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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ
ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ
ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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NUCLEAR-OPTICAL METHOD FOR PRODUCTION OF POLARIZED PHOTONS WITH ENERGIES OF A FEW HUNDREDS GEV

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ЯДЕРНООПТИЧЕСКИЕ МЕТОДЫ ПОЛУЧЕНИЯ ПОЛЯРИЗОВАННЫХ ФОТОНОВ С ЭНЕРГИЯМИ БОЛЬШЕ НЕСКОЛЬКИХ СОТЕН ГэВ

Вычислены коэффициенты поглощения линейно поляризованных фотонов, пролетающих параллельно кристаллическим плоскостям. Описаны способы определения линейной поляризации и потерь пучка в случае, когда поляризованный пучок проходит через различные кристаллы параллельно плоскостям (110). Получена энергетическая зависимость толщин кристаллов-пластин четверть длины волны, необходимых для превращения линейной поляризации в циркулярную.

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NUCLEAR-OPTICAL METHODS FOR PRODUCTION OF POLARIZED PHOTONS WITH ENERGIES OF A FEW HUNDREDS GEV

The absorption coefficients of linearly polarized photons passing through a crystal in parallel to its crystallographic planes are calculated. The methods of determination of the obtainable degree of polarization as well as of the intensity losses for the cases when non-polarized photon beams pass through various crystals in parallel to the planes (110) are described. The energy dependence of the thickness of the quarter-wave plate crystals transforming the linear polarization of the beam into circular one obtained.

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It has been shown by N.Cabibbo et al. [1-3] that when a linearly polarized high energy photon beam passes through a crystal under small angles to the crystallographic planes or axes, the cross section of the coherent $e^+ e^-$-pair production depends on the direction of linear polarization, and the crystal possesses the properties of birefringence (see also [4]). By analogy with optics the authors of [1-3] have suggested to use the crystal targets as "nuclear-optical devices" for production and analysis of polarized photon beams with energies of a few tens GeV. V.Baryshevskii has pointed out [5,6] that one may obtain similar results at smaller angles (channeling angles) for photons of higher energies, which was confirmed by the calculations of the works [7,8].

In connection with the possibilities to observe experimentally the theoretically investigated process of $e^+ e^-$-pair production by photons in external intense fields [9-11], to be more exact, in the fields of crystallographic planes and axes [12-18], in this work more transparent calculations of the effects considered in [5-8] are carried out. The obtained more complete numerical results for various single crystals are of interest for the preparation of experiments for production of polarized photon beams with energies of a few hundreds GeV.

According to the theory (see, e.g. [10]) for photons, passing perpendicularly to the external homogeneous field $\vec{E}$ with direction of polarization parallel ($\parallel$) or perpendicular ($\perp$) to $\vec{E}$, the absorption coefficients due to the e^+e^--pair production are given by the expressions:

$$W^{\parallel} = \frac{1}{8}\sqrt{\frac{3}{2}}\,\frac{\alpha m^2 \chi}{\omega}\exp\left(\frac{-8\chi}{3}\right), \qquad W^{\perp} = 2W^{\parallel}. \tag{1}$$

Here, $\hbar = c = 1$, $\alpha = e \approx 1/137$, m and e are the mass and electric charge of electrons, ω is the primary photon energy. The expressions (1) are valid for the values of the parameter $\chi = E\omega/E_o m \ll 1$, where $E_o = m^2/e = 1.32 \ 10^{16}$.v/cm.

