-n'$)-fold integration of dP_n and summation over n from n' to infinity.

For example, the distribution density of the k-th valence quark ($m' = 1$, $n' = 0$) has the form:

$$f_k^v(x) = Z(P) \frac{V_k(x)}{x} \sum_{n=0}^{\infty} \int \prod_{j=1}^{m}{}' \frac{V_j(x_j)\,dx_j}{\sqrt{x_j^2 + x_T^2}} \times$$

$$\times \prod_a \frac{1}{(n_a)!} \prod_{j=1}^{n_a} \frac{S_a(x_{aj})\,dx_{aj}}{\sqrt{x_{aj}^2 + x_T^2}}\, \delta\Big(1 - \sum_{j=1}^{m} x_j - \sum_a \sum_{j=1}^{n_a} x_{aj}\Big) \qquad \text{(A.4)}$$

The prime at the multiplication (or sum) sign implies that the term with $j = k$ is not taken into account.

Since $\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\xi\, e^{i\xi x}$, we find

$$f_k^v(x) = \frac{Z(P)}{2\pi} \frac{V_k(x)}{x} \times$$

$$\times \int_{-\infty}^{+\infty} d\xi\, e^{i\xi(1-x)} \prod_{j=1}^{m}{}' V_j(x,\xi) \exp\Big(\sum_a g_a^2 S_a(x,\xi)\Big) \qquad \text{(A.5)}$$

where the following notations are introduced:

$$V_j(x,\xi)=\int_0^{1-x}\frac{V_j(y)\,e^{-i\xi y}\,dy}{\sqrt{y^2+x_T^2}} \tag{A.6a}$$

$$S_a(x,\xi)=\frac{1}{g_a^2}\int_0^{1-x}\frac{S_a(y)\,e^{-i\xi y}\,dy}{\sqrt{y^2+x_T^2}} \tag{A.6b}$$

Allowing for the expression (A.2), functions $V_j(x,\xi)$ and $S_a(x,\xi)$ may be presented as

$$V_j(x,\xi)=(1-x)^{\beta_j}\int_0^1 z^{\beta_j}P_j((1-x)z)\,e^{-i\xi(1-x)z}\frac{dz}{z} \tag{A.7a}$$

$$S_a(x,\xi)=\int_0^1\left[P_a((1-x)z)\,e^{-i\xi(1-x)z}-1\right]\frac{dz}{z}+\ln\frac{2(1-x)}{x_T} \tag{A.7b}$$

Then we obtain for the distribution density of the k-th valence quark, allowing for the normalization condition, the following final expression:

$$f_k^V(x)=\frac{V_k(x)}{x}(1-x)^{-1+\gamma+\sum_{j=1}^{m}{}'\beta_j}\,B_{(m-1)}(1;1-x) \tag{A.8}$$

where $\gamma=\sum_a g_a^2$; $B_{(m-1)}(1;x)=C_{(m-1)}(1;x)/C_m(1;1)$

Functions $C_N(\omega;x)$, where $0\leqslant N\leqslant m$, $0\leqslant\omega\leqslant 1$, are defined by the following expression:

$$C_N(\omega;x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\xi\,e^{i\xi}\prod_{j=1}^{N}\int_0^1 z^{\beta_j}P_j(xz)\,e^{-i\xi z}\frac{dz}{z}\times$$
$$\times\exp\left(\omega\sum_a g_a^2\int_0^1\left[P_a(xz)\,e^{-i\xi z}-1\right]\frac{dz}{z}\right) \tag{A.9}$$

The physical meaning of the parameter ω will be explained below. In a similar way arbitrary ($m'+n'$)-parton distribution functions are obtained. In particular, for the distribution density of valence and sea partons in proton we have:

$$u(x)=2\,\frac{V_u(x)}{x}(1-x)^{-1+\gamma+\beta_u+\beta_d}\,B_2^{ud}(1;1-x) \tag{A.10a}$$

$$d(x)=\frac{V_d(x)}{x}(1-x)^{-1+\gamma+2\beta_u}\,B_2^{uu}(1;1-x) \tag{A.10b}$$

$$a_s(x)=\frac{S_a(x)}{x}(1-x)^{-1+\gamma+2\beta_u+\beta_d}\,B_3(1;1-x) \tag{A.10c}$$

where $a=u,\bar{u},d,\bar{d},s,\bar{s},G$. Functions $V_u(x)$, $V_d(x)$ and $S_a(x)$ are defined by the expressions (A.2a) and (A.2b). Superscripts of the function $B_N(\omega,x)$ reflect the fact that primitive distribution functions $V(x)$ for valence u and d -quarks differ.

