Preprint ЕФИ-841(68)-85

ԵՐԵՎԱՆԻ ՖԻԶԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ
ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

L.S.DULYAN, A.G.OGANESSIAN,
A.Yu.KHODJAMIRIAN

RADIATIVE DECAYS OF CHARMONIUM P-LEVELS IN QUANTUM CHROMODYNAMICS

ЦНИИатоминформ

ЕРЕВАН-1985

Препринт ЕФИ-841(68)-85

Л.С.ДУЛЬЯН, А.Г.ОГАНЕСЯН, А.Ю.ХОДЖАМИРЯН

РАДИАЦИОННЫЕ РАСПАДЫ P - УРОВНЕЙ ЧАРМОНИЯ В КХД

Вычислены глюонные степенные поправки к правилам сумм КХД для радиационных распадов $^3P_0$ - уровней чармония χ_0 (3415). Получены предсказания ширины распадов $\chi_0 \to J/\psi\gamma,\ 2\gamma,\ e^+e^-\gamma$.

Ереванский физический институт

Ереван 1985

Preprint ЕФИ-84I(68)-85

L.S. DULYAN, A.G. OGANESSIAN, A.Yu. KHODJAMIRIAN

RADIATIVE DECAYS OF CHARMONIUM P-LEVELS IN QUANTUM CHROMODYNAMICS

The gluonic power corrections to quantum chromodynamics sum rules for $\chi_0(3415)$ charmonium $^3P_0$-levels radiative decays are calculated. The widths of $\chi_0 \to J/\Psi\gamma$, 2γ, $e^+e^-\gamma$ decays are predicted.

Yerevan Physics Institute

Yerevan 1985

1. Introduction

The method of quantum chromodynamics sum rules (QCD SR) [1,2], developed in recent years, permits to calculate a lot of physical parameters: the hadronic masses, widths and form factors, starting just from the quark-gluon interaction Langrangian (see the latest reviews [3,4]).

The first and the most fruitful application of the SR method was the physics of charmonium ($c\bar{c}$ mesons) [1]. It is a unique system of hadrons which, at least qualitatively can be described in "obvious" terms of nonrelativistic potential. Here the orbital and radial excitations of the lowest states are clearly distinguished. At the same time the bound $c\bar{c}$-state is a good probe of the pure gluonic nonperturbative interactions, yielding power $O(\alpha_s \langle G^2 \rangle)$ corrections to the SR, where $\langle G^2 \rangle \equiv \langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle$ is the vacuum gluon condensate density. Taking these corrections into account it became possible to compute the masses of the lowest S- and P-levels of charmonium [1,2,5].

The SR technique is also applicable to the simplest processes with $c\bar{c}$ mesons such as two- photon decays [1] and radiative transitions [6]. Taking the power corrections into account has permitted to calculate the width of the radiative $\eta_c \to 2\gamma$ [7] and $J/\Psi \to \eta_c \gamma$ (magnetic-dipole transition) [8,9] decays involving the $^1S_0$ charmonium level, the $\eta_c(2984)$ pseudoscalar meson.

The aim of this work is the calculation of the gluon power corrections to SR for the charmonium $^3P_0$-level, $\chi_0(3415)$ scalar meson decays. Model independent estimates of $\chi_0 \to J/\Psi\gamma$ ("electric-dipole" transition) and $\chi_0 \to 2\gamma$, $e^+e^-\gamma$ widths will be obtained in result. The essential difference between these processes and η_c-decays is, that the c-quarks in χ_0--meson interact with the gluon condensate, being in the P-wave.

The interest in these χ_0 decays is sharpened by the fact that the QCD prediction [8]

$$\Gamma(J/\Psi \to \eta_c \gamma)_{QCD} = 1.8 \div 2.0 \text{ keV}$$

turned out to be larger than the only measurement of this width [10]

$$\Gamma(J/\Psi \to \eta_c \gamma)_{EXP} = 0.76 \pm 0.? \text{ keV}.$$

The further improvement of this prediction, with account of $O(\alpha_s)$ corrections, carried out recently in Ref. 11 , did not bring theory closer to experiment. It is quite possible that the width $\Gamma(\eta_c \to 2\gamma) = 4.2 \pm 0.4$ keV, predicted by SR [7], may also exceed the experimental value. In fact, the value of $\Gamma(\eta_c \to 2\gamma) = 2.7 \pm 2.3$ keV, extracted from the measurements of the $p\bar{p} \to \eta_c \to 2\gamma$ process [12], points to that. If new measurements do not change these widths we shall face a serious problem of disagreement between QCD and experiment(see also discussion in Ref. [13]).

