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ЦЕНТРАЛЬНЫЙ НАУЧНО-ИССЛЕДОВАТЕЛЬСКИЙ ИНСТИТУТ ИНФОРМАЦИИ И ТЕХНИКО-ЭКОНОМИЧЕСКИХ ИССЛЕДОВАНИЙ ПО АТОМНОЙ НАУКЕ И ТЕХНИКЕ

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PHOTON SPLITTING IN THE FIELDS OF CRYSTALLOGRAPHIC AXES AND PLANES

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PHOTON SPLITTING IN THE FIELDS OF CRYSTALLOGRAPHIC AXES AND PLANES

The non-linear quantum electrodynamic process of photon splitting into two photons in strong fields of the crystallographic axes and planes is investigated. The corresponding absorption coefficients are calculated for the case when a photon beam passes through crystal in parallel to its axes or planes. The obtained results which are applicable at relatively low photon energies, show that the absorption coefficients grow as $\sim\omega^5$ when the photon energy ω increases and, at high values of ω they can exceed the corresponding coefficients for amorphous media.

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РАСЩЕПЛЕНИЕ ФОТОНА В ПОЛЯХ КРИСТАЛЛИЧЕСКИХ ОСЕЙ И ПЛОСКОСТЕЙ

Исследован нелинейный квантовоэлектродинамический процесс расщепления фотона на два фотона в сильных полях кристаллических осей и плоскостей, и вычислены соответствующие коэффициенты поглощения в случае, когда пучок фотонов падает на кристалл параллельно осям или плоскостям. Полученные результаты, применимые при относительно малых энергиях фотонов, показывают, что коэффициенты поглощения растут с энергией фотона как $\sim \omega^5$ и при больших значениях ω могут превосходить соответствующие коэффициенты для аморфных сред.

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1. Introduction

At present among the four non-linear effects of quantum electrodynamics (QED), namely, the processes of light scattering by light, Delbruck scattering, splitting and coalescence of photons in the Coulomb field of nuclei (see e.g. [1]) only the Delbruck scattering is directly investigated experimentally [2]. There are two claims that the splitting process of a photon into two photons, $\gamma Z \to Z\gamma\gamma$, is also observed at low [3] and high [2] energies. However, the number of the events recorded in [3] and [2] on the reaction $\gamma Z \to Z\gamma\gamma$ appeared to be~6 and~200 times greater than the ones predicted by the theory [4] and [5]. respectively. If the results on the Delbruck scattering obtained in the experiment [2] agree with the results of the QED calculations taking into account the Coulomb corrections [6], the events of the same experiment [2] ascribed to splitting processes, appeared to be conditioned by another process [7]. It is true that nobody doubts in the accuracy of QED predictions, nevertheless, the search for new possibilities for their checking is undoubtedly of certain interest. It has been recently pointed out [8] that the enhancement of the Delbruck scattering and photon splitting cross sections due to interference phenomena in crystals at certain conditions may make easier the problem of the experimental investigation of these effects.

On the other hand, in the last decades the theory of the QED processes in strong electromagnetic fields has intensively developed (see e.g. [9]). The fact that the probability of the latter processes grows sharply with the increase in the invariant parameter $\varkappa=(e/m^3)\sqrt{|F_{\mu\nu}K^{\nu}|^2}$ and reaches significant values at $\varkappa\sim 1$, is the characteristic feature of these processes. Here $F_{\mu\nu}$ is the electromagnetic field tensor, K^{ν} is the four-momentum of the primary particle: in the photon case $\varkappa=(e/m^3)[(\vec{K}\times\vec{H}+\omega\vec{E})^2-(\vec{K}\vec{E})^2]^{1/2}$, and for $H=0$ and $\vec{E}\perp\vec{K}$ $\varkappa=\omega E/mE_0$ where $\vec{K}$ is the wave vector of a photon of energy ω, and $E_0=m^2/e=1.32\cdot 10^{16}$ V/cm is the so-called critical field. Therefore, one may hope that it will be possible to investigate these processes experimentally at high energies, when, say, $\omega/m\gg 1$, despite the absence of fields of the order of E_0 in terrestrial conditions. Indeed, as the results of the theoretical works [10-17] show, the probability of pair production by a photon passing through a crystal in parallel to crystallographic axes and planes grows sharply with the increase in ω and for $\omega\gtrsim 100$ GeV exceeds the probability conditioned by the well known Bethe-Heitler pair production. Therefore, the observation in the nearest future of the process of e^+e^--pair production by photons with $\omega\gtrsim 100$ GeV in the fields of crystallographic axes and planes will be a means for the experimental study of QED processes taking place in strong external fields. Of course, one may similarly interpret the radiation of chenneled particles at high values of ω/E_e (also exceeding greatly the corresponding Bethe-Heitler radiation intensity) as synchrotron radiation in strong fields of crystallographic axes or planes. However, such an interpretation needs a correction, since despite the case of pair production, when photons move straightforwardly, the channeled particle on the length of the radiation formation zone m/eE passes through a strongly inhomogeneous external field (the arbitrary parameter L, introduced in [18,19] in a classical way and used further for QED calculations, saves the situation).