The following expressions for primitive distribution functions of quarks and gluons in proton are obtained from data on deep inelastic scattering of leptons [8]:

$$V_u(x)=x^{0,5},\ V_d(x)=x^{0,5}(1-x),\qquad S_a(x)=g_a^2(1-x)^{\kappa_a} \tag{A.11}$$

$$g_u^2=g_{\bar u}^2=g_d^2=g_{\bar d}^2=0,1375;\ g_s^2=g_{\bar s}^2=0,0095;\ g_G^2=3;$$

$$\kappa_u=\kappa_{\bar u}=\kappa_d=\kappa_{\bar d}=1;\ \kappa_s=\kappa_{\bar s}=1,2;\ \kappa_G=2,7;\ \gamma=\sum_a g_a^2\simeq 3,6$$

Distribution densities for valence and sea partons in proton, allowing for (A.10) and (A.11), are given in section 2 (expression (3)). Plots of these functions are presented in fig.2.

Distribution densities of two and more valence quarks are defined by means of the values $B_1^u(1;x)$; $B_1^d(1;x)$ and $B_0(1;x)$. Functions $B_0(\omega,x)$, $B_1^u(\omega;x)$, $B_1^d(\omega;x)$, $B_2^{uu}(\omega;x)$, $B_2^{ud}(\omega;x)$ and $B_3(\omega;x)$ at various values of the parameter $\omega=0,1/3$, $2/3$, 1 are presented in fig.1.

Appendix B

The Kuti-Weisskopf model allows calculation also of the x -distribution of arbitrary multiparton subsystem as a whole. As a particular case serves the valon that contains a valence quark and on the average $1/m$ sea partons of hadron (the probability of the given sea parton belonging to the valon is $W=1/m$). In order to obtain the distribution function of the valon in hadron, one should introduce an additional weight factor $\prod_a C_{n_a}^{n'_a} W^{n'_a}(1-W)^{n_a-n'_a}$, where n'_a is the number of type a sea partons belonging to the valon ($C_{n_a}^{n'_a}$ are the binominal coefficients) when summing expressions (A.1)

over the number of partons that do not enter into the composition of the given valon. Then the distribution density of the quark-gluon subsystem that contains one k-th valence quark and $n'=\sum_a n'_a$ sea partons, may be written down as

$$dF_{n'}=Z(P)\left(\frac{2P}{\mu}\right)^{(1-W)\gamma}(1-x)^{-1+(1-W)\gamma+\sum_{j=1}^{m}{}'\beta_j}C_{(m-1)}(1-W;1-x)$$

$$\times\frac{V_k(x_k)dx_k}{\sqrt{x_k^2+x_T^2}}\prod_a\frac{1}{(n'_a)!}\prod_{j=1}^{n'_a}\frac{WS_a(x_{aj})dx_{aj}}{\sqrt{x_{aj}^2+x_T^2}}\times$$

$$\times\delta\left(x-x_k-\sum_a\sum_{j=1}^{n'_a}x_{aj}\right) \tag{B.1}$$

The distribution density of the k-th valon, as a whole, is obtained from the expression (B.1) after integration over remaining variables and summation over $n'=\sum_a n'_a$ from zero to infinity:

$$G_k^v(x)=x^{-1+W\gamma+\beta_k}(1-x)^{-1+(1-W)\gamma}+\sum_{j=1}^{m}{}'\beta_j\ \times$$
$$\times\ B_1(W,1-W;\ x,\ 1-x) \tag{B.2}$$

where the following notation is introduced:

$$B_{m'}(\omega_1,\omega_2;x_1,x_2)=C_{m'}(\omega_1;x_1)C_{(m-m')}(\omega_2;x_2)/C_m(1;1)\times$$
$$\times\delta(1-\omega_1-\omega_2)\delta(1-x_1-x_2) \tag{B.3}$$

The argument ω of the function $C_N(\omega;x)$ (expression (A.9)) is defined by the value W which is the probability for the sea parton belonging to the separated multiparton state.

