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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ
ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ
ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

PHASE VELOCITY OF WAVE IN IRREGULAR WAVEGUIDE IN PROBLEMS OF SYNTHESIS

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ФАЗОВАЯ СКОРОСТЬ НЕРЕГУЛЯРНОГО ВОЛНОВОДА В ЗАДАЧАХ СИНТЕЗА

В работе рассматриваются вопросы, связанные с выбором фазовой скорости волны в нерегулярных волноводах в задачах синтеза, позволяющих по заданным выходным параметрам пучка заряженных частиц в волноводных ускорителях электронов и протонов или выходной мощности электромагнитного поля излучения в генераторах и усилителях с бегущей волной определять геометрические параметры волновода. Определены фазовые коэффициенты волн действующего электромагнитного поля, являющегося суммой поля, возбужденного пучком, и поля генератора. Полученные выражения для фазовой скорости волны в волноводе позволяют сделать оценки для предельных значений токов и мощностей излучения.

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PHASE VELOCITY OF WAVE IN IRREGULAR WAVEGUIDE IN PROBLEMS OF SYNTHESIS

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The problems related to the choice of phase velocity of waves in irregular waveguides in problems of synthesis are considered that allow determination of the waveguide geometric parameters by the given output parameters of the charged particle beam in waveguide accelerators of electrons and protons or the output power of the electromagnetic field of radiation in travelling wave generators and amplifiers. Phase coefficients of the waves of the effective electromagnetic field, which is the sum of the field excited by the beam and the generator field, are determined. The expressions obtained for the phase velocity of waves in a waveguide allow estimates for limiting values of currents and strengths of radiation.

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The study and calculation of linear waveguide accelerators or emitting devices of the type of traveling-wave tubes or backward-wave tubes are carried out either by the analysis [1] or synthesis method [2,3].

In problems of the analysis, when the waveguide with boundary conditions is considered given, the solution of the wave equation is performed for two regions - with and without charges. By matching particular solutions on the boundary of these regions, a dispersion equation is formed, which allows to determine the longitudinal phase coefficient of propagation [1]

In problems of the synthesis the longitudinal phase coefficient is found from the solution of the phase-energy equation [3], and by means of the transverse phase coefficient the necessary conditions are determined on the interaction space boundary, i.e. parameters of the slow wave waveguide are determined [2]. Methods of analysis and synthesis are in a good agreement with each other [4].

Until presently the effect of the current on phase coefficients of waves and, hence, on the choice of the phase velocity of waves in the waveguide was not taken into account in problems of synthesis.

In order to find the transverse phase coefficient of waves of the radiation field g_u, let us solve Maxwell's equation with currents and charges basing on the following assumptions:

1) the field and current distribution of the beam is symmetric with respect to the longitudinal axis of the cylindrical waveguide;

2) transverse velocities of charges are considerably smaller than longitudinal ones;

3) fields, charges and current vary in time by the cosine law, i.e. $\sim e^{j\omega t}$.

Taking into account the abovestated, we obtain Helmholtz's inhomogeneous equation for the longitudinal component of the wave radiation field E_{01}:

$$\nabla^2 E_{uz} + K^2 E_{uz} = j\omega\mu_0\delta_z + j\frac{1}{\omega\varepsilon_0}\frac{\partial^2\delta_z}{\partial z^2}, \quad (1)$$

where $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial z}\right) + \frac{\partial^2}{\partial z^2}, \quad K^2 = \frac{\omega^2}{c^2}$

ε_0, μ_0 - are the absolute dielectric constant and magnetic permeability, δ_z is the current density in the Z axis.

It should be noted that due to the smallness of transverse variations of the current and charge density, the functional form of E_r and H_θ components of the fields E_{01} in the presence of current vary insignificantly.

Let us present the current density in the waveguide as

$$\delta_z = \delta e^{b} = \delta e^{-j\int h dz}, \quad (2)$$

where δ is the amplitude of the first harmonic of the current density, $h = \omega/v$ is the phase coefficient of the current harmonic, v is the velocity of the charge bunch motion,

$\mathcal{B} = -j\int h\,dz$. The solution of (1) is sought in the form

$$E_{uz} = E_u \mathcal{J} e^{a}, \qquad (3)$$

where $\mathcal{J} = I_0(g_u r)$ is the modified Bessel function of zero order, $a = -j\int h_u\,dz$.

