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H.R.AVAKIAN, V.V.BELOSHITSKY, C.YANG

THE RADIATION AND MULTIPLE SCATTERING OF HIGH ENERGY ELECTRONS AT PLANAR CHANNELING

ЦНИИатоминформ

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А.Р.Авакян, В.В.Белошицкий [illegible]

ИЗЛУЧЕНИЕ И МНОГОКРАТНОЕ РАССЕЯНИЕ ЭЛЕКТРОНОВ ВЫСОКИХ ЭНЕРГИЙ ПРИ ПЛАНАРНОМ КАНАЛИРОВАНИИ

Исследовано многократное рассеяние электронов высоких энергий при прохождении через монокристаллы в условиях планарного (плоскостного) каналирования. Показано его увеличение для частиц, движущихся вблизи потенциального барьера, что проявляется в угловых распределениях прошедших электронов. С учетом этого эффекта рассчитаны спектры излучения. Сравнение теории с экспериментом показало хорошее [illegible]

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THE RADIATION AND MULTIPLE SCATTERING OF HIGH ENERGY ELECTRONS AT PLANAR CHANNELING

The multiple scattering of high energy electrons passing through monocrystals in the case of planar channeling is investigated. It is shown that particles moving near the top of the potential barrier undergo depressed multiple scattering. This displays in the angular distribution of passing electrons. Radiation spectra are calculated taking into account this effect. A comparison of the theory with the experiment shows a good agreement.

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1. Introduction

The radiation of relativistic channeled particles is of great interest due to its high intensity and directivity (see e.g. Ref[1]). At present, the radiation of channeled positrons is investigated in detail both experimentally [2,3] and theoretically [4,5]. Account of particle multiple scattering at the crystal atoms during radiation intensity calculation in real crystal is of great importance. This especially refers to the case of channeled electrons. However, the calculations of electron radiation spectra up to now are carried out either without consideration of multiple scattering [6] or with model distribution of the electron beam [7].

In the present paper we investigate the passage and radiation of electrons at planar channeling by using a realistic atom potential and the solution of the kinetic equation.

2. Multiple Scattering

As is known, charged particles when passing through matter, as a result of their collision with electrons and nuclei, undergo multiple scattering. Consider first the dependence of mean square angle $\langle \Delta \vartheta_c^2 \rangle$ of multiple scattering on transverse energy $\varepsilon_\perp$ of electron in the crystal. $\langle \Delta \vartheta_c^2 \rangle$ is a sum of $\langle \Delta \vartheta_c^2 \rangle_n$ and $\langle \Delta \vartheta_c^2 \rangle_{el}$ determined by the scattering on nuclei and crystal electrons, respectively. $\langle\Delta\vartheta_c^2\rangle_n$ is approximately proportional to the local density of nuclei. In this approximation the ratio of $\langle\Delta\vartheta_c^2\rangle_n$ to mean square angle $\langle\Delta\vartheta_a^2\rangle$ of multiple scattering in an amorphous medium is equal to

$$\frac{\langle\Delta\vartheta_c^2\rangle_n}{\langle\Delta\vartheta_a^2\rangle} \equiv \gamma_n = \frac{Z L'}{Z L' + L''} \int_0^{x_m} \rho_n(x)\, dW(x)\,, \tag{1}$$

where x_m is the maximal distance between the particle and the plane, Z is the nucleus charge number $L' = \ln 184{,}15\, Z^{-1/3}$, $L'' = \ln 1194\, Z^{-2/3}$ are the radiation logarithms [8], $dW(x)$ is the probability to find a particle at a distance x from the plane:

$$dW(x) = \frac{\sqrt{2E}\, dx}{c\, T\sqrt{\varepsilon_\perp - U(x)}}\,, \tag{2}$$

$\rho_n(x)$ is the nuclei distribution with account of thermal oscillation of atoms

$$\rho_n(x) = \frac{d_p}{\sqrt{2\pi}\, u_1} \exp\left(-x^2/2u_1^2\right). \tag{3}$$

Here E is the particle energy, T is the oscillation period, c is the light velocity, $U(x)$ is the averaged planar potential, d_p is the spacing of the lattice planes, u_1^2 is the mean square amplitude of the thermal vibrations.

