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MONTE CARLO CALCULATION OF THE FLUCTUATIONS OF  
THE IONIZATION LOSSES

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ВЫЧИСЛЕНИЕ ФЛУКТУАЦИИ ИОНИЗАЦИОННЫХ ПОТЕРЬ  
МЕТОДОМ МОНТЕ-КАРЛО

Вычисляются флуктуации ионизационных потерь частиц высоких энергий в тонких слоях веществ методом статистических испытаний. Полученное в результате вычислений распределение потерь энергии сравнивается с имеющимися экспериментальными данными. Предложенный метод особенно важен в случае очень тонких слоев веществ.

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MONTE CARLO CALCULATION OF THE FLUCTUATIONS OF THE  
IONIZATION LOSSES

A Monte Carlo method for calculation of the fluctuations of the ionization losses of high energy particles passing through thin layers of matter is suggested. The distribution of the energy losses obtained in the result of the calculations is compared with the existing experimental data.

The suggested method is especially useful in the case of very thin layers of gases.

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## I. INTRODUCTION

It is well known that the ionization losses of charged particles fluctuate, and if the mean energy losses in the given thickness of matter are greater than the maximal possible loss  $\epsilon_{max}$  in a single collision then these fluctuations undergo to the Gauss distribution [1] otherwise the energy loss distribution has a tail [2,3]. Obtaining the energy loss distribution function in [2,3] it is assumed that the energy  $\epsilon$  given in a single collision of the primary particle is distributed according to the law  $1/\epsilon^2$ . The deviation from this distribution is taken into account by Blunck and Leisegang [4] as well as by Shullek et al [5].

In the work [4] it is determined a parameter :  $\epsilon^2 = \bar{\Delta} (eV) Z^{4/3} 20 (eV)^{1/3} / \beta^2$  where  $\bar{\Delta}$  is the mean energy loss in a thickness  $x \text{ gcm}^{-2} = 0,300 (Z/A) x (m_e c^2 / \beta^2)$ ,  $Z$  and  $A$  are the atomic number and weight of the matter,  $m_e$  is the electron rest mass and  $\beta$  is the primary particle velocity. According to the theory if  $\epsilon^2 \ll 3$  it takes place the Landau distribution [2,3], and if  $\epsilon^2 \gg 3$  the Blunck-Leisegang distribution describes the energy loss fluctuation.

The results of the recent experiments [6,7] carried out by means of gas proportional counters with thicknesses  $10^{-3} \div 10^{-2} \text{ gcm}^{-2}$ , when the value of the above given parameter is  $\epsilon^2 \sim 10 \div 100$ , are in disagreement with Landau as well as with Blunck-Leisegang distributions: the experimental distributions are narrower than Blunck-Leisegang gives and broader than the Landau theory predicts.

Accurately taking into account all the physical processes involved one may in principle solve this problem by the random walk method of statistical tests: this is important at small thicknesses when the number of the collisions is small.

In this paper a Monte Carlo method for the calculation of the ionization

loss fluctuations is developed and the dependence of the energy loss distribution on the thickness is investigated on the basis of the obtained results.

## 2. PROBLEM MODELLING BY THE MONTE CARLO METHOD

The energy losses of the charged particles passing through a matter is mainly due to excitations and ionizations of the atoms<sup>[8]</sup>. The solving of the problem requires the knowledge of the analytical form of the probability distribution function of the energy losses. To obtain the exact form of this function it is necessary to take into account the atomic form factors and the polarization effects which is connected with certain difficulties. In order to simplify the problem the following assumptions are made:

1. The probability  $\tau_i$  for the excitation of the  $i$ -th level of the atom in a thickness of matter of  $1 \text{ gcm}^{-2}$  is inverse proportional to the energy of the given level and proportional to the number of the electrons in this level:

$$\tau_i = \frac{af_i}{\epsilon_i},$$

where  $f_i = \frac{n_i}{Z}$ ,  $n_i$  and  $\epsilon_i$  are the number and the energy of the  $i$ -th level.

2. The distribution of the energy losses due to the ionization has the form  $1/\epsilon^2$ . The probability  $\sigma_i$  of the ionization of the  $i$ -th atomic level in a thickness of matter of  $1 \text{ gcm}^{-2}$  is determined by expression

$$\sigma_i = \frac{C}{\beta^2} \frac{f_i}{\epsilon_i},$$

where  $C=0,1536(Z/A) \text{ Mev/gcm}^{-2}$ . Therefore, the total probability  $\sigma$  of atom ionization when a charged particle passes through  $1 \text{ gcm}^{-2}$  of matter will be

$$\sigma = \frac{C}{\beta^2} \sum_i^n \frac{f_i}{\epsilon_i},$$

and the energy losses  $E_1$  due to ionization is determined by expression

$$E_1 = \frac{C}{\beta^2} \sum_i^n f_i \int_{\epsilon_i}^{\epsilon_{\max}} \frac{\epsilon d\epsilon}{\epsilon^2} = \frac{C}{\beta^2} \left[ \ln \epsilon_{\max} - \sum_i^n f_i \ln \epsilon_i \right].$$

Here,  $\epsilon_{\max}$  is the maximal possible energy transferred to the atomic electrons in a single collision. In the case of a heavy primary particle with a mass  $m_0$ , momentum  $p$  and energy  $E$ <sup>[9]</sup>

$$\epsilon_{\max} = p^2/m_0 \left[ m_0/2m_e + (m_e/2m_0) + (E/m_0c^2) \right],$$

while when the primary particle is an electron  $\epsilon_{\max} = E/2$ . In the calculation of the energy losses due to the ionization the polarization effects have not been taken into account separately.