Note that at $W \to 0$ the expression (B.2) transforms into the expression for the distribution density of the k-th valence quark-parton (A.8), since

$$C_1(0;x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\xi\, e^{i\xi} \int_0^1 z^{\beta_k} P_k(xz) e^{-i\xi z} \frac{dz}{z} =$$

$$= \int_0^1 z^{\beta_k} P_k(xz)\,\delta(1-z)\frac{dz}{z} = P_k(x) \qquad \text{(B.4)}$$

Substituting values of parameters (A.11) in the expression (B.2), we obtain distribution densities for U and D -valons in proton (at $W=1/3$):

$$U(x) = x^{0,7}(1-x)^{2,4} B_1^u\left(\frac{1}{3}, \frac{2}{3}; x, 1-x\right) \qquad \text{(B.5a)}$$

$$D(x) = x^{0,7}(1-x)^{2,4} B_1^d\left(\frac{1}{3}, \frac{2}{3}; x, 1-x\right) \qquad \text{(B.5b)}$$

In [6,7] the following expressions are obtained for distribution densities of U and D -valons:

$$U_H(x) = \frac{2\Gamma(a+1)}{\Gamma(a+4)} x^a (1-x)^2 \qquad \text{(B.6a)}$$

$$D_H(x) = \frac{\Gamma(2-a)\Gamma(2a+2)}{\Gamma(a+4)} x^{1-a}(1-x)^{2a+1} \qquad \text{(B.6b)}$$

where $a = 0.65$, $\Gamma(x)$ are the Euler's gamma functions. Functions $U(x)$, $D(x)$ and $U_H(x)$, $D_H(x)$ are presented in fig.3.

Consider the behaviour of the functions $C_N(\omega;x)$ at $\omega \to 0$. It is easy to check that

$$C_0(\omega;x) \to \omega \sum_a g_a^2 P_a(x) \qquad \text{(B.7a)}$$

$$C_1(0;x) = P_k(x) \qquad \text{(B.7b)}$$

$$C_2(0;x) = \int_0^1 z^{-1+\beta_k}(1-z)^{-1+\beta_j} P_k(xz) P_j(x(1-z))\,dz \qquad \text{(B.7c)}$$

$$C_3(0;x) = \int_0^1\int_0^1\int_0^1 z_1^{-1+\beta_k} z_2^{-1+\beta_j} z_3^{-1+\beta_i} \times$$

$$\times P_k(xz_1) P_j(xz_2) P_i(xz_3)\,\delta(1-z_1-z_2-z_3)\,dz_1 dz_2 dz_3 \qquad \text{(B.7d)}$$

Let us define also the functions $C_0^a(\omega;x)$:

$$C_0^a(\omega;x) = \frac{g_a^2}{2\pi}\int_{-\infty}^{+\infty} d\xi\, e^{i\xi} \int_0^1 \frac{P_a(xz)e^{-i\xi z}dz}{\sqrt{z^2 + \left(\frac{x_T}{x}\right)^2}} \times$$

$$\times \exp\left(\omega \sum_a g_a^2 \int_0^1 \left[P_a(xz)e^{-i\xi z} - 1\right]\frac{dz}{z}\right) \qquad \text{(B.8)}$$

Then $C_0^a(0;x) = g_0^2 P_a(x)$ and, hence

$$\lim_{\omega\to 0} \frac{C_0(\omega;x)}{\omega} = \sum_a C_0^a(0;x) \qquad \text{(B.9)}$$

Consider now the proton fragmentation into the hadron h having m' common with the proton valence quarks and n'' valence quarks that are sea quarks in proton ($m'+n'' = 2$ if

h is a meson, and $m'+n''=3$ if h is a baryon).

Let us separate in proton a substate containing m' valence quarks, n'' sea quarks (antiquarks) of the given type as well as an arbitrary number $n'-n''=\sum_a (n'_a - n''_a)$ of sea partons belonging to the considered substate with the probability W. The distribution function of such a substate is defined by the expression:

$$dF_{n'}^{n''} = z(P)\left(\frac{2P}{\mu}\right)^{(1-W)\gamma} \frac{1}{W^{n''}} (1-x)^{-1+(1-W)\gamma+\sum_{j=m'+1}^{m}\beta_j} \times$$

$$\times C_{(m-m')}(1-W;1-x)\prod_{j=1}^{m'} \frac{V_j(x_j)\,dx_j}{\sqrt{x_j^2+x_T^2}} \prod_a \frac{1}{(n'_a)!} \prod^{n_a} \frac{W S_a(x_{aj})\,dx_{aj}}{\sqrt{x_{aj}^2+x_T^2}} \times$$

$$\times \delta\Big(x-\sum_{j=1}^{m'} x_j - \sum_a \sum_{j=1}^{n'_a} x_{aj}\Big) \qquad \text{(B.10)}$$