Does not this disagreement suggest that there is gluon admixture in η_c-meson, which effectively reduces the radiative widths? Indeed, the investigation of two-gluon states with $J^P = 0^-$ quantum numbers (see e.g. review [14]) shows evidence for strong interaction of these states with the physical vacuum and, in particular, for vigorous mixing with the quark sector. At the same time, not less strong effects are predicted for 0^+-two-gluon states too. Consequently, the check of the obtained below QCD SR predictions for "pure" $c\bar{c}$ scalar states can give an additional information about possible gluon admixture in charmonium.

2. SR for electric-dipole transition $\chi_0 \to J/\Psi\gamma$

The deduction of SR has been presented many times in [6-9], therefore we shall briefly mention the main points only.

If we are interested in the radiative transition $\chi_0 \to J/\Psi\gamma$, then we should consider the three-current vacuum amplitude

$$\Delta_{\mu\nu}(q,p) = \int dx\,dy \exp[-i(qx+py)] \cdot$$

$$\langle 0|T\{j^S(0)\,j_\mu^{em}(x)\,j_\nu^V(y)\}|0\rangle \,. \qquad (1)$$

where $j^S = \bar{c}c$ is the c-quark scalar current; $j_\nu^V = \bar{c}\gamma_\nu c$, $j_\mu^{em} = \bar{c}\gamma_\mu c$ are the c-quark vector and electromagnetic currents with momenta q+p; p,q ($q^2=0$), respectively. The j^S current corresponds to the creation of the χ_0 meson and higher $c\bar{c}$ states with 0^{++} quantum numbers from vacuum. Similarly, the j^V current corresponds to the creation of J/ψ, ψ', ... mesons.

Let us extract from (1) the invariant amplitude, as a function of two kinematic invariants $s_1 = (q+p)^2$ and $s_2 = p^2$:

$$\Delta_{\mu\nu}(q,p) = [\delta_{\mu\nu}(qp) - q_\nu p_\mu]\Delta(s_1,s_2) + \cdots \quad (2)$$

In Eq. (2) we have written out only the kinematical structure which contributes into the $\chi_0 \to J/\psi\gamma$ decay amplitude.

In unphysical region $s_1, s_2 \ll 4m_c^2$, where QCD asymptotic freedom takes place, the amplitude (1) can be calculated in $O(\alpha_s)$ as a sum of a bare c-quark triangle diagram (Fig.1a), diagrams corresponding to the perturbative corrections (one of such diagrams is presented in Fig.1b) and diagrams giving the main power corrections $\sim \alpha_s \langle G^2 \rangle$ (Fig.1c).

The best way of representing the c-quark bare loop contribution is the double dispersion integral over the region $s_1, s_2 \geq 4m_c^2$

$$\Delta(s_1,s_2) = \frac{1}{\pi^2} \iint \frac{\rho(\tilde{s}_1,\tilde{s}_2)\, d\tilde{s}_1\, d\tilde{s}_2}{(s_1 - \tilde{s}_1)(s_2 - \tilde{s}_2)} \quad (3)$$

In the definition of invariant amplitude adopted in (2), the corresponding double spectral function is

$$\rho(s_1,s_2) = \frac{3m_c}{2}\left\{\left[2vs_2 - 4m_c^2 \ln\frac{1+v}{1-v}\right]\delta'(s_1 - s_2) + \right.$$

$$\left. + \ln\frac{1+v}{1-v}\,\delta(s_1 - s_2)\right\}\vartheta(s_1 - 4m_c^2)\,\vartheta(s_2 - 4m_c^2) \quad (4)$$

where $v=(1-4m_c^2/s_1)^{1/2}$, δ' is a derivative of δ -function.