The energy losses due to the excitation of the atoms in a thickness of matter of  $1 \text{ gcm}^{-2}$  can be determined from the expression

$$E_2 = \epsilon_{av} - E_1,$$

where  $\epsilon_{av}$  is the average energy losses taking into account the density effect<sup>[9]</sup>. On the other hand  $E_2 = \sum_i \epsilon_i \tau_i$ . Since  $\tau_i = af_i/\epsilon_i$ , then  $Q = E_2$ . Therefore the total probability of atom excitation in a thickness of matter of  $1 \text{ gcm}^{-2}$  will be.

$$\tau = \sum_i \tau_i = E_2 \sum_i f_i / \epsilon_i.$$

Thus, one may calculate  $E_1$ ,  $E_2$ ,  $\sigma$  and  $\tau$  using the values of  $f_i$ ,  $\epsilon_i$  for the given substance and  $\epsilon_{av}$ <sup>[9]</sup> for a particle with certain energy.

The process of the computation of the particle energy loss distribution in various substances has been modelled in the following way:

a) The distance  $x$  at which a collision takes place is found using the distribution  $w(x) = 1 - e^{-Sx}$ , where  $S = \tau + \sigma$  is the total probability of the inelastic collision in a thickness of  $1 \text{ gcm}^{-2}$ .

b) The type of the inelastic collision (excitation or ionization) is

found according to their weights.

c) The energy loss of the primary particle in the given collision is found : In the case of excitation the energy loss is equal to the energy of the binding level, in the case of ionization its magnitude is found taking the distribution  $1/\epsilon^2$ .

This operations are repeated until the particle comes out from the substance. The energy loss distribution is determined repeating this process  $N$  times. All the constants necessary for the computations and the formula for the average ionization losses are taken from [9]. Following to [9], as a value of the binding energy of the  $i$ -th level it is taken the effective value  $\epsilon_i = \rho h \omega_i$ , where  $h \omega_i$  is the energy of the given level and  $\rho$  ( $\rho > 1$ ) takes into account the transition from  $i$ -th level to the free state.

### 3. RESULTS AND DISCUSSION

In order to check the method of calculations and to compare the results with the existing experimental data [7] the ionization losses distribution of 1,59 GeV pions in 15 cm thick argon at atmosphere pressure has been calculated by the above described method. In Fig. 1 the comparison of the experimental data [7] with the Landau and Blunck-Leisegang distributions as well as with the distribution obtained in the result of our Monte Carlo calculations is given. As it is seen the experimental distribution is between the Landau and Blunck-Leisegang distributions, the last one being wider than the experimental one even without taking into account the counter resolution. The distribution obtained by the Monte Carlo method is wider than the Landau and narrower than the experimental distributions. But if one takes into account the counter resolution, then the Monte Carlo distribution will become wider especially in the part of the lower energy losses, and the agreement between the experimental and Monte Carlo distributions will be better than the one between the exper-

imental and Blunck-Leisegang distributions.

By the above described method we have calculated the distribution of the ionization losses of ultrahigh energy particles in various thicknesses of argon and xenon with a purpose to identify the particles according to their masses by means of energy separation method [10, 11] of the detection of transition radiation. The calculation of the energy loss distribution by the Monte Carlo method is especially useful at small thicknesses of matter when the Landau and Blunck-Leisegang distributions are invalid since in this case the number of the collisions is small and the energy losses are of order of the atomic energies. Figures 2-7 show the energy loss (keV) distribution of 10 GeV pions passing through argon at atmospheric pressure having thicknesses 30; 6; 3; 1; 2; 0,6 and 0,3 cm, respectively. In the case of relatively large thicknesses the calculated distributions have a Landau form, while decreasing the thickness they take the Poisson distribution form. For the distributions shown in Figs 2-7 the relative width (the ratios of the full widths at half height, FWHH, to the most probable energy losses) are about 31; 50; 52; 91; 120 and 230%, respectively. A sharper increase of these ratios with decreasing the thicknesses is expected according to Blunck-Leisegang while according to Landau they weakly depend on the thickness. At very small thicknesses, about  $10^{-1}$  cm and smaller, when the number of the collisions  $\leq 1$ , separate energy levels of atoms appear in the calculated energy loss distributions.

In connection with the wide application of the thin proportional counters the method suggested in this work for calculation of the ionization loss distribution can be useful in high energy physics.