Substituting (2) and (3) in eq.(1), we obtain

$$g_u = (h^2\Delta - K)\left[\frac{h_u^2\nu - K^2}{h^2\Delta - K^2} - j\frac{\delta Z_o}{K E_u \mathcal{J}} e^{\mathcal{B}-a}\right]$$

$$Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}}\ , \quad \Delta = 1 + \frac{\ddot{\delta}}{\delta\dot{\mathcal{B}}^2} + 2\frac{\dot{\delta}\dot{\mathcal{B}}}{\delta\dot{\mathcal{B}}^2} + \frac{\ddot{\mathcal{B}}}{\dot{\mathcal{B}}^2}\ , \qquad (4)$$

$$\nu = 1 - \frac{1}{h_u^2}\left(\frac{\ddot{E}_u}{E_u} + 2\frac{\dot{E}_u\dot{\mathcal{J}}}{E_u\mathcal{J}} + \frac{\ddot{\mathcal{J}}}{\mathcal{J}} + 2\dot{a}\frac{\dot{E}_u}{E_u} + 2\dot{a}\frac{\dot{\mathcal{J}}}{\mathcal{J}} + \ddot{a}\right),$$

where points denote derivatives by the Z coordinate. Equation (4) allows estimate of the transverse phase coefficient as a function of variation of the phase velocity of the wave and charge current. This equation is similar in form to those for transverse wave numbers obtained by many investigators (see e.g. [5]).

The relations $|\nu| \simeq 1$ and $|\Delta| \simeq 1$ are the condition of smallness of admissible longitudinal variations of E, V_ϕ and δ . These conditions are better observed with the increase in the kinetic energy of charges, i.e. at $V \simeq C$.

For simplicity let us write down the expression (4) as

$$\mathcal{B} - a = \int (h_u - h)\,dz = \omega\int\left(\frac{1}{V_u} - \frac{1}{V}\right)dz = -\varphi_u\ , \qquad (5)$$

where φ_u is the phase of the radiation field, where the charge center is located.

Let us write down also [3]

$$h^2 - \kappa^2 = \kappa^2/(u^2+2u),\ z_0/\kappa = 60\lambda,\ \delta = \frac{I_1}{\pi r_0^2} = \frac{2I\sin\Psi}{\pi r_0^2 \Psi}, \quad (6)$$

where $u = \frac{U}{U_0}$ is the relative kinetic energy of charges, U_0 is the electron rest energy, λ is the field wavelength in free space, Ψ is the phase length of the bunch, I_1 is the amplitude of the first harmonic of current, r_0 is the bunch radius. Taking account of (5)-(6) and assuming $I_0(g_u r) \simeq 1$, which is valid provided $g_u r < 1$ that is fulfilled in practice, the expression (4) takes the form

$$g_u^2 = g^2\left[1 + \frac{120}{\pi\lambda}\left(\frac{\lambda}{r_0}\right)^2 \frac{\sin\Psi}{\Psi}\ \frac{I\sin\varphi_u}{E_u}\right]. \quad (7)$$

Using the last formula let us determine the transverse phase coefficient g_∂ of the effective field. The effective field is a sum of the generator field and the field excited by the beam itself [2]:

$$E I_0(g_\partial r)\sin\varphi_p = E_c I_0(g r)\sin\varphi_c + E_u I_0(g r)\sin\varphi_u \,. \quad (8)$$

Provided $g_\partial r < 1,\ g_u r < 1,\ g r < 1$, let us write down (8) in the form

$$\left[E + E\left(\frac{g_\partial r}{2}\right)^2\right]\sin\varphi_p = \left[E_c + E_c\left(\frac{g r}{2}\right)^2\right]\sin\varphi_c + \left[E_u + E_u\left(\frac{g_u r}{2}\right)^2\right]\sin\varphi_u \quad (9)$$