The physical representation of amplitude $\Delta(s_1,s_2)$, following from double dispersion relations on s_1 and s_2 and from unitarity condition, consists of the lowest resonance contribution (χ_0 in 0^{++} channel, J/ψ , ψ' in 1^{--} channel):

$$\Delta^{res}(s_1,s_2)=\frac{\langle 0|j^S|\chi_0\rangle\langle\chi_0|j^{em}|J/\psi\rangle\langle J/\psi|j^V|0\rangle}{(s_1-m_{\chi_0}^2)(s_2-m_\psi^2)}$$

$$+\frac{\langle 0|j^S|\chi_0\rangle\langle\chi_0|j^{em}|\psi'\rangle\langle\psi'|j^V|0\rangle}{(s_1-m_{\chi_0}^2)(s_2-m_{\psi'}^2)} \qquad (5)$$

(Lorentz indices are omitted) and of analogous contribution of continuums with quantum numbers 0^{++} and 1^{--}, sited above the threshold of charmed particles production by j^S and j^V currents, respectively. Following to [1,6] , we replace the continuum contribution by the integral (3) from the bare quark loop over the region of s_1, $s_2 > s_0$ (see Fig.2).

There are several ways to express the approximate coincidence of two representations of amplitude Δ (s_1, s_2): chromodynamical one ((3)+ α_s corrections) and physical one ((5) + contribution of continuum) in the region of asymptotic freedom s_1, $s_2 \ll 4m_c^2$. In the case with heavy quarks it is most convenient to equate a few initial derivatives on s_1 and s_2 of both representations, taken, generally speaking, at an arbitrary point of this region [1,2,5].

For this purpose we introduce the operator

$$\mathcal{D}(n,k;\xi_1,\xi_2)=\frac{1}{n!\,k!}\;\frac{\partial^n}{\partial s_1^n}\;\frac{\partial^k}{\partial s_2^k}\bigg|_{s_1=-\xi_1 m_c^2,\; s_2=-\xi_2 m_c^2} \qquad (6)$$

The action of this operator on both representations of amplitude Δ (s_1, s_2) at each n and k gives the l.h.s. and r.h.s. of (nk)-th moment of double SR. The exact form of these SR is the following:

$$A(\chi_0 \to J/\psi\gamma) + \frac{g_{\psi'}}{g_\psi}\left(\frac{m_\psi}{m_{\psi'}}\right)^{2K}\left(\frac{1+m_c^2\xi_2/m_\psi^2}{1+m_c^2\xi_2/m_{\psi'}^2}\right)^{K+1} A(\psi' \to \chi_0\gamma) =$$

$$\frac{3m_\psi^{2K}\, m_{\chi_0}^{2n+1}}{\pi^2 g_{\chi_0} g_\psi\, m_c^{2n+2K+1}}\left(1+m_c^2\xi_1/m_{\chi_0}^2\right)^{n+1}\left(1+m_c^2\xi_2/m_\psi^2\right)^{K+1} A_{nK}(\xi_1,\xi_2)\times\left\{1+O(\alpha_s)\right\} \tag{7}$$

where the coefficients A_{nk} are determined by the integration of double spectral function (4) with a weight of $(s_1+\xi_1 m_c^2)^{-n-1}\times$ $(s_2+\xi_2 m_c^2)^{-K-1}$ over the region $4\,m_c^2 \leqslant s_1,\ s_2 \leqslant s_0$. So in these coefficients the continuum contribution is already taken into account.

In the l.h.s. of (7) the following notations of physical matrix elements included in representation (5), have been introduced

$$\begin{aligned}
\langle 0|j^S|\chi_0\rangle &= g_{\chi_0} m_{\chi_0}^2 \\
\langle 0|j_\nu^V|J/\psi\rangle &= g_\psi m_\psi^2 \psi_\nu \\
\langle 0|j_\nu^V|\psi'\rangle &= g_{\psi'} m_{\psi'}^2 \psi'_\nu
\end{aligned} \tag{8}$$

