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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

R.L.MKRTCHYAN

THERMODYNAMICS OF (SUPERSYMMETRIC) THEORIES AT LOW TEMPERATURES

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Introduction

The first part (Sec.1) of our work deals with the derivation of the new representation for the free energy (or effective potential) of general relativistic field theory at finite temperature. The derivation is based on the previously developed new diagram technique for the calculations at finite temperature [1]. The new representation for the free energy at finite temperature derived here is in some sense an expansion over non-negative powers of the distribution function $\beta(E_K)=1/(e^{\beta E_K}-1)$, $E_K=\sqrt{\vec{K}^2+m^2}$, $\beta=1/T$ with coefficients which are zero-temperature amputated causal Green functions. Since $\beta(E)\to 0$ at $T\to 0$, this expansion immediately generates a systematic low-temperature expansion of the free energy, this being absent in the usual Matsubara approach. The immediate interesting consequence of this expansion is the statement that the first temperature correction to the free energy is exactly calculable in arbitrary theories, while the second one requires summation of all the orders of the loop expansion.

In Sec.2 we discuss some implications of the results of Sec.1 on the behaviour of the supersymmetric theories with the weak breaking of supersymmetry. In particular, the case of the small non-zero temperature T as the supersymmetry breaking parameter is considered.

1. Free Energy at Low Temperatures

We firstly remind the reader the real-time diagram technique developed in our previous work [1]. At zero temperature the free energy, as is well known, is given by the sum of all connected vacuum diagrams (with coefficients $1/n$, n is the order of the diagram) plus $-\frac{1}{2}Sp\ell n D_0$, where D_0 is the free propagator. At finite temperature T each zero temperature diagram must be replaced [1] by the sum, each member of which is the initial diagram with some Feynman propagators (A) replaced by new ones (B):

$$(A) = \frac{i}{K^2 - m} \quad \text{by}$$

$$(B) = 2\pi\delta(K^2 - m^2)\, \beta(E_K)$$

for bosons, and

$$(A') = \frac{i}{\hat{K} - m} \quad \text{by}$$

$$(B') = -2\pi\,\delta(K^2 - m^2)(\hat{K} + m) f(E_K), \qquad f(z) = \frac{1}{e^{\beta z} + 1}$$

for fermions.

Also, in the course of generating the new diagrams, the following important rule must hold: only those new diagrams must be included, which remain connected after cutting of all their (B)-type propagators. The $Sp\ell n D_0$ at $T \neq 0$ is replaced by the $Sp\ell n D_T$, where D_T is free propagator at $T \neq 0$ (the calculation of $Sp\ell n D_T$ see, e.g. in [2]).

As we have already pointed out, $\beta(E_K) \to 0$ (and also $f(E_K) \to 0$) at $T \to 0$, hence, to get a low-temperature expansion, it is reasonable to group together the diagrams with equal number of the (B)-type propagators.

For simplicity, in what follows we consider the case of one self-interacting scalar field, the generalization for other theories being obvious.

Let's consider firstly the sum of all diagrams with one (B)-insertion. It may be written as

$$\int \frac{dp}{(2\pi)^4}\, 2\pi\delta(p^2 - m^2)\, \beta(E_p)\, G_2(p, -p) \tag{1}$$

G_2 is the sum of diagrams like those for amputated propagator. It is not difficult to realize that G_2 indeed is an amputated causal zero-temperature two-point function without its first term - inverse free propagator $\frac{1}{i}(p^2 - m^2)$. This is because the variation of the sum of vacuum diagrams over the parameters of free propagator gives the exact propagator. The formal proof in this and the following cases is based on the path-integral representation of the free energy $F(\mathcal{J})$ of the theory considered with external two-point source term: $\frac{1}{2}\int dx\,dy\, \varphi(x)\, \mathcal{J}(x,y)\, \varphi(y)$

$$e^{iF(\mathcal{J})} = \int \mathcal{D}\varphi \exp\left\{ iS(\varphi) + \frac{1}{2}\int \varphi \mathcal{J} \varphi \right\} \tag{2a}$$

$$\left.\frac{\delta F}{\delta \mathcal{J}(x,y)}\right|_{\mathcal{J}=0} = \langle \varphi(x)\, \varphi(y) \rangle_{\text{connected}} \tag{2b}$$