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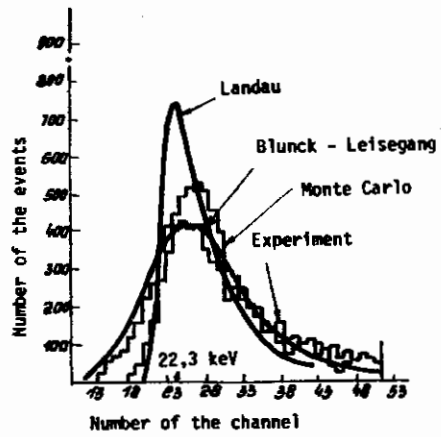


Fig.1

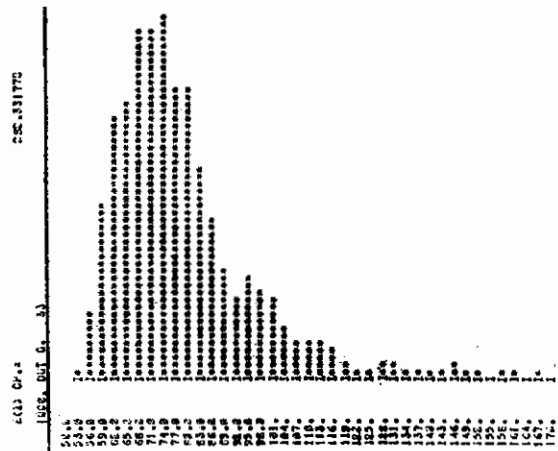


Fig.2

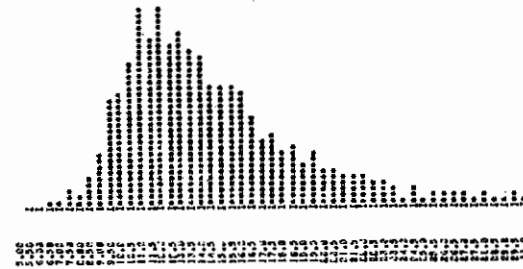


Fig.3

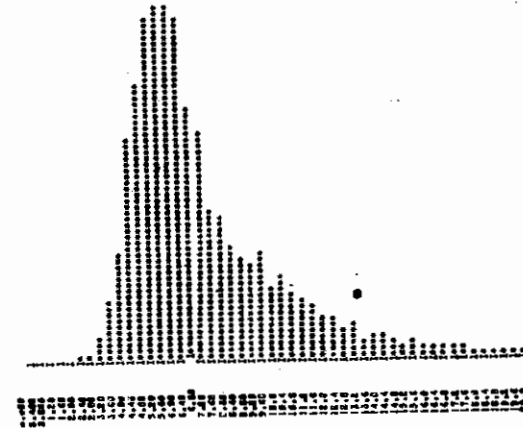


Fig.4

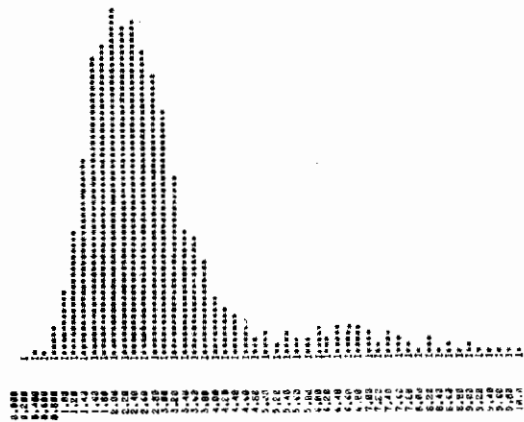


Fig.5

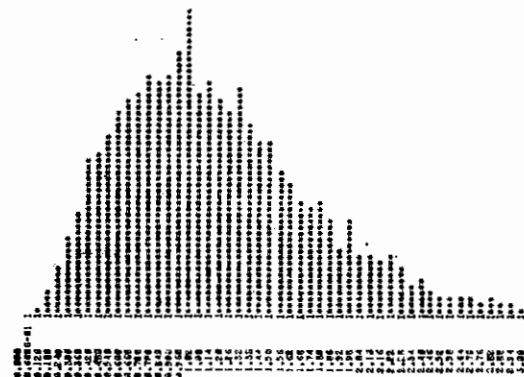


Fig.6

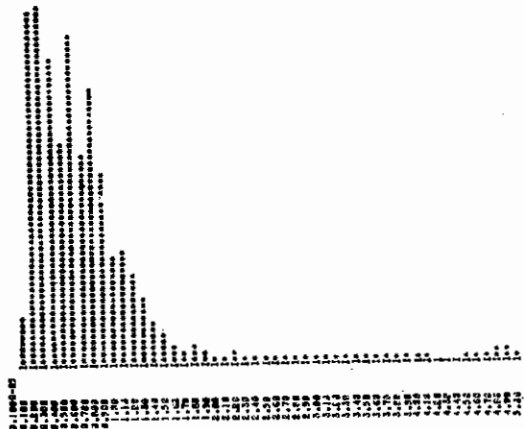


Fig.7

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