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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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STOCHASTIC QUANTIZATION OF YANG-MILLS THEORY

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The problem of quantization of Yang-Mills theories in the stochastic quantization (SQ) scheme of Parisi-Wu /1/ discussed in Refs./2-5/. The results of works /2-4/ can be formulated as the following sequence of operations:

1. The Yang-Mills field $A^a_\mu(x,t)$ is determined as a functional of the external source $\mathcal{J}^a_\mu(x,t)$ by the equation

$$\partial_t A^a_\mu(x,t) + \frac{\delta S_{cl}}{\delta A^a_\mu(x,t)} + D^{ab}_\mu \mathcal{U}^b(A) = \mathcal{J}^a_\mu(x,t) \tag{1}$$

where x is a four-dimensional coordinate. t -additional time, S_{cl} - Euclidean classical Yang-Mills action, $\mathcal{U}$ -arbitrary functional of A^a_μ.

2. The stochastic average of a gauge invariant functional $F_{GI}(A)$ is determined by the formula

$$\langle F_{GI}(t)\rangle_{\mathcal{J}} \equiv \int F_{GI}(A^{\mathcal{J}}) \exp - \int \mathcal{J}^2 dt\, \mathcal{D}\mathcal{J} \tag{2}$$

3.The limit of eq.(2) at $t \to \infty$ is independent of the functional $\mathcal{U}^a$ and equal to usual quantum average $\langle F\rangle_q$ of the gauge invariant functional F with due regard for Faddeev-Popov ghosts:

$$\lim_{t\to\infty} \langle F_{GI}(t)\rangle_{\mathcal{J}} = \langle F_{GI}\rangle_Q \tag{3}$$

In eq.(1) the term $\partial_\mu \mathcal{U}$ actually fix the gauge, but the Faddeev-Popov construction is absent.

Our purpose is to liberate eq.(1) from $\partial_\mu \mathcal{U}$ term, i.e. we begin with naive Langevin equation

$$\partial_t A^a_\mu(x,t) + \frac{\delta S_{cl}}{\delta A^a_\mu(x,t)} = \mathcal{J}^a_\mu(x,t) \tag{4}$$

We will prove that instead of $A^{\mathcal{J}}$ in eq.(2) one can put the solution eq.(4) with zero initial data:

$$A^a_\mu(x,0) = 0 \tag{5}$$

and then eq.(3) holds for such a solution. This means that the singularities, which exist in the limit $t\to\infty$ for non-gauge invariant functionals if one starts from naive Langevin equation (4), cancel in gauge invariant functionals, and the Faddeev-Popov contribution arises automatically.

Note, that one can't put $\mathcal{U}=0$ in eq.(1) to obtain our result, because there are singularities in stochastic averages in the limit

A similar problem is discussed in Ref./5/, where it is proved by means of obvious calculation that eqs.(4), (5), (2), (3) give usual quantum average for $F^2_{\mu\nu}$ in g^2 approximation.

Note, that eq.(5) actually fixes the gauge under t-independent gauge transformations.

Turn now to the proof of our statement (eqs.(4), (5), (2), (3)). Latent supersymmetry, inherent in the SQ scheme, plays and important part in our proof. To make this supersymmetry evident we will pass to the superfield formalism of SQ /6-8/. Begin with cutting the additional time at T and put the following conditions on $\mathcal{J}$:

$$\mathcal{J}^a_\mu(x,T) = 0 \tag{6}$$

Then we will take the limit $T\to\infty$. The stochastic average of a gauge invariant functional defined by eqs. (4), (5), (2) one can express in superfields /7/:

$$\langle F_{GI}(T)\rangle_{\mathcal{J}} = \int F_{GI}(A_\mu(T))\exp -S(\Phi_\mu)\,\mathcal{D}\Phi_\mu \tag{7}$$

where

$$\Phi^a_\mu = A^a_\mu + \bar\theta\psi^a_\mu + \bar\psi^a_\mu\theta + \bar\theta\theta C^a_\mu$$

$$S = \int_0^T \mathcal{L}(\Phi)\,dt\,d\theta\,d\bar\theta \tag{8}$$

$$\mathcal{L}(\Phi) = \mathcal{L}_{cl}(\Phi) - \Phi\left(\frac{\partial^2}{\partial\theta\partial\bar\theta} + \theta\frac{\partial}{\partial\theta}\partial_t\right)\Phi$$

$\mathcal{L}_{cl}$ is the Euclidean Yang-Mills Lagrangian. Four dimensional integration is implicated in action in eq.(8). Later on, isotopic indices of fields are omitted as well for simplicity.

The following boundary conditions for functional integral (7) come out of eqs. (5), (6) and the reality of action in the exponent of eq.(7)

$$
\begin{aligned}
&A_\mu(0)=0 \\
&\Psi_\mu(0)=\Psi_\mu(T)=\bar{\Psi}_\mu(0)=\bar{\Psi}_\mu(T)=0 \\
&C_\mu(T)=0
\end{aligned}
\tag{9}
$$

Functionals F and S in integral (7) are invariant under the t , θ , $\bar{\theta}$ independent gauge transformations:

$$\Phi_\mu \to \Omega^{-1}\Phi_\mu\Omega + \Omega^{-1}\partial_\mu\Omega \tag{10}$$

It follows from (10) that we can put the following condi - tion on the first component of Φ_μ at fixed point over t (for example at $t=T$):

$$\partial_\mu A_\mu(T)=0$$

This is achieved by means of the Faddeev-Popov trick due to which the following expression for integral (7) is obtained:

$$\langle F_{GI}(T)\rangle_{\mathcal{Y}}=\int F_{GI}(A_\mu(T))\delta(\partial_\mu A_\mu(T))\Delta_{FP}(A_\mu(T))\exp-S(\Phi)\mathcal{D}\Phi_\mu \tag{11}$$

where Δ_{FP} is the Faddeev-Popov determinant.

