ЕФИ-815(42)-85

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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

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QUANTUM GEOMETRY OF COVARIANT SUPERSTRING WITH N=1 GLOBAL SUPERSYMMETRY

ЦНИИатоминформ

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[illegible] quantization of the superstring [illegible] is considered in the [illegible]tional integral [illegible]. [illegible] the effective [illegible] conformal anomaly [illegible] [illegible] determined [illegible] $h_{\alpha\beta}$ [illegible] face [illegible]

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А.Р.КАВАЛОВ, А.Г.СЕДРАКЯН

КВАНТОВАЯ ГЕОМЕТРИЯ КОВАРИАНТНОЙ СУПЕРСТРУНЫ С $N = 1$ ГЛОБАЛЬНОЙ СУПЕРСИММЕТРИЕЙ

Рассмотрено квантование суперструны с $N = 1$ глобальной суперсимметрией в формализме континуального интегрирования. Показано, что интеграл по антикоммутирующим переменным определяется конформной аномалией индуцированной метрики. Возникающее эффективное действие является комбинацией действия струны и действия модели Лиувилля.

Ереванский физический институт

Ереван 1985

Recently, Green and Schwarz have constructed the N=2 superstring theory which is now highly attractive. Their theory is formulated in the critical dimension D=10 in which the canonical quantization is possible [1,6] . After the compactification of extra dimensions the superstring may provide a fundamental theory of matter interacting with gauge and gravitational fields [2,3] . Remarkable features of the theories with gauge groups SO(32) and $E_8 \times E_8$ were also discovered [4,5] . The covariant superstring action of Green and Schwarz [6] has a local supersymmetry and in light-cone gauge reduces to non-covariant light-cone superstring action [1] . We think that it is possible to fix another gauge, namely the gauge in which $\theta^1 = \theta^2$ (it is possible at least classically). (In the work [7] , however, the impossibility of this gauge fixing is discussed but not proved). In the gauge $\theta^1 = \theta^2$ the N=2 superstring action reduces to the superstring action with N=1 global supersymmetry, considered previously by Brink, Green, Julia, Rocek, Schwarz, Siegel and di Vecchia [8] . Note also that Siegel has proved the equivalence between the above-mentioned superstrings, i.e. the N=2 superstring of Green and Schwarz and the N=1 superstring in three-dimensional space [9] . In the present paper we shall consider the N=1 superstring theory.

There exists also another approach to superstring theory in which the string is quantized directly in the physical dimension, D=4. Polyakov has shown [10] that in this case one must take into account the conformal anomaly giving rise to additional "Liouville" modes. The resulting theory is expected to describe the physics of strong interactions. Note also that in three-dimensional space the N=1 superstring is connected with the continuum limit of three-dimensional Ising model [11] .

In the present work the partition function of the N=1 superstring in arbitrary dimension is considered. We have calculated the part of the functional integral over anticommuting variables determined by the conformal anomaly. We have shown that the conformal anomaly determines the fermionic determinant only up to a (nonlocal) functional over $\Delta^{-1} H$ (where Δ^{-1} is the Green function of the Laplace operator acting on [illegible] and H is the mean curvature of the surface). [illegible] N=1 superstring is determined [illegible] by the intrinsic [illegible] of the surface [illegible] depends on the way the surfaces are embedded [illegible]

[illegible] string action has the form [8]

$$S = \int d^2\xi \sqrt{g}\, g^{\alpha\beta} \left[-\frac{1}{2} p_\alpha^\mu p_\beta^\mu + p_\alpha^\mu \left(\partial_\beta x^\mu - i \bar{\theta} \gamma^\mu \partial_\beta \theta \right) \right] \qquad (1)$$

where x^μ are the coordinates of [illegible]-dimensional space, θ [illegible] Majorana spinor. The Dirac fermions may be considered similarly with the substitution $\bar{\theta}\gamma^\mu \partial_\alpha \theta \longrightarrow \frac{1}{2} \bar{\theta} \gamma^\mu \overleftrightarrow{\partial_\alpha} \theta$

Consider the partition function

$$Z = \int \mathcal{D}x^\mu \, \mathcal{D}\theta \, \mathcal{D}g_{\alpha\beta} \, \mathcal{D}p_\alpha^\mu \, e^{-S} \qquad (2)$$

The integration over X^μ yields the constraint

$$\delta\left(\partial_\beta \sqrt{g}\, g^{\alpha\beta} p_\alpha^\mu \right) \qquad (3)$$