Since $d_p \gg u_1$ the integral (1) may be calculated by the method of steepest descent provided $x_m \gg u_1$. In the case of parabolic approximation for the channel potential [9] one can obtain

$$\gamma_n = z L' \left\{ \sqrt{\varepsilon_\perp / U_0}\, \ln \left[(\sqrt{U_0} + \sqrt{\varepsilon_\perp}) / \sqrt{|\varepsilon_\perp - U_0|} \right] (z L' + L'') \right\}^{-1}, \quad (4)$$

where $\varepsilon_\perp$ is counted from the bottom of the potential well, U_0 is the depth of the well. For the particles in the very bottom of the potential well ($\varepsilon_\perp \approx 0$) for which $x_m \ll u_1$ we get from (1) that multiple scattering in crystal is

$$z L' d_p \left[\sqrt{2\pi}\, u_1 (z L' + L'') \right]^{-1}$$

times as great as that in an amorphous medium.

To calculate the $\langle \Delta \vartheta_c^2 \rangle_{el}$ one may replace in (1) $\rho_n(x)$ by $\rho_{el}(x) \alpha L'' / z L'$ where $\rho_{el}(x)$ is the electron density distribution in the transverse plane, $\alpha \approx 0.5$ is a separation factor of the ionization loss for contributions of short- and long-range collisions [10]. The distribution $\rho_{el}(x)$ may be obtained from the potential according to Poisson's equation. For parabolic potential it is independent of x, i.e. in this approximation the multiple scattering on crystal electrons is the same as in an amorphous medium.

The calculation of γ_n as a function of transverse energy is carried out for general conditions by numerical integration of formula (1) by using Molier's averaged potential $U(x)$ in (2).

The value of $\gamma_{el} = \langle \Delta \vartheta_c^2 \rangle_{el} / \langle \Delta \vartheta_a^2 \rangle$ is calculated analogously.

Fig. 1 presents dependence of γ_{el} and $\gamma_s = \gamma_{el} + \gamma_n$ on transverse energy $\varepsilon_\perp$. Analogous curve for γ_s calculated in the parabolic approximation of the potential is adduced for comparison.

One can see the essential dependence of γ_s on transverse energy. Using the calculated values of γ_s one may obtain the particle distribution function $F(\varepsilon_\perp, \ell)$ in transverse energies for a crystal thickness ℓ from the kinetic equation [11]

$$\frac{\partial F(\varepsilon_\perp,t)}{\partial t} = \frac{\partial}{\partial \varepsilon_\perp}\left[\mathcal{D}(\varepsilon_\perp)T(\varepsilon_\perp)\frac{\partial}{\partial \varepsilon_\perp}\left(\frac{F(\varepsilon_\perp,t)}{T(\varepsilon_\perp)}\right)\right] - \frac{\partial}{\partial \varepsilon_\perp}\left\langle\frac{\Delta\varepsilon_\perp}{\Delta t}\right\rangle_{loss} F(\varepsilon_\perp,t), \quad (5)$$

where

$$\mathcal{D}(\varepsilon_\perp) = \frac{1}{T(\varepsilon_\perp)}\int_0^{\varepsilon_\perp} T(\varepsilon_\perp)\left\langle\frac{\Delta\varepsilon_\perp}{\Delta t}\right\rangle d\varepsilon_\perp ,$$

$$\left\langle\frac{\Delta\varepsilon_\perp}{\Delta t}\right\rangle = \frac{E\gamma_s}{2}\left\langle\frac{\Delta\theta_a^2}{\Delta t}\right\rangle,$$

$$\left\langle\frac{\Delta\varepsilon_\perp}{\Delta t}\right\rangle_{loss} - \frac{\alpha}{E}\left\langle\left(\varepsilon_\perp - U(x)\right)\frac{dE}{\alpha t}\right\rangle .$$