$$\begin{aligned}
\langle\chi_0|j_\mu^{em}|J/\psi\rangle &= [\delta_{\mu\nu}(qp)-q_\nu p_\mu]\,A(\chi_0\to J/\psi\gamma)\,m_{\chi_0}^{-1}\psi_\nu \\
\langle\chi_0|j_\mu^{em}|\psi'\rangle &= [\delta_{\mu\nu}(qp)-q_\nu p_\mu]\,A(\psi'\to\chi_0\gamma)\,m_{\chi_0}^{-1}\psi'_\nu
\end{aligned} \tag{9}$$

where $\Psi_V(\Psi'_V)$ is a wave function of $J/\Psi(\Psi')$. g_Ψ, g_{χ_0} constants are independently determined from one dimensional SR [2,5] for two-current correlators ($\langle j^V j^V \rangle$ and $\langle j^S j^S \rangle$ correspondingly). Using the results of [5] we have calculated these constants with the required accuracy $O(\alpha_s)$

$$|g_\Psi| = 0.115, \quad |g_{\chi_0}| = 0.095 \tag{10}$$

The ratio $|g_{\Psi'}/g_\Psi| = 0.6$ can be extracted from the experimental values of leptonic widths $J/\psi \to e^+e^-$, $\Psi' \to e^+e^-$.

The amplitudes $A(\chi_0 \to J/\Psi\gamma)$ and $A(\Psi' \to \chi_0\gamma)$ determine the widths of radiative transitions we are interested in:

$$\Gamma(\chi_0 \to J/\Psi\gamma) = \frac{\alpha Q_c^2}{8} m_{\chi_0}(1 - m_\Psi^2/m_{\chi_0}^2)^3 |A(\chi_0 \to J/\Psi\gamma)|^2$$

$$\Gamma(\Psi' \to \chi_0\gamma) = \frac{\alpha Q_c^2}{24} \frac{m_{\Psi'}^3}{m_{\chi_0}^2}(1 - m_{\chi_0}^2/m_{\Psi'}^2)^3 |A(\Psi' \to \chi_0\gamma)|^2 \tag{11}$$

($Q_c = 2/3$ is the charge of c-quark).

The use of SR may be considered as successful if there exists a set of optimal moments in physical part of which one lowest amplitude dominates (in (7) it is $A(\chi_0 \to J/\Psi\gamma)$, while in their QCD part $O(\alpha_s)$ corrections are not so large yet. In order to find out if there are such moments for transitions considered here, it is necessary, first of all, to calculate the gluon power corrections which strongly depend on the momen. numbers (n k).

3. The calculation of gluon condensate contribution

The diagrams in Fig.1c, corresponding to the interaction of virtual c-quark with the vacuum gluon field, give the main power corrections of $\alpha_s\langle G^2\rangle/m_c^4$ order to r.h.s. of SR (7). We shall calculate these diagrams by the standard method of fixed point gauge (see e.g. Ref.[15]) which allows rather quickly derive an expression explicitly containing the vacuum average $\langle G^2\rangle$. The general form of the sum over six diagrams shown in Fig.1c is

$$\Delta^G_{\mu\nu} = \left(-\frac{\langle\alpha_s G^2\rangle}{3\cdot 2^6\cdot\pi^3}\right)\left(\delta_{\lambda\rho}\delta_{\alpha\beta}-\delta_{\lambda\beta}\delta_{\alpha\rho}\right)\times$$

$$\times\int d^4f\left\{\frac{\partial^2}{\partial u_\lambda\partial v_\rho}\sum_{i=1}^{6} Sp_{(i)\mu\nu\alpha\beta}(f,u,v,q,p)\right\}\Bigg|_{u=v=0} \qquad (12)$$

where, e.g., for the first diagram in Fig.1c

$$Sp_{(1)\mu\nu\alpha\beta} = Sp\left\{\gamma_\mu\frac{1}{\hat f+\hat q+\hat p+\hat v-m_c}\gamma_\alpha\frac{1}{\hat f+\hat q+\hat p+\hat u+\hat v-m_c}\cdot\frac{1}{\hat f-m_c}\times\right.$$

$$\left.\gamma_\nu\frac{1}{\hat f+\hat p-m_c}\;\gamma_\beta\;\frac{1}{\hat f+\hat p+\hat v-m_c}\right\}$$

etc. Then the problem comes to the calculation of Feynman integrals included in (12) which we have carried out using the " REDUCE" analytic manipulation programme.