Using the paraboilic dependence of the crystallographic plane potential, U , upon the distance, y , from the plane

$$U(y) = U_o\left[1 - \frac{y^2}{(d_p/2)^2}\right], \tag{2}$$

where d_p is the distance between the neighbouring crystallographic planes, and carrying out y-averaging of (1), one obtains the absorption coefficients $W^{\parallel}_{p\ell}$ and $W^{\perp}_{p\ell}$ for the photon beams passing under angles $\theta = 0$ to the crystallographic planes (The axial case is not of interest for the below considered effects, though the absorption coefficients will be greater due to the presence of stronger fields):

$$W^{\parallel}_{p\ell} = A\left[\left(1 - \frac{B}{\omega}\right)\exp\left(-\frac{B}{\omega}\right) - \frac{B^2}{\omega^2}\,Ei\left(-\frac{B}{\omega}\right)\right]; \qquad W^{\perp}_{p\ell} = 2W^{\parallel}_{p\ell}, \tag{3}$$

where

$$A = \frac{\sqrt{3}}{4\sqrt{2}} \frac{\alpha U_0}{m d_p} ; \quad B = \frac{2}{3} \frac{m^3 d_p}{U_0} ,$$

and $Ei(z)$ is the integral exponent. Let us note that in the case of $W_{pe}^{\parallel}$ ($W_{pe}^{\perp}$) the photon polarization vector is perpendicular (parallel) to the crystallographic planes.

The values of U_0 , d_p . A and B for the (110) planes of various crystals are given in the Table. The dependence of $W_{pe}^{\parallel}$ upon ω calculated according to (3) is shown in Fig.1 by solid curves. As it is seen, with the increase of the photon energy, $W_{pe}^{\parallel}$, consequently also the absorption coefficients for non-polarized photons, $W_{pe} = (W_{pe}^{\parallel} + W_{pe}^{\perp}) / 2 = 3 W_{pe}^{\parallel} / 2$ grow in the beginning very sharply; then at $\omega \approx 200 \div 800$ GeV they become of the order of the corresponding absorption coefficients W_{BH} , calculated by the Bethe-Heitler formulae of pair production $W_{BH} = n r_0^2 Z^2 \alpha \left(\frac{28}{9} \ln 183 Z^{-1/3} - \frac{2}{27} \right)$ and shown in Fig.1 by short lines; and finally, $W_{pe}^{\parallel} \to A$ when $\omega \to \infty$ In the Table we give the values of photon energies ω_1 and ω_3 at which $W_{pe} = W_{BH}$ and $W_{pe} = 3 W_{BH}$, respectively.

Let us consider the processes which take place when a non-polarized high energy photon beam passes in parallel to the crystallographic planes. At relatively small energies the photons will be absorbed due to the well known Bethe-Heitler pair production mechanism which is independent of photon polarization. With increasing photon energy (see Fig.1), when the above-discussed mechanism of pair creation becomes essential and then dominant, the non-polarized beam with initial intensity at the entrance into the crystal $I(0)$ acquires some polarization after passing a crystal thickness x , since due to the difference between $W_{pe}^{\parallel}$ and $W_{pe}^{\perp}$ the beam components with parallel and perpendicular polarization will undergo various absorptions, and $I^{\parallel}(x) \neq I^{\perp}(x)$. Instead of x it is convenient to use dimensionless thickness $y = x/x_e$, i.e. to measure the thickness in units of $x_e = 1/W_{pe}^{\parallel}$. x_e is such a crystal thickness after which the intensity of the photons with the given energy and parallel polarization decreases e -times. With increasing ω the growth of x_e slows down, as it is seen from the dashed curves in Fig.1.

Following the works [1-3] let us determine:

a) The parameter characterizing the difference between the absorption coefficients:

$$R = \frac{W^{\perp} - W^{\parallel}}{W^{\perp} + W^{\parallel}} = \frac{W_{pe}^{\parallel}}{W^{\perp} + W^{\parallel}} \quad . \qquad (4)$$

For $\omega \gtrsim \omega_3$, $R \approx 1/3$ and for $\omega \lesssim \omega_1$ one may assume $W^{\parallel,\perp} \approx W_{BH} + W_{pe}^{\parallel,\perp}$ and, therefore, $R \approx W_{pe}^{\parallel}/(2W_{BH} + 3W_{pe}^{\parallel})$

b) The beam polarization at the depth x or $y = x/x_e$.