Note that at $m'=1$, $n''=C$ the expression (B.10) transforms into the expression (B.1) ror a valon. It is assumed, as in [6,7], that the formation of the hadron h occurs due to recombination of constituent system-valons. The expression (B.10), describing the separated multiparton state, may be transformed into the expression describing the ($m'+n''$)-valon state. Assume, for example, that this substate has the valence composition of the π^+-meson ($m'=1$, $n''=n''_{\bar d}=1$). Let us integrate the expression (B.10) over the remaining x-variables, requiring that each sea parton belongs either to U-valon or to $\bar{D}$-valon with equal probability $W/2$ and sum it over the number of sea partons in each valon from zero to infinity. After such an integration and summation we find for the two-valon distribution density of the separated multiparton substate:

$$F_{u\bar D}(x_1,x_2) = 2(1-x)^{-1+(1-W)\gamma+\beta_u+\beta_d}\, x_1^{-1+\frac{W}{2}\gamma+\beta_u}\, x_2^{-1+\frac{W}{2}\gamma} \times$$

$$\times C_2^{ud}(1-W;1-x)\, C_1^{u}\left(\frac{W}{2};x_1\right) C_0^{\bar d}\left(\frac{W}{2};x_2\right)\Big/ C_3(1;1) \qquad \text{(B.11)}$$

where x_1 and x_2 are the momenta of U and $\bar{D}$-valons, respectively, $x=x_1+x_2$. Note that at $W\to 0$

$$F_{u\bar D}(x_1,x_2) =$$

$$= 2(1-x)^{-1+\gamma+\beta_u+\beta_d} x_1^{-1+\beta_u} g_{\bar d}^2 x_2^{-1} (1-x_2)^{K_{\bar d}} B_2^{ud}(1;1-x) \qquad \text{(B.12)}$$

which coincides with the two-parton distribution density of u and $\bar d$-quark-partons in a proton.

Similarly one may obtain arbitrary ($m'+n''$)-valon distribution densities.

Table*

$p \to h$	W_{exp}	$W=0$	$W=1/3$	$W=2/3$	$W=1$	m'
$p \to p$	-1	2.6	1.4	0.2	-1	3
$p \to n$	0.8±0.2					
$n \to p$	0.8±0.1					
$p \to \Lambda$	0.5±0.1					
	0.7±0.1					
	1.0±0.2					
	1.2±0.2	3.1	1.9	0.7	-0.5	2
$n \to \Lambda$	0.9±0.3					
	1.5±0.4					
$\bar{p} \to \bar{\Lambda}$	0.9±0.2					
$p \to \Delta^{++}$	0.35±0.15					
	0.5±0.1					
$p \to \Lambda_{1520}$	0.7±0.1					
$p \to \Sigma^{+}_{1385}$	1.0±0.3					
$p \to \pi^{+}$	3.0±0.2					
	3.2±0.3					
	3.39±0.05					
	3.43±0.11					
	3.5±0.03					
	3.7±0.5					
	3.9±0.3					
$n \to \pi^{-}$	4.0±0.2					
$\bar{p} \to \pi^{-}$	3.00±0.19					
	3.10±0.36					
	3.59±0.36	3.6	2.4	1.2	0	1
$p \to K^{+}$	2.56±0.20					
	2.77±0.10					

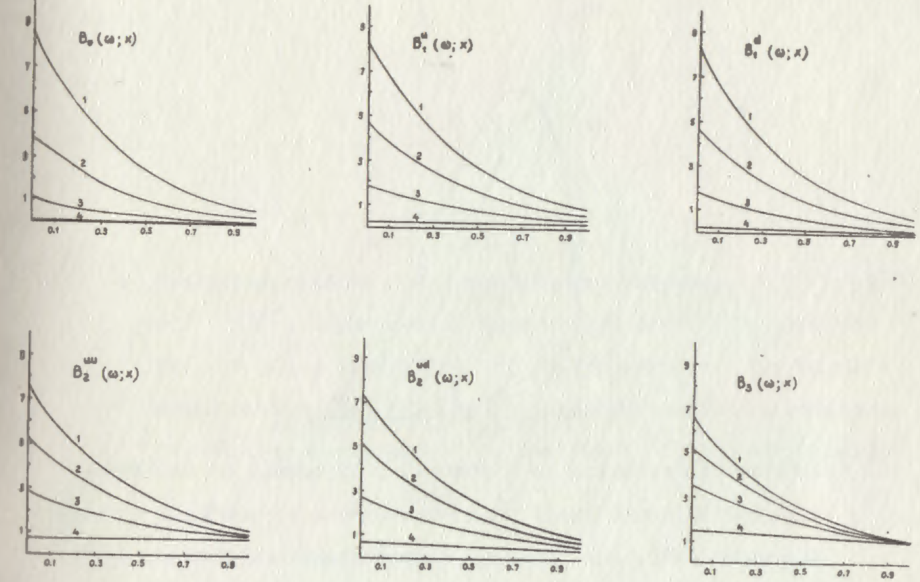
* Experimental values for exponents β are taken from [10,12-14].