In order to integrate the δ -function out we present P_α^μ in the form

$$P_\alpha^\mu = g^{\alpha\beta}\partial_\beta \chi^\mu + \frac{\varepsilon^{\alpha\beta}}{\sqrt{g}}\partial_\beta \phi^\mu \tag{4}$$

and decompose the measure

$$\mathcal{D}P_\alpha^\mu = \left[\det(g^{\alpha\beta}\partial_\beta)^2\right]^{\frac{D}{2}} \left[\det\left(\frac{\varepsilon^{\alpha\beta}}{\sqrt{g}}\partial_\beta\right)^2\right]^{\frac{D}{2}} \mathcal{D}\chi^\mu \mathcal{D}\phi^\mu$$

$$= (\det\Delta)^D \mathcal{D}\chi^\mu \mathcal{D}\phi^\mu \tag{5}$$

where Δ is the Laplace operator on 0-forms. Substituting (4) into (2) and integrating over χ^μ one obtains

$$\int \mathcal{D}\chi^\mu \; \delta(\Delta_0 \chi^\mu) = (\det\Delta)^{-D} \tag{6}$$

so that the partition function takes the form

$$Z = \int \mathcal{D}\phi^\mu \mathcal{D}g_{\alpha\beta} \mathcal{D}\theta \, e^{-S_1} \tag{7}$$

$$S_1 = \int d^2\xi \sqrt{g}\, g^{\alpha\beta}\frac{1}{2}\partial_\alpha\phi^\mu\partial_\beta\phi^\mu + i\varepsilon^{\alpha\beta}\theta\gamma^\mu\partial_\alpha\phi^\mu\partial_\beta\theta \tag{8}$$

Now the integration over θ may be performed:

$$\int \mathcal{D}\theta\, e^{-i\int d^2\xi\, \varepsilon^{\alpha\beta}\bar{\theta}\gamma^\mu\partial_\alpha\phi^\mu\partial_\beta\theta} = \det D = e^{W} \tag{9}$$

$$W = \mathrm{Tr}\,\ln D = \frac{1}{2}\mathrm{Tr}\,\ln D^2 = -\frac{1}{2}\mathrm{tr}\int_0^\infty \frac{ds}{s} e^{-D^2 s} \tag{10}$$

Here $D = i\frac{\varepsilon^{\alpha\beta}}{\sqrt{h}}\partial_\alpha\hat{\phi}\partial_\beta$. $h_{\alpha\beta} = \partial_\alpha\phi^\mu\partial_\beta\phi^\mu$ is the induced metric.

To calculate the W we shall use the well-known method based on the variational equation for W [12] . Under variation of the field

$\phi^\mu_\alpha \equiv \partial_\alpha \phi^\mu$ the fermionic part of the effective action W changes by

$$\delta W = tr \int_0^\infty ds\ \delta D \cdot D e^{-D^2 s} \tag{11}$$

$$\delta \mathcal{D} = i \varepsilon^{\alpha\beta} \delta \left(\frac{\hat{\phi}_\alpha}{\sqrt{h}} \right) \partial_\beta \tag{12}$$

We restrict ourselves to the variations of the form

$$\delta \left(\frac{\hat{\phi}_\alpha}{\sqrt{h}} \right) = A \frac{\hat{\phi}_\alpha}{\sqrt{h}} \tag{13}$$

with some matrix A . Hence we deal with the restricted class of the variations. This is the reason why we cannot calculate W exactly. So,

$$\delta W = tr A \int ds\, D^2 e^{-D^2 s} \equiv tr A\, \psi_0 (D^2) \tag{14}$$

where

$$\psi_0 = \frac{1}{48\pi} \sqrt{h}\, R + \frac{1}{16\pi} \sqrt{h} \left(\nabla_\alpha \phi^{\mu\alpha} \right)^2 \tag{15}$$

is the Seeley's coefficient of the operator D^2 . In (13) R is the scalar curvature of the metric $h_{\alpha\beta}$. Note that the expression $(\nabla_\alpha \phi^\mu_\alpha)^2$ may be interpreted as the mean curvature of the surface $\phi^\mu(\xi)$. Indeed, $\nabla_\alpha \phi^\mu_\beta = b^i_{\alpha\beta} \rho^\mu_i$, where ρ^μ_i are the unit normal vectors of the surface satisfying $\rho^\mu_i \rho^\mu_j = \delta_{ij}$. Then $(\nabla_\alpha \phi^\mu_\alpha)^2 = (b^i_{\alpha\beta} h^{\alpha\beta})^2 \equiv H^2$. To solve Eq.(14) we rewrite W as a sum of two terms corresponding to two terms in ψ_0 (15):

$$W = W_1 + W_2 \tag{16}$$

$$\delta W_1 = \frac{1}{48\pi} \operatorname{tr} A \sqrt{h}\, R \tag{17}$$

$$\delta W_2 = \frac{1}{16\pi} \operatorname{tr} A \sqrt{h} \left(\nabla_\alpha \phi^{\mu\alpha}\right)^2 \tag{18}$$

The solution of Eq.(17) is the well-known Liouville action [10]

$$W_1 = \frac{1}{192\pi} 2^{[\frac{D}{2}]} \int d^2\xi \sqrt{h}\, R\, \Delta^{-1} R$$

The factor $2^{[\frac{D}{2}]}$ comes from the trace over spinorial indices.