$$P(x) = \frac{I^{\parallel}(x) - I^{\perp}(x)}{I^{\parallel}(x) + I^{\perp}(x)} = th\left(\frac{x W_{pe}^{\parallel}}{2}\right) = th\left(\frac{y}{2}\right) . \qquad (5)$$

At the depth $y = 1$, when the intensity of the photon beam component with parallel polarization decreases e times, the degree of the linear polarization of the initially non-polarized beam becomes equal to $P(y=1) = th\ (0.5) = 0.462$.

c) The decrease of the beam intensity at the depth x or y :

$$K = \frac{I(x)}{I(0)} = \exp\left(-R^{-1} th^{-1} P\right)\left(1 - P^2\right) = \frac{(1-P)^{(1-R)/2R}}{(1+P)^{(1+R)/2R}} = \exp\left(-\frac{y}{2R}\right) ch\left(\frac{y}{2}\right) . \qquad (6)$$

The energy dependence of R and K (the last one for $y = 1$) for two single crystals W and Si are shown in Fig.2, while Fig.3 presents the dependence of both P and K (for $R = 1/3$) on the dimensionless thickness y, the last two dependences being universal for all single crystals.

At relatively low energies $\omega \lesssim 100 \div 200$ GeV, when still $W''_{pe} < W_{BH}$, the non-polarized beam will be absorbed mainly due to the Bethe-Heitler process, and though one may achieve high degrees of polarization, nevertheless the beam intensity losses will be significant. Therefore the application of such a method for production of polarized photons at such energies seems to be irrational. The application of the method is reasonable at photon energies when already $W''_{pe} \gtrsim W_{BH}$. For this purpose, first for the given energy and available single crystal one determines X_e using the dependence of X_e upon ω (Fig.1). Then, with the help of Fig.3 one determines y, i.e. the crystal thickness $X = yX_e$, for the required degree of polarization. If the photon energy is sufficiently high ($R \gtrsim 1/3$), it is possible to determine easily the beam losses from the curve $K(y)$, otherwise it is necessary to determine R and then the losses by the formula (6).

Just in the same way that is described in the work [1] the single crystal may be used as polarimeter measuring the linear polarization of a given beam. Without discussing this possibility let us consider the method for obtaining circularly polarized photons when a linearly polarized photon beam passes through a crystal operating as a quarter-wave plate.

By analogy with the optics [2] the single crystal will work as a plate converting the linear polarization into circular one (or vice versa) if its thickness is equal to:

$$\chi_{1/4} = \frac{\pi}{2\omega \,\mathrm{Re}\left[n^{\perp}(\omega) - n^{\parallel}(\omega)\right]}, \qquad (7)$$

where the real part of the differences between the refraction indices $n^{\perp,\parallel}$ for photon with $\perp$ and $\parallel$ polarization is equal to:

$$\mathrm{Re}\left[n^{\perp}(\omega)-n^{\parallel}(\omega)\right]=\frac{1}{\pi}P\int_0^{\infty}\frac{W^{\perp}(\omega')-W^{\parallel}(\omega')}{\omega'^2-\omega^2}d\omega'=\frac{1}{\pi}P\int_0^{\infty}\frac{W^{\parallel}_{p\ell}(\omega')}{\omega'^2-\omega^2}d\omega'. \qquad (8)$$

Using the expression (3) one can calculate the Cauchy integral (8) and obtain:

$$\mathrm{Re}\left[n^{\perp}(\omega)-n^{\parallel}(\omega)\right]=\frac{A}{\pi a}\left\{\frac{1-a}{2}\exp(-a)\,Ei(a)-\frac{1+a}{2}\exp(a)\,Ei(-a)-\right.$$
$$-a^2\left[\frac{1}{a}+\frac{1}{2}\left(Ei(a)\,Ei(-a)-\ell n^2 a-2\check{C}\ell n a\right)-\frac{1}{2}\left(\check{C}^2+\frac{\pi^2}{2}\right)+\right. \qquad (9)$$
$$\left.\left.+\sum_{k=1}^{\infty}\frac{2k^{-1}+\Psi(k)-\ell n a}{k!\,k}\,a^k\right]\right\}$$

where $a=B/\omega$, $\check{C}=0.5772$ is the Euler's constant, $\Psi(k)=\Gamma'(k)/\Gamma(k)$ is the digamma function. The results of the numerical calculations of the expressions (8) and (9) with the help of a computer presented in Fig.4 show that $\mathrm{Re}\left[n^{\perp}(\omega)-n^{\parallel}(\omega)\right]$ is a smooth function of ω. The dependence of the thickness of various quarter-wave plate crystals upon ω, calculated by (7) is shown in Fig.5.