Table (continuation)

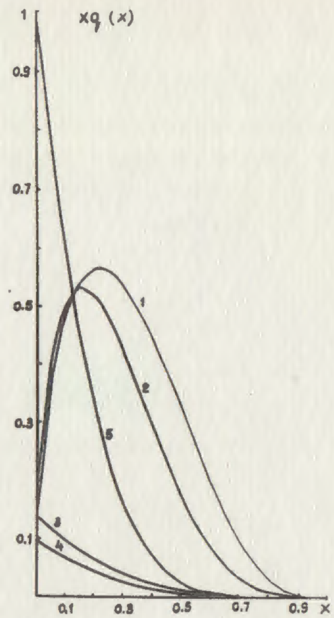
$p \to h$	W_{exp}	$W=0$	$W=1/3$	$W=2/3$	$W=1$	m'
	2.87±0.19					
	3.0±0.2					
$n \to K^{\circ}$	3.8±0.9					
$p \to \rho^{\circ}$	2.8±0.4					
	2.9±0.6					
	3.0±0.3					
	3.1±0.4					
$p \to \pi^{-}$	3.6±0.5					
	3.9±0.2					
	4.0±0.2					
	4.3±0.2					
	4.36±0.05					
	4.39±0.10					
	4.53±0.16					
	4.8±0.6					
$n \to \pi^{+}$	5.0±0.2					
$\bar{p} \to \pi^{+}$	3.56±0.43	3.6	2.4	1.2	0	1
	3.71±0.45					
	3.72±0.87					
	4.0±0.1					
$p \to K^{\circ}$	3.4±0.2					
	3.7±0.1					
	3.9±0.3					
	4.3±0.4					
	4.5±0.6					
$\bar{p} \to \bar{K}^{\circ}$	3.2±0.4					
$p \to \Xi^{-}$	3.0±0.5					
	4.1±0.5					
$p \to \Sigma^{-}_{1385}$	2.65±0.7					
$p \to K^{-}$	5.75±0.15					
$p \to \varphi$	4.0±0.1					
$p \to \bar{p}$	7.8±0.3					

Table (continuation)

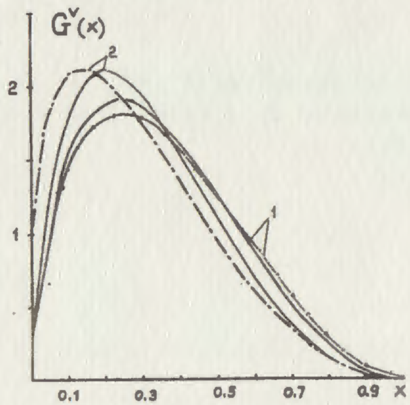
$p \to h$	W_{exp}	$W=0$	$W=1/3$	$W=2/3$	$W=1$	m'
$p \to \bar{\Lambda}$	8.03±0.44					
	8.09±0.57	4.1	2.9	1.7	0.5	0
	8.1±1.4					
	3.2±0.7					
	3.8±1.0					
	7.1±0.4					
	7.6±2.4					



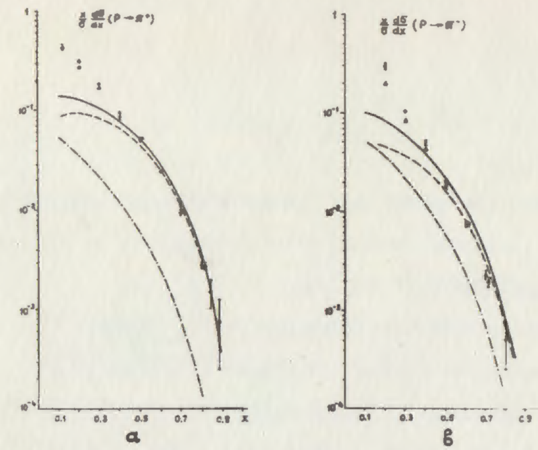
1. Dependence of functions $B_N(\omega; x)$ on x at various values of the parameter ω ; curve $1-\omega=1$, $2-\omega=2/3$, $3-\omega=1/3$, $4-\omega=0$.