Among the non-linear effects of QED only the photon splitting* in intense external field is investigated theoretically [20-24]. Using the results of [23-24] it will be shown here that by analogy with the pair production [10-15], the probability of the photon splitting in fields of crystallographic axes and planes grows sufficiently sharply with the increase in the photon energy and may exceed the corresponding probability in the amorphous medium, that opens new possibilities for the observation of this interesting non-linear QED effect.

2. Calculations

According to [23] in the case of the non-polarized photon beam passing perpendicularly to an external homogeneous electric field $\vec{E}$, the total $W_{\gamma\to\gamma\gamma}$ and differential with respect to the energy of one splitted photon $dW_{\gamma\to\gamma\gamma}/d\xi$ ($\xi = \omega'/\omega$) absorption coefficients due to splitting are defined by the expressions:

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* From the experimental viewpoint it should be better to deal with the Delbruck scattering in a strong external field. As is pointed out in [23,24], the Delbruck scattering is described by the same third order polarization operator with one virtual and two real photons.

$$W_{\gamma\to\gamma\gamma}(\omega) = \frac{361}{10\cdot 315^2}\;\frac{\alpha^3}{\pi^2}\left(\frac{\omega}{m}\right)^5\left(\frac{E}{E_0}\right)^6 m\,, \tag{1}$$

$$dW_{\gamma\to\gamma\gamma}(\omega,\xi)/d\xi = 30\,W_{\gamma\to\gamma\gamma}(\omega)\,\xi^2(1-\xi)^2\,, \tag{2}$$

where m is the electron mass and $\alpha = 1/137$. Note that the expressions (1) and (2) are obtained for $\varkappa = \omega E/mE_0 \ll 1$ without taking into account the photon mass arising due to the photon absorption process of e^+e^- pair production.

Now let the photon beam pass a single crystal parallel to the crystallographic axes (or planes). This means that each photon before interacting passes at a certain distance ρ from an axis (or y from the middle of neighbouring planes) and undergoes constant perpendicular electric field $E(\rho)$ of an axis (or $E(y)$ of a plane). Therefore, in order to derive the absorption coefficients for a photon beam it is necessary to carry out the ρ (or y) averaging of the expressions (1) and (2).

From many model potentials, that satisfactorily describe the potentials calculated in the Moliere approximation and frequently used for the investigation of the channeled particle radiation and pair production in single monocrystals we use the following ones:

for the axial case [15]

$$U^{ax}(x) = U_0^{ax}\left[\ln\left(1+\frac{1}{x+\eta}\right) - \ln\left(1+\frac{1}{x_0+\eta}\right)\right], \tag{3}$$

for the planar case [14]

$$U^{pl}(v) = U_0^{pl}\, v^2. \tag{4}$$

In these expressions $x = \rho^2/a_s^2$, $x_0^{-1} = \pi a_s^2 n d$, $\eta = 2u_1^2/a_s^2$, $v = y/(d_p/2)$, a_s is the screening radius, n is the atomic density, d and d_p

are the distances between the atoms in the axis and between the planes, u_1 is the amplitude of thermal oscillations of the crystal atoms. The values of the parameters [14,15] entering into (3) and (4) are given in the table for [111] axes and (110) planes of various single crystals.

After determining the fields $\vec{E} = -\mathrm{grad}\, U$ from (3) and (4) and substituting them into (1) and (2), as has been mentioned above, in order to derive the absorption coefficients for a photon beam, it is necessary to carry out the averaging over the transverse coordinate, i.e. to calculate the integrals $n \cdot d \cdot \int d^2\rho\, W_{\gamma\to\gamma\gamma}(\rho)$ and $d_p^{-1} \int dy\, W_{\gamma\to\gamma\gamma}(y)$ for the axial and planar cases, respectively. In the first case one carries out the integration from 0 to infinity, since for the potential (3) the region $x \sim \eta$ gives the main contribution into the integral, while in the second case the limits of the integral are $-d_p/2$ and $d_p/2$. After averaging one obtains the following expressions for the photon splitting per unit length (or the photon absorption coefficients due to splitting in the field of crystallographic axes and planes):

$$\overline{W}^{ax}_{\gamma\to\gamma\gamma}(\omega) = \frac{32\cdot 361}{5\cdot 315^2}\,\frac{\alpha^3}{\pi}\,\frac{nd}{m^2}\left(\frac{U_0^{ax}}{m}\right)^6 \frac{1}{m^4 a_s^4}\, I\cdot\left(\frac{\omega}{m}\right)^5 m = B^{ax}\omega^5, \qquad (5)$$