The procedure of the (B)-propagator insertion differs from the variation in two respects. First, in the former case, free propagators don't appear on lines, becoming external (if discarding (B)-propagator). Second, in the latter case, the first term of $F(\mathcal{J})$, $-\frac{1}{2}Sp\ell n D(\mathcal{J})$ ($D(\mathcal{J})$ is the propagator from the quadratic part of the action in (2a): $S(\varphi) + \frac{1}{2}\int \varphi \mathcal{J} \varphi$) is varied and gives at $\mathcal{J} = 0$ the free propagator $i/(p^2 - m^2)$ which is the first term in the perturbation theory expansion of the exact two-point function. This term, if amputated, becomes $\frac{1}{i}(p^2-m^2)$. So we conclude, that G_2 differs from the exact amputated causal two-point function only by the absence of $\frac{1}{i}(p^2-m^2)$. But since in (1) G_2 is in product with the on-shell δ -function $\delta(p^2-m^2)$, the term $\frac{1}{i}(p^2-m^2)$ is inessential and may be included in G_2 , the latter becoming an exact amputated two-point Green function.

In the same way one may understand, that the sum of all diagrams with two (B)-insertions may be represented as

$$\frac{1}{2!\,2^2}\int\frac{dp_1}{(2\pi)^4}\,\frac{dp_1}{(2\pi)^4}\,(2\pi)^2\delta(p_1^2-m^2)\delta(p_2^2-m^2)\,\mathcal{B}(E_{p_1})\,\mathcal{B}(E_{p_2})\cdot$$

$$\cdot\, G_4(p_1,-p_1,p_2,-p_2)$$

It is easy to understand that G_4 only by the combinatorial factor (and amputation of the external lines) may differ from the second variation of $F(\mathcal{J})$ at $\mathcal{J}=0$, which coincides with the causal four-point function. Actually, the factor $1/2!\,2^2$ is chosen so that G_4 is exactly the amputated four-point function (the explanation see below). In the general case the sum of all diagrams with K (B)-insertions may be represented as

$$\frac{1}{K!}\,\frac{1}{2^K}\int\frac{dp_1}{(2\pi)^4}\cdots\frac{dp_K}{(2\pi)^4}\prod_{i=1}^{K}2\pi\,\delta(p_i^2-m^2)\,\mathcal{B}(E_{p_i})\cdot \qquad (3)$$

$$\cdot\, G_{2K}(p_1,-p_1,p_2,-p_2\,\cdots\,p_K,-p_K)$$

where G_{2K} is amputated $2K$ -point function.

The factor $1/K!$ is due to the fact that at variation the same set of propagators may be chosen by $K!$ ways. Besides, due to the symmetry of

$\mathcal{J}(x,y)$, each variation gives factor 2. Combining (3) with the known result for $Sp\,\ell n\,D_T$ (D_T is the free propagator at $T\neq 0$) [2], we get finally the desired representation of the free energy at finite temperature T :

$$\frac{1}{V_4}F(T)=\frac{1}{V_4}F(0)+T\int\frac{d^3k}{(2\pi)^3}\,\ell n\left(1-e^{-\beta E_k}\right)+$$

$$+\sum_{K=1}^{\infty}\frac{1}{K!\,2^K}\int\frac{dp_1}{(2\pi)^4}\cdots\frac{dp_K}{(2\pi)^4}\prod_{i=1}^{K}2\pi\,\delta(p_i^2-m^2)\,\mathcal{B}(E_{p_i})\cdot \qquad (4)$$

$$\cdot\, G_{2K}(p_1,-p_1,p_2,-p_2,\cdots,p_K,-p_K)\cdot \qquad V_4 - 4\text{-volume}$$

Remembering that the effective potential for the scalar fields actually coincides with the free energy, taken with shifted masses (taken from the quadratic part of the action with the shifted fields: $S(\varphi+\varphi_0)$, φ_0 constant), one understands, that (4) gives also a representation of the effective potential for scalar field at $T\neq 0$.