In conclusion let us discuss the validity of the calculations and some of the obtained results. It has been already mentioned that formula (1) is valid for $\chi \ll 1$, while for greater values of ω or E the values of the parameter χ may become $\chi \gg 1$. The values of the photon energies ω_e, which at maximal crystallic field values E^{max} gives $\chi = 1$ for various single crystals, are given in the Table. Taking into account the facts that E sharply decreases in the neighbourhood of the crystallographic planes where $E \approx E^{max}$ and that formula (1) provides an accuracy not worse than 15% even for $\chi \approx 2$ (see [17]), one may show that the application of the formula (1) provides sufficient accuracy up to photon energies $\omega \approx 2000$ GeV

As to the results obtained, let us first note that the numerical results on the absorption coefficients calculated by the simple formula (3) are in good agreement with the numerical data of the work [16]. The crystal thicknesses $X_e(1000)$ and $X_{1/4}(1000)$ necessary for production of a linearly polarized photon beam with $P \approx 0.5$ using a non-polarized one and for converting a linearly polarized photon beam into a circularly polarized one at $\omega = 1000$ GeV, respectively, are given in the Table. Except diamond, crystals of such thickness are available. However, as it is seen from the curves $X_e(\omega)$ and $X_{1/4}(\omega)$ (Figs. 3 and 5), at lower energies the required crystal thicknesses become non-realistic for $\omega \lesssim 400$ GeV (perhaps tungsten crystals may be useful for $\omega \gtrsim 200$ GeV). These circumstances along with the large intensity losses ([illegible]) at $\omega \lesssim 200$ GeV prove that it will be unreasonable to use the methods under discussion for $\omega \lesssim 200$ GeV.

It is of interest to compare the results obtained in this work with those obtained by other methods [1-3] or similar calculations [7,8]. As it is already pointed out in [7,8], the required crystal thicknesses X_e and $X_{1/4}$ for reasonable energies are less than the thicknesses necessary for the method of coherent pair production [1-3] at lower energies. This fact can be explained by the higher values of R : the maximal value in the case of coherent pair production is $R_{coh}^{max} \approx 0.16$ for optimal conditions of Cu single crystal at $\omega = 40$ GeV, while in the case of the [7,8] and this paper $R^{max} = 1/3$. Comparing the results of the present work with those of [7,8] it is necessary to point out that similar dependences are obtained by various approaches. However there are some disagreements (especially at relatively lower ω when the method is ineffective) between the numerical results. For instance, in the case of tungsten crystals $X_e \approx 10$, 1 and 0.15 cm for $\omega = 100$, 200 and 1000 GeV and $X_{1/4} \approx 20$ and 2 cm for $\omega = 17$ and 170 GeV according to [7,8] . while according to this work $X_e \approx 80$, 2 and 0.085 cm for $\omega = 100$, 200 and 1000 GeV and $X_{1/4} \approx 0.82$ cm for $\omega = 170$ GeV.

Thus, while the nuclear-optical methods suggested in [1-3] have been used scarcely [19-21] , we hope that the method considered in this work will find wide application for production of polarized photon beams at $\omega \gtrsim 200$ GeV.

Table

Some parameters and results for (110) planes of

various single crystals.