$$d\overline{W}^{ax}_{\gamma\to\gamma\gamma}(\omega,\xi)/d\xi = 30\,\overline{W}^{ax}_{\gamma\to\gamma\gamma}(\omega)\,\xi^2(1-\xi)^2 = 30B^{ax}\omega^5\xi^2(1-\xi)^2, \qquad (6)$$

$$\overline{W}^{pl}_{\gamma\to\gamma\gamma}(\omega) = \frac{2^{11}\cdot 361}{5\cdot 7\cdot 315^2}\,\frac{\alpha^3}{\pi^2}\left(\frac{U_0^{pl}}{m}\right)^6 \frac{1}{m^6 d_p^6}\left(\frac{\omega}{m}\right)^5 m = B^{pl}\omega^5, \qquad (7)$$

$$d\overline{W}^{ax}_{\gamma\to\gamma\gamma}(\omega,\xi)/d\xi = 30\,\overline{W}^{pl}_{\gamma\to\gamma\gamma}(\omega)\,\xi^2(1-\xi)^2 = 30B^{pl}\omega^5\xi^2(1-\xi)^2, \qquad (8)$$

where

$$I = \frac{1}{20\eta^2} - \frac{3}{2\eta} - 63 - 252\eta(1+\eta) + \frac{1}{20(1+\eta)^2} + \frac{3}{2(1+\eta)} - 21(1+8\eta+18\eta^2+12\eta^3)\ln\frac{\eta}{1+\eta}\,. \qquad (9)$$

The values of the constants B^{ax} and B^{pl} are given in the table.

3. Discussion

Let us analyse the validity regions and consequences of the obtained results. Note first that we have assumed the parallelism between the photon beam and crystallographic axes (or planes). Nevertheless, by analogy with [11,12,16] the formulae (5-8) are applicable for angles $\max\left(\frac{m}{\omega}, \theta_0\right) < U_0/m$ where θ_0 is the divergence angle of the beam, since the formation zone of the processes in the external field is of the order of m/eE and it is required that the field variation on this length be small.

It is especially worth discussing in more detail the condition $\varkappa = \omega E/mE_0 \ll 1$ and the validity of energy regions of the obtained formulae following from this condition. Let us denote by E_{max} the maximum value of the field E of crystallographic axes (or planes). For the used model potentials (3-4) the fields reach their maximum values E_{max} at $\rho \approx U_1$ and $y = d_p/2$, i.e. at distances very close to the axes and planes. Let us denote by ω_ℓ (see the table) the photon energy for which $\varkappa = \omega_\ell E_{max}/mE_0 = 1$. It is clear that besides the point $\rho_{E=E_{max}}$ (or $y = y_{E=E_{max}}$), i.e. for all the region ρ (or y) $\varkappa < 1$, and the obtained formulae are valid for photon energies $\omega \lesssim \omega_\ell$.

As is seen from (5-8), in the region $\omega \lesssim \omega_\ell$ the absorption coefficients due to splitting in crystallographic axes and planes depend on ω quite strongly ($\sim \omega^5$). Therefore, there is a hope that just as the absorption coefficients of photons in crystals due to pair production [10-17],

$\overline{W}^{ax,p\ell}_{\gamma\to\gamma\gamma}$ also may become (at higher energies) greater than the corresponding absorption coefficients in amorphous media given by the expression [5]:

$$W^{am}_{\gamma\to\gamma\gamma} = n\,\sigma_{\gamma\to\gamma\gamma} = \frac{nZ^2\alpha}{\pi}\,\ell n\,(183\,Z^{-1/3})\cdot 10^{-29}\,cm^{-1}, \qquad (10)$$

where n and Z are the density and charge of the nuclei. As the calculations, however, show, at $\omega \lessdot \omega_\ell$, $\overline{W}^{ax,p\ell}_{\gamma\to\gamma\gamma}$ are still less than $W^{am}_{\gamma\to\gamma\gamma}$, which is independent of ω. It is clear that the growth of $\overline{W}_{\gamma\to\gamma\gamma}$ will continue with the increase in ω and if one assumes that for $\omega > \omega_\ell$ the growth of $\overline{W}_{\gamma\to\gamma\gamma}$ is still given by the expression (5-8) then, for instance, $\overline{W}^{ax}_{\gamma\to\gamma\gamma}$ will be of the order of $W^{am}_{\gamma\to\gamma\gamma}$ at ω_{eq} = = 264 and 48.8 GeV for diamond and tungsten crystals, respectively, i.e. at energies $\sim$ 2-3 times higher than ω_ℓ.