The angular brackets denote the averaging over an oscillation period

$$\langle X\rangle = \frac{1}{T}\int_0^T X(t)\,dt .$$

Using the distribution function $F(\varepsilon_\perp,\ell)$ one may get the velocity mean square components $\bar{v}_x^2(\ell)$ and $\bar{v}_y^2(\ell)$, respectively perpendicular and parrallel to channeling plane, for the thickness ℓ

$$\bar{v}_x^2(\ell) = \int_0^\infty F(\varepsilon_\perp,\ell)\langle v_x^2\rangle d\varepsilon_\perp , \quad (6)$$

$$\bar{v}_y^2(\ell) = c^2\langle\Delta\vartheta_a^2\rangle_\ell \int_0^\infty \frac{\gamma_s d\varepsilon_\perp}{\ell}\int_0^\ell F(\varepsilon_\perp,\ell)d\ell . \quad (7)$$

Fig. 2 presents the dependence of $\sigma_x = \bar{v}_x^2(\ell)/c^2\langle\Delta\vartheta_a^2\rangle_\ell$ and $\sigma_y = \bar{v}_y^2(\ell)/c^2\langle\Delta\vartheta_a^2\rangle_\ell$ on thickness ℓ where

$$\langle\Delta\vartheta_a^2\rangle_\ell = \left(\frac{14{,}1}{E}\right)^2 \frac{\ell}{\ell_{Rad}}\left(1 + \frac{1}{9}\ln\frac{\ell}{\ell_{Rad}}\right),$$

(ℓ_{Rad} is the radiation length).

At small thickness the dispersion $\overline{v_x^2}(\ell)$ is associated with the particle scattering at averaged potential $U(x)$. The value of $\overline{v_x^2}(\ell)$ for the thickness more than 150 μm is less than that for an amorphous medium. This result is qualitativly confor- med experimentally [12].

3. Radiation Spectra

We have calculated the radiation spectra for various va- ues of particle transverse energy [6] using Molier's averaged potential by a well-known formula of electromagnetics

$$\frac{dW}{\ell\hbar d\omega}=\frac{e^2\omega}{2\pi\hbar c^2}\sum_{K=1,2\ldots}^{\infty}|\vec{n}\times\vec{j}|^2\delta\left(1-\frac{\vec{n}\cdot\langle\vec{v}\rangle}{c}-\frac{2\pi K}{\omega T}\right)d\Omega=\frac{2e^2\omega}{\pi\hbar c^2}\sum_{K=K_0,K_0+1,\ldots}^{\infty}\int_0^{\infty}\Phi(\theta_K,\varphi)d\varphi \quad (8)$$

where

$$\Phi(\theta_K,\varphi)=(j_z\theta_K\sin\varphi-j_x)^2+j_z^2\,\theta_K^2\cos^2\varphi,\quad \vec{n}=\{\theta\cos\varphi,\ \theta\sin\varphi,\ 1-\theta^2/2\}$$

$$\theta_K=\sqrt{4\pi K/\omega T-\gamma^{-2}-\langle v_x^2\rangle/c}\ .$$

For channeled particles

$$j_K=4(cT)^{-1}\int_0^{x_m}\cos(A_K(x)+\delta_K)\cos(B_K(x)+\delta_K)dx\ ,\quad \delta_K=\begin{cases}0\ , & K=2m\\ \pi/2\ , & K=2m+1,\end{cases}$$

$$j_z=4T^{-1}\int_0^{x_m}\sin(A_K(x)+\delta_K)\sin(B_K(x)+\delta_K)\,v_x^{-1}\,dx\ ;$$

for above-barrier particles

$$j_x=2(cT)^{-1}\int_0^{d_p/2}\cos(A_K(x)-C_K(x))dx\ ,\quad j_z=2T^{-1}\int_0^{d_p/2}\cos(A_K(x)-C_K(x))\,v_x^{-1}dx.$$

Here

$$A_K(x)=-\frac{2\pi K\,t(x)}{T}+\frac{\omega}{c^2}\left[\int_0^x v_x dx-\langle v_x^2\rangle\int_0^x\frac{dx}{v_x}\right],\quad B_K(x)=\frac{\omega x}{c}\theta_K\sin\varphi,$$

$$C_K(x)=\frac{\omega}{c}\left(x-\frac{d_p}{T}\int_0^x\frac{dx}{v_x}\right)\theta_K\sin\varphi,$$