Since g_∂ varies only due to g_u, then

$$E\sin\varphi_p = E_c\sin\varphi_c + E_u\sin\varphi_u \,. \quad (10)$$

Substituting (7) in (9) and taking into account (10) we obtain

$$g_\partial = g\sqrt{1 + \frac{120}{\pi\lambda}\left(\frac{\lambda}{r_0}\right)^2 \frac{\sin\Psi}{\Psi}\ \frac{I\sin^2\varphi_u}{E\sin\varphi_p}} \quad (11)$$

At $I \to 0$ the formulae (7) and (11) are transformed into obvious equalities:

$$g_\partial = g_u = g$$

The variations of the transverse phase coefficient g_{∂} is most substantial in processes of pure radiation, whereas at acceleration, when the generator field is large and the current is small, g_{∂} is practically equal to g .

The phase coefficient of the wave of the generator field is determined by the expression [2]

$$h = \frac{\omega}{v} - \frac{d\varphi}{dz} = \frac{\kappa}{\beta} - \frac{d\varphi}{dz} \quad , \qquad (12)$$

where $\beta = \frac{v}{c}$ is the relative velocity of charges, and the variation of the particle phase is determined from the phase-energy equation presented in [3,6] and having the form

$$\frac{d\varphi}{dz} = \frac{d\varphi_p}{dz} + \frac{1}{2}\frac{\sin\Psi}{\Psi}\frac{IE\cos\varphi_p}{P_c \pm IU} \quad , \qquad (13)$$

where the sign (+) concerns the radiation mode, (-) - the acceleration, and P_c is the generator power. Hence, for the transverse phase coefficient we obtain from (12)-(13)

$$g^2 = h^2 - \kappa^2 = \left(\frac{\kappa}{\beta} - \frac{d\varphi_p}{dz} - \frac{1}{2}\frac{\sin\Psi}{\Psi}\frac{IE\cos\varphi_p}{P_c \pm IU}\right)^2 - \kappa^2 \qquad (14)$$

Taking account of (14) the expression for the phase coefficient of the effective field (11) takes the form

$$g_{\partial} = \left[\left(\frac{\kappa}{\beta} - \frac{d\varphi_p}{dz} - \frac{1}{2}\frac{\sin\Psi}{\Psi}\frac{IE\cos\varphi_p}{P_c \pm IU}\right)^2 - \kappa^2\right]\left[1 + \frac{120}{\pi\lambda}\left(\frac{\lambda}{r_0}\right)\frac{2\sin\Psi}{\Psi}\frac{I\sin^2\varphi_u}{E\sin\varphi_p}\right] \qquad (15)$$

As is seen from (15), the influence of the current on the phase coefficient of the effective field is of a double nature. The first square bracket describes the influence of the current proceding from the energetics of the interaction process, and the second bracket reflects the variation of space properties in the waveguide due to the presence of electric charges in it.

Note that at Ψ close to π, the influence of the current on $g_{\text{э}}$ abruptly decreases. For simplicity we will later on assume that $\Psi < 1$ and $\sin\Psi/\Psi < 1$.

For the radiation process in the synchronous mode, when

$$\varphi_u = -\pi/2\,, \quad E\sin\varphi_p = E_u \sin\varphi_u\,, \quad \sin^2\varphi_u / E\sin\varphi_p = -1/E_u$$

the formula (15) takes the form

$$g_{\text{э}}^2 = \left[\left(\frac{\kappa}{\beta}\right)^2 - \kappa^2\right]\left[1 - \frac{120}{\pi\lambda}\left(\frac{\lambda}{r_0}\right)^2 \frac{I}{E_u}\right] = \frac{\kappa^2}{u^2 + 2u}\left[1 - \frac{120}{\pi\lambda}\left(\frac{\lambda}{r_0}\right)^2 \frac{I}{E_u}\right]. \qquad (16)$$

