

ВФМ-783(10)-85

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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ  
ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ  
ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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TOPOLOGICAL ASPECTS OF THE BIRTH OF THE UNIVERSE

ЕРЕВАН-1985

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и технико-экономических исследований по атомной науке  
и технике (ЦНИИатоминформ) 1985г.

EDM-763(10)-83

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TOPOLOGICAL ASPECTS OF THE BIRTH OF THE UNIVERSE

It is noted that in Hartly-hawking's approach to the calculation of the universe birth amplitude there are topological restrictions on the properties of born Universe and matter fields in it. These restrictions are obtained in explicit form in particular cases. The possible consequence is the statement that the baryon charge of our Universe would be zero in the absence of the baryon number violating interactions.

Yerevan Physics Institute

Yerevan 1985

БМ-763(10)-85

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ТОПОЛОГИЧЕСКИЕ АСПЕКТЫ РОЖДЕНИЯ ВСЕЛЕННОЙ

Отмечено, что в подходе Хартли-Хокинга есть топологические ограничения на свойства рождающейся Вселенной и полей материи в ней. Ограничения явно получены в частных случаях. Возможным следствием является утверждение, что барионный заряд Вселенной был бы равен нулю в отсутствие несохраняющих барионное число взаимодействий.

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Ереван 1985

1. One of the principle problems of cosmology attracting recently great attention is the problem of the quantum birth of the Universe [1-4]. In the works of Hartly and Hawking [1,2] (see also [3,4]) there was given the definition of the amplitude of the Universe tunneling "from nothing" (or according to [1,2] the wave function of the ground state of the Universe) based on the Feynman functional integral in Euclidean regime. In the following we shall consider only the case of closed Universe with compact connected space-like section  $V$

The amplitude  $\Psi(V, h_{ij}, \varphi)$  of the birth of the Universe  $V$  with metric  $h_{ij}$  and configuration  $\varphi$  of matter fields on  $V$  is given according to [1,2] by the functional integral

$$\Psi(V, h_{ij}, \varphi) = \sum_M \int \mathcal{D}g_{\mu\nu} \mathcal{D}\bar{\varphi} e^{-I(g_{\mu\nu}, \bar{\varphi})} \quad (1)$$

The summation (integration) goes over all compact Riemannian manifolds  $M$ , the boundary of which is  $V$  ( $\partial M = V$ ), over all metrics  $g_{\mu\nu}$  such that the induced metric on  $V$  coincides with  $h_{ij}$ , and over all regular on  $M$  matter fields  $\bar{\varphi}$ , which coincide with  $\varphi$  on  $V$ :  $\bar{\varphi}|_V = \varphi$ .  $I(g_{\mu\nu}, \varphi)$  is the Euclidean action [1,3,4], the precise form of which is inessential for us.

In this work we shall study only the topological aspects of (1) assuming validity of our conclusions also after precise definition of r.h.s. of (1).

To be more concrete, we shall study the question: when, at given  $(V, h_{ij}, \varphi)$ , do  $(M, g_{\mu\nu}, \bar{\varphi})$  satisfying the conditions in the definition (1) exist? In other words, the question under study is: for which Universes  $(V, h_{ij}, \varphi)$  the amplitude of their creation "from nothing" is nonzero?

2. Firstly, let's discard variables  $h_{ij}$ ,  $\varphi$ ,  $g_{\mu\nu}$ ,  $\bar{\varphi}$  and consider the question: when, at given manifold  $V$ , does manifold  $M$  exist, whose boundary is  $V$  ( $\partial M = V$ )? This problem, in different variations, is the main problem of the theory of cobordisms (see, e.g. [5]). In the case when such  $M$  exists, manifold  $V$  is said to be boundary or " $V$  is bordant to zero".

Let's consider now the most interesting case:  $V$  is three-dimensional (and closed, i.e. compact and without boundary), and do not restrict the orientability of  $V$  and  $M$ .

Then, as was noted in [1], it is known that

a) Any  $V$  is the boundary, i.e. for any three-dimensional closed manifold  $V$  there exists four-dimensional one  $M$  with boundary, diffeomorphic to  $V$ :  $\partial M = V$  [6].

We may assume that for orientable  $V$  the main (or maybe the only) contribution in (1) comes from the orientable  $M$ . Then, the following statement of [6] is also important:

a') Any orientable  $V$  is the boundary of orientable manifold  $M$ .

All these means that in both cases there may be born the Universe  $V$  with arbitrary topology.

3. The problem becomes more interesting when considering the Kaluza-Klein theories. In that case, dimension of  $V$  is greater than three - up to ten, provided one considers only supergravity theories. In higher dimensions there exist manifolds which are not boundaries.