ω is the radiation frequency, θ and φ are polar and azimuthal angles of radiation, K_0 is a minimal integer satisfying the inequality

$$4\pi K_0/\omega T \geq \gamma^{-2} + \langle v_x^2 \rangle / c^2 .$$

Fig. 3 presents the theoretical radiation spectra for some typical values of particle transverse energy. The averaged radiation spectra of electrons are calculated using the distribution function $F(\varepsilon_\perp, \rho)$. Fig.4 illustrates radiation spectra of 4.5 GeV electrons channeled in (110) planes of a 100 μm thick diamond. Initial beam divergence (5.10^{-5} rad) is taken into account. This figure shows a good agreement of calculations with experiment [13]. Calculation using approximate distribution function [7] gives larger contribution of high-energetic photons. Since comparatively low-energetic photons are generated by particles with transverse energy close to the potential barrier top (see fig.3), one can expect from the comparison of theory and experiment [13] that such particles are of grater part in the experiment [13].

The calculation is also compared with the experiment on silicon for 7 GeV electrons. The ratio of radiation intensities in crystal and in amorphous matter in this case is illustrated in Fig.5. Spectrum for amorphous matter is calculated by Bethe-Heitler formula. As is seen, theoretical spectra for silicon are also in good agreement with the experiment.

Thus in conclusion we can say that multiple scattering of channeling particles, which strongly differs from the scattering in amorphous matter, when correctly taken into account, may give a good quantitative agreement between theory and experiment. In this case an essential difference in angular distribution of electrons passing through the crystal in a regime of planar channeling and amorphous matter also takes place.

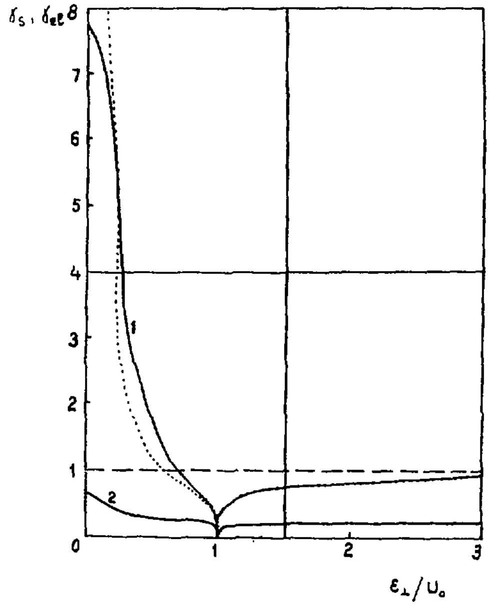

Fig.1 Dependence of γ_s (solid curve 1) and γ_{el} (solid curve 2) on transverse energy $\varepsilon_\perp$ (in units of U_0) of electrons with total energy E=4.5 GeV channeled in diamond planes (110), calculated by means of Molier's potential. The dependence of γ_s calculated in parabolic potential approximation using formula (4) is adduced for comparison. In the case of nonoriented crystal γ_s =1 (dotted line).

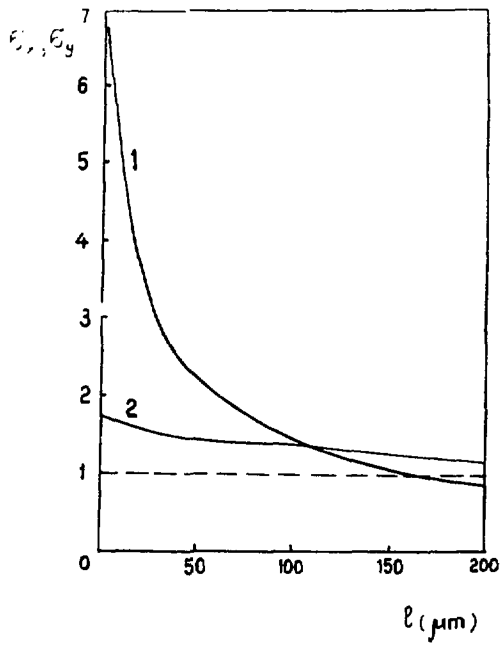

Fig.2 Dependence of σ_x (curve 1) and σ_y (curve 2) on penetration thickness ℓ for electrons with total energy E=4.5 GeV channeled in planes (110) of diamond. In nonoriented crystal $\sigma_x = \sigma_y = 1$ (dotted line).