Eq.(18) can be written in conformal gauge as

$$\delta W = \frac{2^{[\frac{D}{2}]}}{16\pi} \int d^2\xi \frac{1}{\rho} (\partial_\alpha \phi^\mu_\alpha)^2 \operatorname{tr} A \tag{19}$$

It follows from (13) that A must be equal to $-\frac{1}{2} \hat{\phi}^\alpha \delta \hat{\phi}_\alpha$ and

$$\operatorname{tr} A = -\frac{1}{2} \frac{\delta\rho}{\rho} \operatorname{tr} \mathbb{1} \tag{20}$$

(this means that the transformation $\phi^\mu_\alpha \to \phi^\mu_\alpha + \delta\phi^\mu_\alpha$ with $\delta\phi^\mu_\alpha$ satisfying (13) is the conformal transformation).

The solution of Eq.(19) is

$$W_2 = -\frac{1}{32\pi} \int d^2\xi \frac{1}{\rho} \left(\partial_\alpha \phi^\mu_\alpha\right)^2 \ln\rho = \tag{21}$$

$$= -\frac{1}{32\pi} \int d^2\xi \sqrt{h} \left(\nabla_\alpha \phi^{\mu\alpha}\right)^2 \Delta^{-1} R$$

where R is the curvature scalar, built from the metric $h_{\alpha\beta}$, Δ is the Laplace operator acting on scalars.

Thus the effective action has the form

$$W = \frac{2^{[\frac{D}{2}]}}{192\pi} \int d^2\xi \sqrt{h}\, R \Delta^{-1} R - \frac{2^{[\frac{D}{2}]}}{32\pi} \int d^2\xi \sqrt{h} \left(\nabla_\alpha \phi^{\mu\alpha}\right)^2 \Delta^{-1} R \quad (22)$$

So we have calculated the part of the effective action which is determined by the conformal anomaly. The total effective action may contain additional terms of the type:

$$\int d^2\xi \frac{1}{\rho} (\partial_\alpha \phi^{\mu\alpha})^2$$

$$\int d^2\xi \frac{1}{\rho} (\partial_\alpha \phi^{\mu\alpha})^2 \frac{1}{\partial^2} \frac{1}{\rho} (\partial_\beta \phi^{\mu\beta})^2 \quad (23)$$

$$\int d^2\xi \frac{1}{\rho} (\partial_\alpha \phi^{\mu\alpha})^2 \frac{1}{\partial^2} \frac{1}{\rho} (\partial_\beta \phi^{\mu\beta})^2 \frac{1}{\partial^2} \frac{1}{\rho} (\partial_\gamma \phi^{\mu\gamma})^2 ,$$

Such terms are invariant under the class of variations we have considered. The total effective action has the form

$$\begin{aligned} L_{ef} &= \sqrt{g}\, g^{\alpha\beta} \partial_\alpha \phi^\mu \partial_\beta \phi^\mu + \frac{2^{[\frac{D}{2}]}}{192\pi} \sqrt{h}\, R \Delta^{-1} R \\ &- \frac{2^{[\frac{D}{2}]}}{32\pi} \sqrt{h}\, H^2 \Delta^{-1} R + \\ &+ \sqrt{h}\, H^2 f(\Delta^{-1} H^2, \xi) + \mu^2 \sqrt{h} \end{aligned} \quad (24)$$

where $h_{\alpha\beta} = \partial_\alpha \phi^\mu \partial_\beta \phi^\mu$ is the metric induced on the surface $\phi^\mu(\xi)$, R is the scalar curvature built from that metric. $H^2 = (\nabla_\alpha \phi^{\mu\alpha})^2$ is the mean curvature of the surface $\phi^\mu(\xi)$ $\Delta = \frac{1}{\sqrt{h}} \partial_\alpha \sqrt{h}\, h^{\alpha\beta} \partial_\beta$ is the Laplace operator acting on scalars, and f is the general reparametrization invariant function.

The presence of the last term in the expression (24) shows that the dynamics of N=1 superstring is determined not only by the intrinsic geometry of the surfaces (as it was the case with bosonic and Neveu-Schwarz strings) but also by the external geometry, i.e. by the way the surface is embedded in D-dimensional space.

The authors express their gratitude to S.G. Matinyan, G.M. Asatryan, R.L. Mkrtchyan, and especially to A.M. Polyakov for valuable discussions.

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The manuscript was received 17 May 1985

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(на английском языке, перевод З.Н.Асланян)

Редактор Л.П.Мукаян
Технический редактор А.С.Абрамян

Подписано в печать II/Х-85г. ВФ-09037 Формат 60х84/I6
Офсетная печать. Уч.-изд.л.0,7 Тираж 299 экз Ц.I0 к.
Зак.тип.№ 436 Индекс 3624

Отпечатано в Ереванском физическом институте
Ереван 36, Маркаряна 2

индекс 3624

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