Single crystal	U_o (eV)	d_p (Å)	A (cm^{-1})	B (GeV)	ω_1 (GeV)	ω_3 (GeV)	ω_ℓ (GeV)	X_e (cm)	$X_{1/4}$ (cm)
C	26	1.26	9.25	2180	510	660	820	2.0	1.8
Si	30	1.92	6.85	2880	770	1010	1100	7.0	3.1
Cu	46	1.28	15.7	1252	440	630	520	0.4	0.7
Ge	54	2.0	11.8	1666	560	800	625	0.9	1.0
W	160	2.24	31.2	630	290	480	230	0.08	0.27

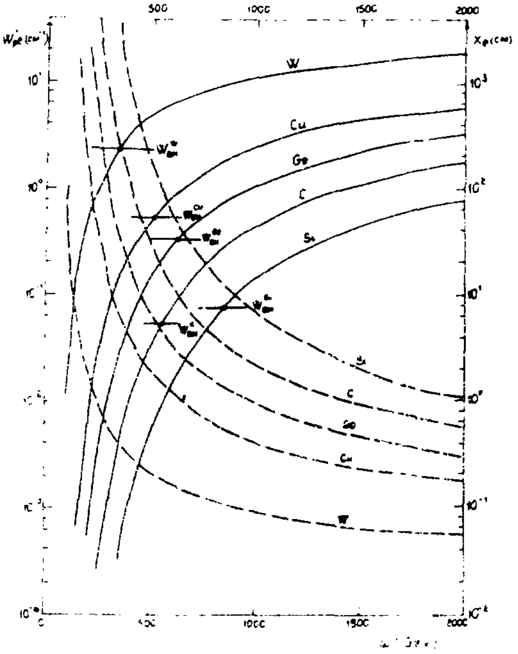
500
1000
1500
2000
W'_{pe} (cm⁻¹)
X_e (cm)
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
W
Cu
Ge
C
Si
W^W_{ex}
W^{Cu}_{ex}
W^{Ge}_{ex}
W^{Si}_{ex}
W^C_{ex}
Si
C
Ge
Cu
W
0
500
1000
1500
2000