The accurate calculation of $\overline{W}^{ax,p\ell}_{\gamma\to\gamma\gamma}$ at energies $\omega > \omega_\ell$ when in certain regions of ρ and y the values of $\varkappa$ are greater than 1, represent a very cumbersome task which may be solved with the help of computers. This is connected with the fact that according to [23,24] the photon absorption coefficients in homogeneous external fields in the case of $\varkappa \sim 1$ are given by very complicated expressions which are simplified only for $\varkappa \gg 1$: $W_{\gamma\to\gamma\gamma}(\varkappa \gg 1) \sim \varkappa^{2/3}/\omega \sim \omega^{-1/3}$.

However, one may give some simple arguments proving that at energies $\omega > \omega_\ell$ the increase in $\overline{W}^{ax,p\ell}_{\gamma\to\gamma\gamma}$ still continues in a certain region of ω, and, therefore, just as in the case of pair production, $\overline{W}^{ax,p\ell}_{\gamma\to\gamma\gamma}$ will be greater than $W^{am}_{\gamma\to\gamma\gamma}$ at large ω. Indeed for $\omega > \omega_\ell$ one obtains $\varkappa = 1$ at certain values $\rho_{\varkappa=1}$ and $y_{\varkappa=1}$. Then $\varkappa < 1$ in the whole region $\rho > \rho_{\varkappa=1}$ (or $y > y_{\varkappa=1}$) and $\varkappa > 1$ in the region $\rho < \rho_{\varkappa=1}$ (or $y < y_{\varkappa=1}$). Therefore, one may divide the integrals of averaging $nd\int d^2\rho\, W_{\gamma\to\gamma\gamma}(\rho)$ or $d_p^{-1}\int dy\, W_{\gamma\to\gamma\gamma}(y)$ into three parts $\int W(\text{æ}\ll 1)+\int W(\text{æ}\sim 1)+\int W(\text{æ}\gg 1)$. When ω becomes greater than ω_ℓ, the second and third parts of the integral begin to give contribution, and the increase in $\overline{W}_{\gamma\to\gamma\gamma}$ is due to the increase in the effective integration (radial in the axial case) volume. At very high energies, when $\omega \gg \omega_\ell$, the increase in $\overline{W}_{\gamma\to\gamma\gamma}$ slows down, since as it has been already mentioned for the given field, $W(\text{æ}\sim 1)$ and $W(\text{æ}\gg 1)$ increase weakly or do not increase with ω. Besides, the æ-dependence of $W(\text{æ}\ll 1)$ and $W(\text{æ}\gg 1)$, more accurately the æ-dependences of the functions $W(\text{æ}\ll 1)\pi^2\omega/\alpha^3 m^2$ and $W(\text{æ}\gg 1)\pi^2\omega/\alpha^3 m^2$ intersect with each other at æ = 3.654. This circumstance also is in favour of the fact that by analogy with the pair production [16] the formula (1) for $W(\text{æ}\ll 1)$ has a satisfactory accuracy of the order of (20-30)% up to æ ~ 2 ÷ 3.

Thus, just in the case of pair production [16,17], one may undoubtedly expect that the increase in $\overline{W}_{\gamma\to\gamma\gamma}$ will continue still at $\omega > \omega_\ell$, and $\overline{W}_{\gamma\to\gamma\gamma}$ will exceed $W^{am}_{\gamma\to\gamma\gamma}$. In more detail the results applicable at higher photon energies will be published in subsequent papers.

TABLE

Single crystal	$U_o[111]$ (eV)	$U_o(110)$ (eV)	$d[111]$ (Å)	$d_p(110)$ (Å)	a_s (Å)	X_o	η	$B[111]$ ($GeV^{-5} \cdot cm^{-1}$)	$B(110)$ ($GeV^{-5} \cdot cm^{-1}$)	$\omega_c[111]$ (GeV)	$\omega_c(110)$ (GeV)
C	29	26	3.089	1.26	0.326	5.5	0.025	$5.31 \cdot 10^{-19}$	$8.9 \cdot 10^{-23}$	126	917
Si	54	30	4.703	1.92	0.3	15	0.150	$7.3 \cdot 10^{-20}$	$1.68 \cdot 10^{-23}$	185	1079
Ge	91	54	4.9	2	0.3	16	0.13	$2.77 \cdot 10^{-16}$	$4.48 \cdot 10^{-22}$	99	625
W	417	160	2.741	2.24	0.215	40	0.115	$1.1 \cdot 10^{-13}$	$1.54 \cdot 10^{-13}$	14.3	220

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РАСЩЕПЛЕНИЕ ФОТОНА В ПОЛЯХ КРИСТАЛЛИЧЕСКИХ ОСЕЙ И ПЛОСКОСТЕЙ

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