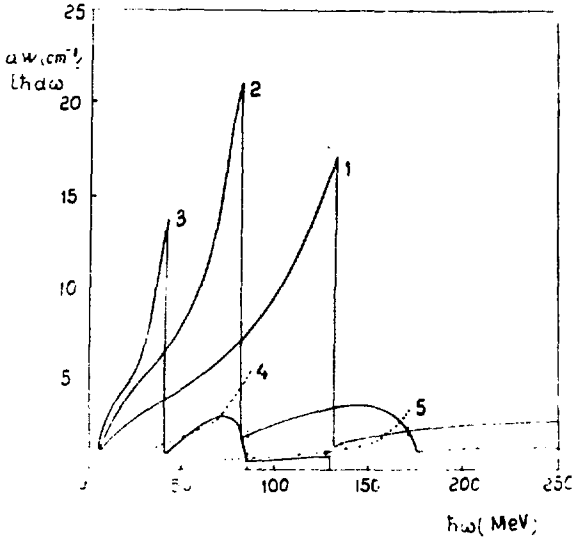

Fig.3 Radiation spectra for some typical values of transverse energy. Curves 1-3 correspond to channeled electrons $\varepsilon_\perp/U_0$ = 0.2; 0.5; 0.85; curves 4 and 5 - to overbarier electrons: $\varepsilon_\perp/U_0$ = 1.1; 2.

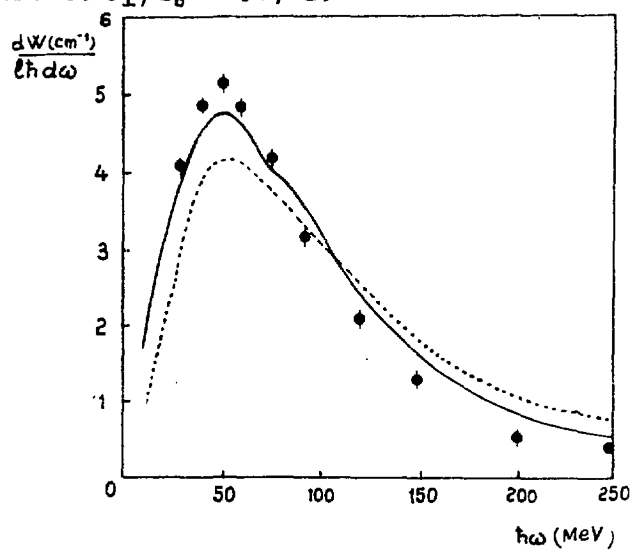

Fig.4 Averaged radiation spectra of electrons with E=4.5 GeV channeled in planes (110) of 100μm thick diamond. Initial beam divergence is 5.10^{-5}rad. Dotted curve corresponds to theoretical calculation [7]. Points correspond to experimental data [13].

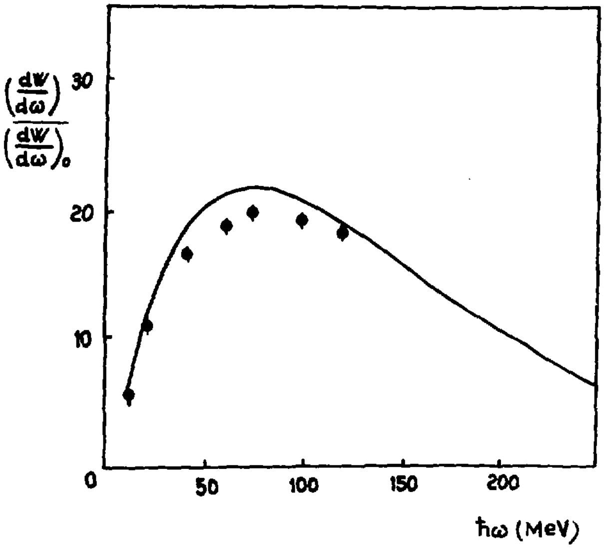

Fig.5 The ratio of averaged radiation spectra of electrons with E = 7 GeV channeled in (110) planes of 100μm thick silicon to analogous spectra in amorphous matter. Points correspond to experimental data [14].

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