Fig.1

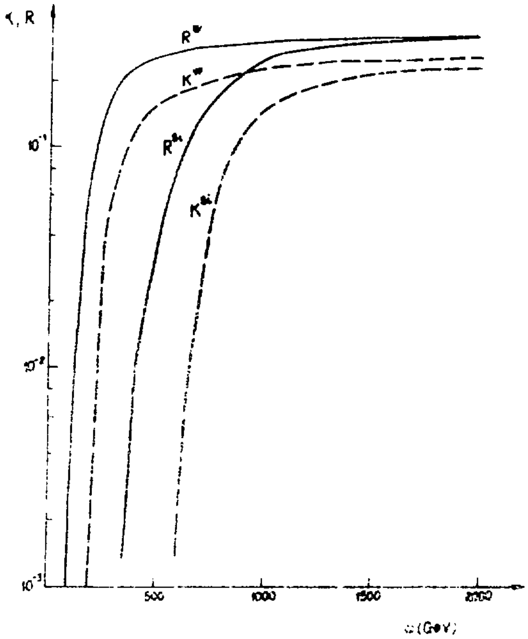

Fig. 2

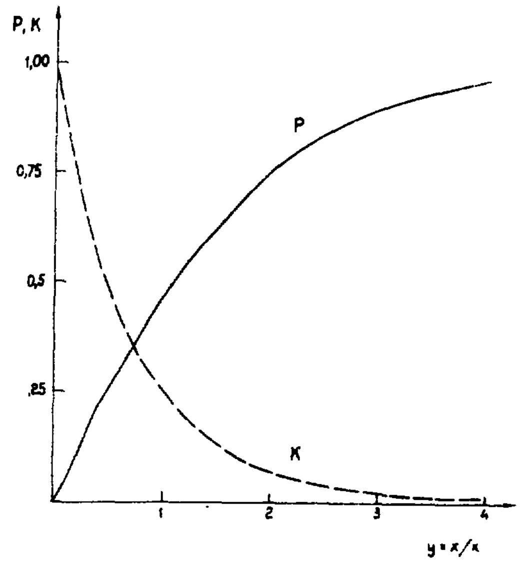

Fig.3

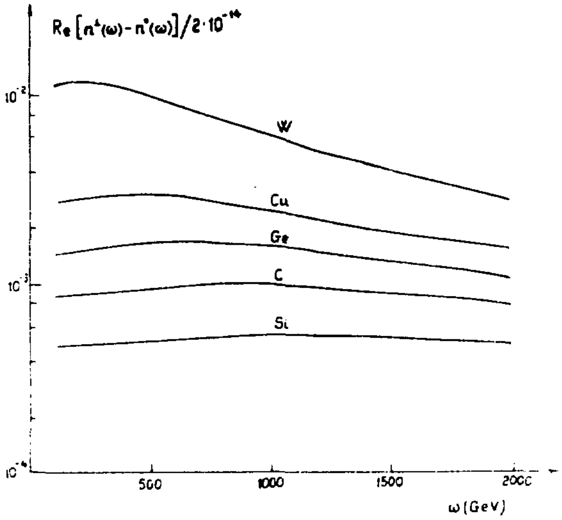
$Re\left[n^{\perp}(\omega)-n^{\parallel}(\omega)\right]/2\cdot 10^{-14}$
10^{-2}
10^{-3}
10^{-4}
W
Cu
Ge
C
Si
500
1000
1500
2000
ω (GeV)

Fig.4

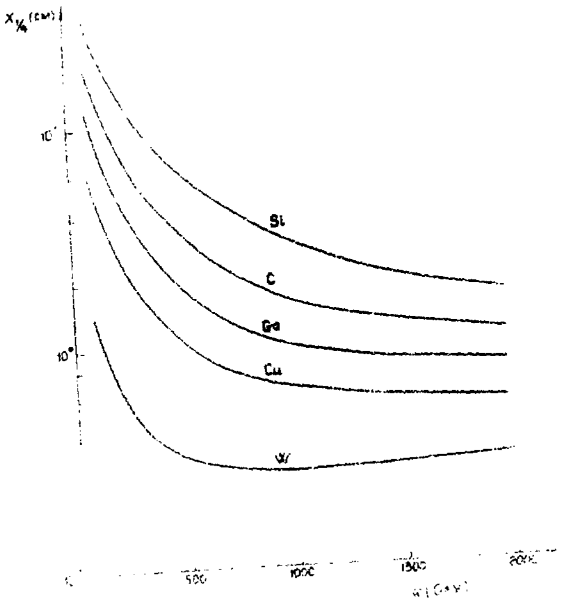

Fig.5

Figure Captions

Fig.1. The dependence of the photon absorption coefficients $W^{\parallel}_{pe}$ (cm^{-1}, solid curves) and crystal thicknesses X_e (cm, dashed curves) on the photon energy ω (GeV) for the (110) planes of the single crystals of diamond (C), silicon (Si), germanium (Ge), copper (Cu) and tungsten (W). The absorption coefficients W_{BH} (cm^{-1}) corresponding to the Bethe-Heitler cross sections for pair production in case of complete screening are shown by short solid lines.

Fig.2. The dependence of R and K (at $X = X_e$) on ω (GeV) for W and Si.

Fig 3. The dependence of P and K (at $R = 1/2$) on $y = X/X_e$.

Fig.4. The dependence of $Re\left[n^{\perp}(\omega) - n^{\parallel}(\omega)\right] /2\cdot 10^{-14}$ on (GeV) for various single crystals.

Fig.5. The dependence of $X_{1/4}$ (cm) on ω (GeV) for various single crystals.

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ЯДЕРНООПТИЧЕСКИЕ МЕТОДЫ ПОЛУЧЕНИЯ ПОЛЯРИЗОВАННЫХ ФОТОНОВ С ЭНЕРГИЯМИ БОЛЬШЕ НЕСКОЛЬКИХ СОТЕН ГЭВ

(на английском языке, перевод З.Н.Асланян)

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