The low-temperature expansion follows from (4) immediately, if one expands all $\mathcal{B}(E_p)$:

$$\mathcal{B}(E_p)=e^{-\beta E_p}\left(1+e^{-\beta E_p}-e^{-2\beta E_p}+\dots\right)$$

$$\ell n\left(1-e^{-\beta E_p}\right)=-e^{-\beta E_p}+e^{-2\beta E_p}-\frac{1}{2}e^{-3\beta E_p}+\dots$$

We shall prove now for the free energy that

$$\int dp\;\delta(p^2-m^2)\,\mathcal{B}(E_p)\,G_2(p,-p)=0 \qquad (5)$$

Hence the first temperature correction of order $\exp(-\beta E_p)$ comes entirely from the $Sp\,\ell n\,D_T$ (one-loop term) and is given by

$$\frac{1}{V_4} F(T) = \frac{1}{V_4} F(0) - T \int \frac{d^3 k}{(2\pi)^3} e^{-\beta E_k} + O(e^{-2\beta E_p}) \tag{6}$$

The statement claimed above is obvious: due to the normalization condition. $G_2 \longrightarrow (p^2 - m^2)$ at $p^2 \to m^2$, so the product $\delta(p^2 - m^2)\, G_2(p_1 - p)$ is equal to zero.

The generalization of (4) and (6) for the other theories is straightforward and will be used in the next Section.

2. Weakly Broken Supersymmetry

In general, we want to study the speed of supersymmetry restoration in N-extended supersymmetric theories with external (i.e. depending on external parameter) supersymmetry breaking, when this parameter tends to zero. To be more concrete, we study the density of the free energy in the N-extended supersymmetric theories. The relevant history explaining our interest to this situation is the following.

In 1975, Zumino [3] had pointed out that free energy (at $T \neq 0$) of supersymmetric theories with unbroken supersymmetry is zero. This follows directly from the characteristic relation of the supersymmetry algebra:

$$H = \{Q, \bar{Q}\}$$

H is the Hamiltonian, Q, $\bar{Q}$ are sypersymmetry generators. Indeed,

$$F = -T \ln Z = -T \sum_n e^{-\beta E_n} \xrightarrow[T \to 0]{} E_0$$

$$E_0 = \langle 0 | H | 0 \rangle = \langle 0 | \{Q, \bar{Q}\} | 0 \rangle = 0$$

Actually, exact consideration shows [3] that if we introduce a supersymmetry breaking by the X -dependent coupling constant (such that it

smmothly goes to zero outside the region with size of order R), then the density of the free energy goes to zero when $R \to \infty$ (and supersymmetry is restored) as $1/R^2$. More recently, in 1983, Bengtsson and Lindgren [4] considered the same quantity in the N-extended (N = 1, 2, 4) supersymmetric theories and found that the density of the free energy in those theories goes to zero as $1/R^{2N}$ at $R \to \infty$.

The aim of this Section is to call attention to the quoted results. I believe that they possess some generality. If so, this effect - the unexpectedly high speed of supersymmetry restoration in theories with initial high supersymmetry (N) - must be of great importance in the realistic supersymmetric GUTs with extended supersymmetry. In the latters this effect must manifest itself as abnormally small value of observed deviations from supersymmetry compared with the value of the initial supersymmetry-breaking parameters.

Unfortunately, at present, I have no precise formulation of the effect discussed. We hope to return to this subject in future studies. Now I only wish to show the necessity of a more precise formulation, namely I shall give a counterexample.

The latter follows from the consideration of the supersymmetry breaking by the introduction of small temperature T. The breaking of supersymmetry in this case is well-known [5-7], hence $F(T) \neq 0$ in such a theory. At $T \to 0$ the first (i.e. the main) temperature correction is given according to Sec.1, by (6), generalized to include fermions. The answer is (note that $\frac{1}{V_4} F(0) = 0$)

$$\frac{1}{V_4} F(T) = \sum_{bosons} (-T) \int \frac{d^3 k}{(2\pi)^3} e^{-\beta E_k} + \sum_{fermions} (-T) \int \frac{d^3 k}{(2\pi)^3} e^{-\beta E_k} + O(e^{-2\beta E_p})$$

There is not any fermi-bose cancellation, and $F(T)$ is of the same order $(e^{-\beta E_p})$ for all N .

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