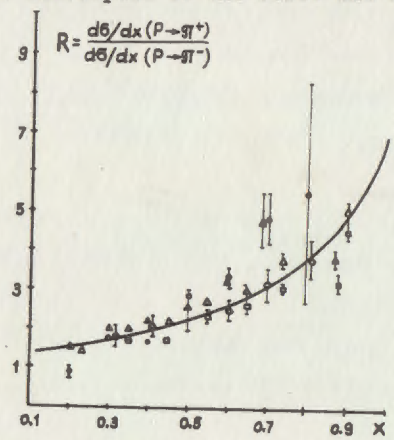
2. Parton distribution on a proton as dependent on the Feynman variable X. Curve 1 is the valence u-quark, 2 - valence d-quark (x2), 3 - sea u, d-quarks and antiquarks, 4 - s-quark and antiquark (x10), 5 - gluons ($\times 1/3$).



3. Distribution of U (curve 1) and D (curve 2) valons in a proton. Dash-dot line is for the distribution of U and D-valons in a proton from [6,7].



4. Inclusive spectra for fragmentation processes: a) $P \to \pi^+$ b) $P \to \pi^-$. Experimental data at 100 GeV (●) and 175 GeV (▲) are taken from [12]. Solid curve is for calculations by formulae (10a) and (10b). The dashed and dot-and-dashed curves correspond to the first and second summands in the expressions (10a) and (10b) (see the text).



5. Ratio of inclusive spectra of $\pi^{\pm}$-mesons in pp-interactions at (100-400) GeV. Experimental points are from [8, 12]. The curve is for calculations by formulae (10a) and (10b).

References

1. Van Hove L., Pokorski S. High-energy hadron-hadron collisions and internal hadron structure. - Nucl.Phys.,1975, vol.B86, p.243-253.
2. Van Hove L. The quark recombination mechanism in multihadron production. Preprint TH.2628-CERN, 1979.
3. Das K.P., Hwa R.C. Quark-antiquark recombination on the fragmentation region. - Phys.Lett., 1977, vol.68B, p.459-462.
4. Takasugi E. et al. Fast-meson production and the recombination model. - Phys.Rev., 1979, vol.D20, p.211-220.
5. Takasugi E., Tata X. Particle correlations in the recombination model with modified Kuti-Weisskopf structure functions. - Phys.Rev.,1982, vol.D26, p.120-142.
6. Hwa R.C. Clustering and hadronization of quarks: A treatment of the low-P_T problem. - Phys.Rev., 1980, vol.D22, p.1593-1608.
7. Hwa R.C., Zahir M.S. Parton and valon distributions in the nucleon. - Phys.Rev., 1981, vol.D23, p.2539-2553.
8. Buschbeck B., Dibon H., Gerhold H.R. An extended quark recombination model with valence quark annihilation compared to inclusive π^+/π^- ratios. - Z.Phys., 1980, vol.C7, p.73-80.
9. Anisovich V.V. et al. An experimental check of the quark-model relations for inclusive reactions. - Nucl.Phys.,1973, vol.B55, p.474-492.

10. Анисович В.В. и др. Описание спектров вторичных частиц в pp_-, $\bar{p}\bar{p}_-$, πp_- и Kp_- соударениях в рамках аддитивной модели кварков и правил кварковой статистики. ЯФ, I983, т.38, с.425-436.
11. Kuti T., Weisskopf V.F. Inelastic lepton-nucleon scattering and lepton pair production in the relativistic quark-parton model. - Phys.Rev., 1971, vol.D4, p.3418-3439.
12. Brenner A.E. et al. Experimental study of single particle inclusive hadron scattering and associated multiplicities. - Preprint FERMILAB - 81/82-EXP, 1981.
13. Gottgens R. et al. Proton fragmentation into pions and pion systems in $K\bar{p}$ interactions at 110 GeV/c and comparison to model predictions. - Z.Phys., 1984, vol.C92, p.205-218.
14. Fialkowski K., Kittel W. Parton models of low momentum transfer processes. - Rep.Prog.Phys., 1983, vol.46, p.1283-1392.

The manuscript was received 5 July 1984