The boundary condition for A_μ in eq. (11) now is

$$A_\mu(0)=\Omega^{-1}(x)\partial_\mu\Omega(x) \tag{12}$$

where Ω is arbitrary matrix of gauge transformation (actually there is an integration over $\Omega(x)$). The boundary conditions

for the other components of Φ_μ remain the same as in eq.(9).

Let us introduce a production functional for the calculation of averages of the type of (11):

$$
\begin{aligned}
&Z(h_\mu)=\int\delta(\partial_\mu A_\mu(T))\Delta_{FP}(A_\mu(T))\exp-\int_0^T dt\,d\theta\,d\bar{\theta}(\mathcal{L}(\Phi)+H_\mu\Phi_\mu)\mathcal{D}\Phi_\mu \\
&H_\mu=\bar{\theta}\theta\delta(t-T)h_\mu(x)
\end{aligned}
\tag{13}
$$

The boundary conditions of integral (13) coincide with those of (11), i.e.

$$
\begin{aligned}
&A_\mu(0)=\Omega^{-1}\partial_\mu\Omega \\
&\Psi_\mu(0)=\Psi_\mu(T)=\bar{\Psi}_\mu(0)=\bar{\Psi}_\mu(T)=0 \\
&C_\mu(T)=0
\end{aligned}
\tag{14}
$$

Let us introduce, just as in Refs. /9,10/, one-parameter production functionals Z^λ with the same boundary conditions (14):

$$Z^\lambda(h_\mu)=\int\delta(\partial_\mu A_\mu(T))\Delta_{FP}(A_\mu(T))\exp-\int_0^T dt\,d\theta\,d\bar{\theta}(\mathcal{L}^\lambda(\Phi)+H_\mu\Phi_\mu)\mathcal{D}\Phi_\mu$$

$$\mathcal{L}^\lambda(\Phi)=\{\lambda+(1-\lambda)\bar{\theta}\theta\delta(t-T)\}\mathcal{L}(\Phi) \tag{15}$$

At $\lambda=1$ we have

$$Z^1(h_\mu)=Z(h_\mu) \tag{16}$$

At $\lambda=0$ we have

$$Z^0(h_\mu)=\int\delta(\partial_\mu A_\mu(T))\Delta_{FP}(A_\mu(T))\exp-(\mathcal{L}_{cl}(A_\mu(T)+h_\mu A_\mu(T))\mathcal{D}A_\mu(T)$$

$$Z^0(h_\mu) = Z^{FP}(h_\mu) \tag{17}$$

i.e. Z^0 coincides with usual Faddeev-Popov production functional. We will prove that Z^λ in the limit $T\to\infty$ will be independent of λ , and hence, from eqs.(16), (17) the desired equation will follow:

$$\lim_{T\to\infty} Z(h_\mu) = Z^{FP}(h_\mu) \tag{18}$$

Let us calculate the following quantity:

$$\frac{\partial \ln Z^\lambda}{\partial\lambda} = -\int_0^T dt\,d\theta\,d\bar\theta\,(1-\bar\theta\theta\delta(t-T))\langle\mathcal{L}(\varphi)\rangle \tag{19}$$

The production functionals Z^λ are invariant under the supertransformation:

$$t\to t-\bar\theta\varepsilon$$
$$\theta\to\theta+\varepsilon \tag{20}$$

Owing to this, $\langle\mathcal{L}(\varphi)\rangle$ in eq.(19) depends on $t,\theta,\bar\theta$ only via the invariant τ :

$$\tau = t+\bar\theta\theta$$

Then eq. (19) reduces to

$$\frac{\partial \ln Z^\lambda}{\partial\lambda} = -\langle\mathcal{L}(\varphi)\rangle_{t=\theta=\bar\theta=0} \tag{21}$$

The source term in $\langle\mathcal{L}(\varphi)\rangle$ acts at time T and in the limit $T\to\infty$ the right-hand side of (21) becomes independent of h_μ , because the source acts at infinite time distances from the instant $t=0$. Thus in the limit $T\to\infty$ the right-hand side of eq. (21) is proportional to

$$\frac{1}{N}\int\mathcal{L}(\varphi(0))\delta(\partial_\mu A_\mu(T))\Delta_{FP}(A_\mu(T))\exp-S^\lambda\mathcal{D}\varphi_\mu\,\mathcal{D}\Omega \tag{22}$$

where

$$N=\int\delta(\partial_\mu A_\mu(T))\Delta_{FP}(A_\mu(T))\exp-S^\lambda\mathcal{D}\varphi_\mu\,\mathcal{D}\Omega$$

In eq.(22) we have written out the $\Omega(x)$ integration manifestly. Let us make the inverse to eq.(10) gauge transformation in eq.(22). The "gauge fixing" terms disappear and we return from boundary condition (14) to original ones (9). But on the boundary conditions (9)

$$\langle\mathcal{L}(\varphi)\rangle_{t=\theta=\bar\theta=0} = 0$$

so the right-hand side of eq.(21) in the limit $T\to\infty$ is zero and the desired eq. (18) is proved.

Thus the naive Langevin eq.(4) together with the initial data (5) give usual quantum averages for gauge invariant functionals.

Eqs.(4), (5), as distinct from eq.(1) are easy to extend to the non-abelian supergauge theories. It is sufficient to replace A_μ and the action in eqs.(4), (5), (2), (3) by the superfield and superaction, respectively.

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