But in the most widely considered case, when  $V$  is a direct product  $V = V_3 \times V_n$  of two closed manifolds, three-dimensional  $V_3$  and  $n$ -dimensional  $V_n$  (with small radius), the situation is trivial, i.e. there are no topological restrictions on the birth of such Universe  $V$ . This follows from the statements

b) Any  $V = V_3 \times V_n$  is the boundary.

b') Any orientable  $V = V_3 \times V_n$  is the boundary of some orientable manifold.

b) and b') follow directly from a) and a'): let  $V_3$  be the boundary of  $M$  ( $\partial M = V_3$ ), then  $M \times V_n$  has the boundary  $V_3 \times V_n$ .

So  $V$  may not be the boundary, only if the relation  $V = V_3 \times V_n$  holds locally, not globally, or in other words,  $V$  is non-trivial fiber bundle over base  $V_3$  with the fiber  $V_n$ .

4. Non-trivial restrictions arise when considering the amplitude (1) of the birth of the Universe with given configuration of the matter fields  $\varphi$ .

In the following we denote the space in which fields  $\varphi(x)$  take values at each  $x$  by  $\Phi$ , so  $\varphi$  is the mapping of  $V$  into  $\Phi$ .

The general idea is that in this situation the amplitude (1) may be nonzero only if there exist manifolds  $M$  such that the mapping of their boundary  $\varphi : V = \partial M \rightarrow \Phi$  may be extended continuously to the mapping  $\bar{\varphi} : M \rightarrow \Phi$  of the entire manifold  $M$  into  $\Phi$ . Generally speaking, it may not be the case. Here we shall not consider this problem in all

its generality, rather we shall solve it in the particular case, which may be interesting by itself.

Let the Universe  $V$  be  $S_3$  - the three-dimensional sphere. By the fields  $\varphi$  we take fields  $g$ , taking their value in the group manifold  $G$ , i.e.  $\Phi = G$ . For definiteness we take  $G = SU(2)$  or  $SU(3)$ . Such fields  $g$  arise in the supergravity theories and also in the effective chiral lagrangian of QCD [7,8]. The mappings  $g: V = S_3 \rightarrow G$  fall into homotopic classes characterized by one integer  $n$ , which we shall call solitonic charge. In the works [7,8] it is argued that solitons of the effective chiral lagrangian of QCD are baryons, so in that case  $n$  is baryonic charge.

Let also  $M$  in (1) be the orientable manifolds. Then the following statement may be proved:

c) The manifold  $M$  and the mapping  $\bar{g}: M \rightarrow G$  satisfying conditions in (1) (i.e.  $\partial M = S_3$ ,  $\bar{g}|_{S_3} = g$ ) exist iff the solitonic charge of the mapping  $g: \partial M = S_3 \rightarrow G$  is equal to zero.

In other words, only the Universe with zero solitonic charge may be born.

In the nonorientable case one may prove the similar statement c') which differs from c) only in that solitonic number must be taken modulo two.

Hence, if one applies c) to the solitonic model of baryons [7,8] and disregards the baryon-number violating interactions, then it follows that the baryonic charge of the Universe must be zero.

Looking at the matter the other way round, the conclusion is that all baryonic asymmetry of the Universe is due to baryon number violating interactions.

5. It is worth noting the following principle possibility. Let topological considerations, similar to the above one, forbid the birth of the

Universe (  $V^1, h_{ij}^1, \varphi^1$  ) (for example, in the model of sect.4  $g$  has nonzero solitonic charge). Nevertheless, one may show easily that such Universe may be born together with the "sister" - some other Universe (  $V^2, h_{ij}^2, \varphi^2$  ), whose topology is correlated with the topology of (  $V^1, h_{ij}^1, \varphi^1$  ) (in the example - the solitonic charge of the second universe is opposite to the charge of the first one).

The amplitude of simultaneous birth of the two universes is also given by (1) [1], where  $M$  has to be a connected manifold whose boundary consists of two pieces  $V_1$  and  $V_2$ . Estimates [1] show that such amplitudes are relatively small.

The author is indebted to Asatryan H.M., Gurzadyan V.G., Matinyan S.G. and Khudaverdyan O.M. for the interest in this work and discussions.

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The manuscript was received 5 February 1985

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ТОПОЛОГИЧЕСКИЕ АСПЕКТЫ РОЖДЕНИЯ ВСЕЛЕННОЙ

(на английском языке, перевод Э.Н. Асланян)

Редактор Л. П. Мукаян

Технический редактор А. С. Абрамян

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Подписано в печать 26/IV-85г.

Офсетная печать. Уч. изд. л. 0,5  
Зак. тип. № 179

ВФ-00906 Формат 60x84/16

Тираж 299 экз. Ц. 7 к.  
Индекс 3624

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Отпечатано в Ереванском физическом институте

Ереван 36, Маркаряна 2

индекс 3624



ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