For the charge acceleration mode the formula (15) is written in the form

$$g_{\text{э}}^2 = \left[\left(\frac{\kappa}{\beta} - \frac{d\varphi_p}{dz} - \frac{1}{2}\,\frac{I E \cos\varphi_p}{P_c - IU}\right)^2 - \kappa^2\right]\left[1 + \frac{120}{\pi\lambda}\left(\frac{\lambda}{r_0}\right)^2 \frac{I\sin^2\varphi_u}{E\sin\varphi_p}\right] \qquad (17)$$

The formulae (15)-(17) define the connection of the given dynamics of the charge bunch - the longitudinal phase coefficient of wave of the effective field $h_{\text{э}} = \frac{\omega}{V_{\Phi\text{э}}} = \frac{\omega}{V} - \frac{d\varphi_p}{dz}$, with the convection current, amplitude and the equilibrium phase of the effective field.

Let us define the connection between the phase velocity of the effective field wave $V_{\Phi\text{э}}$ and that of the wave in the waveguide without current V_Φ. Since $g_{\text{э}}^2 = h_{\text{э}}^2 - \kappa^2$, then taking into account (15) one may easily obtain

$$h = h_{\text{э}} \sqrt{(1 + \mu\beta_{\Phi\text{э}}^2)/(1 + \mu)} \qquad (18)$$

where $\mu = (120/\pi\lambda)(\lambda/r_0)^2 (I\sin^2\varphi_u)/(E\sin\varphi_p)$

The phase velocity of the wave in the waveguide without current required to provide the given phase velocity of the effective field wave, is

$$V_\Phi = \frac{V}{(1 - \dot{\varphi}_p \beta/\kappa)} \sqrt{\frac{1+\mu}{1 + \mu\beta_{\Phi\text{э}}^2}}\,, \qquad (19)$$

where $\beta_{\varphi э} = \beta / (1 - \dot{\varphi}_p \beta / \kappa)$

At $I = 0$ we come to the following formula [2]

$$V_\varphi = V / (1 - \dot{\varphi}_p \beta / \kappa) . \tag{20}$$

For pure radiation at $E_c = 0$, $\varphi_u = \varphi_p = -\pi/2$,

$$V_\varphi = V \sqrt{(1+\mu)/(1+\mu\beta^2)} , \tag{21}$$

where $\mu = -\frac{120}{\pi\lambda}\left(\frac{\lambda}{r_0}\right)^2 \frac{I}{E_u}$.

For the charge acceleration mode the formula (19) stays unchanged. The relations (19),(21) are used for the calculation of phase coefficients: $h = \omega / V_\varphi$, $g^2 = (\omega/V_\varphi)^2 - (\omega/c)^2$, and the transverse phase coefficient g is introduced in the characteristic equation to calculate geometric parameters of the slow wave waveguide [2].

In problems of the analysis [1] one should know the phase velocity of the effective field, which may be easily found from the relation (19)

$$V_{\varphi э} = V_\varphi / \sqrt{1 + \mu - \mu\beta_\varphi^2} . \tag{22}$$

Let us estimate the influence of the charge current on the phase velocity in the waveguide. For instance, at $I =$ [illegible] $r_0 = 1$ cm, $\lambda = 10$ cm, $E \sin\varphi_p = 10$ kV/cm, $V_\varphi = 0.9\,c$ the difference of $V_{\varphi э}$ from V_φ in (22) is $\sim 7\%$.

In radiation modes, as follows from (22), $|\mu| \lesssim 1$ where $\mu = -(120/\pi\lambda)(\lambda/r_0)^2(I/E_u)$. From this condition one may obtain helpful estimates for limiting values of radiation currents and powers. Assuming $\lambda/r_0 \approx 10$ in the expression for μ , let us write $I/(\lambda E_u) \lesssim 10^{-3}$. On the other hand, the specific radiation power is [2]

$$dP_u/dz = I E_u ,$$

whence we obtain $dP_u/dz = 10^{-3}\lambda E_u^2 = 10^{-3} I^2/\lambda$. For instance, for the radiator for λ = 10 cm and at E_u = 10 kV/cm, the limiting value of the current is 100 A, and dP_u/dz = 100 MV/M. With the increase in the wavelength the radiation power and admissible current grow.

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