After extraction of the invariant amplitude (2) we succeeded in getting the result i.e. the r.h.s. of (12) as a double integral over parameters

$$\Delta^G(s_1,s_2) = -\frac{3\Phi}{\pi^2 m_c}\int_0^1 dx\int_0^x dy$$

$$\left\{\frac{1}{\text{æ}^3}\left[-4(101)+2(020)+8(110)+8(011)-6(300)+\right.\right.$$

$$+(120)-20(111)+4(201)-6(210)+4(102)-$$

$$-6(012)+(021)-6(003)+12(301)+12(121)+$$
$$+24(211)+24(112)+24(202)+12(103)]+$$
$$+\frac{1}{\ae^4}[6(300)+6(003)-3(030)-9(120)-9(021)-$$
$$-18(111)+48(211)-48(121)+48(112)+36(122)+$$
$$+72(212)+36(221)-24(401)+12(131)-24(104)]-$$
$$-\frac{\xi_1}{\ae^4}[-6(111)+3(221)+3(122)-24(212)-$$
$$-\frac{\xi_2}{\ae^4}[-3(221)-15(122)+9(021)+12(112)+3(121)-3(022)]-$$
$$-\frac{\xi_1-\xi_2}{\ae^5}[24(121)-96(222)]\Big\} \qquad (13)$$

where the following definitions are introduced:

$$\Phi=\frac{4\pi^2}{9}\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle(4m_c^2)^{-2},$$

$$\ae=1-\xi_1(x-y)(1-x)-\xi_2(x-y)y,$$

$$\xi_{1,2}=-s_{1,2}/m_c^2\ ;\quad(\alpha,\beta,\gamma)=(1-x)^\alpha y^\beta(x-y)^\gamma$$

Then it is necessary to act with operator (6) on this integral. It is more convinient to do this before the integration over the parameters. At $s_1=s_2=0$ the result may be obtained in analytical form. We present the final expression for SR moments (7) at $s_1=s_2=0$ accounting for gluon power corrections

$$A(\chi_0\to J/\psi\gamma)+\frac{g_{\psi'}}{g_\psi}\left(\frac{m_\psi}{m_{\psi'}}\right)^{2k}A(\psi'\to\chi_0\gamma)+\ \ldots=$$

$$= \frac{3}{\pi^2} \frac{m_\psi^{2k} m_{\chi_0}^{2n+1}}{g_\psi g_{\chi_0} m_c^{2n+2k+1}} \frac{(n+k)!\,(n+k)!}{(2n+2k+4)!}\left[(2k+3)(n+k+1)+1\right] \cdot \quad (14)$$

$$\left\{ 1 + c_{nk}^{S}\,\phi + \cdots \right\}$$

$$c_{nk} = -(n+k+1)(n+k+2)(2n+2k+5)^{-1}\left[(2k+3)(n+k+1)+1\right]^{-1} \cdot$$

$$\left[2k^4 + 10k^3 n + 14k^2 n^2 + 6kn^3 + 17k^3 + 63k^2 n + \right.$$

$$\left. + 55kn^2 + 9n^3 + 53k^2 + 124kn + 43n^2 + 68k + 64n + 24\right] \quad (15)$$

As follows from (15), the values of c_{nk}^{S} at large n,k grow $\sim(n+k)^3$ and have an opposite sign with respect to zeroth approximation. There is a full correspondence with the analogous coefficients c_{nk}^{P} which determine the power corrections to pseudoscalar- vector triangle $J/\Psi \to \eta_c \gamma$ [8,9]. At the same time, at equal n,k , the numerical values $c_{nk}^{S} \gg c_{nk}^{P}$ (see Fig.5). This fact agrees to the relation of power corrections to scalar and pseudoscalar polarization operators obtained in Ref.[2] . It means, in essence, that nonperturbative effects for c-quarks in P-wave (χ_0) manifest themselves stronger than in S-wave (η_c).

The dots in l.h.s. and r.h.s. of Eq. (14) stand correspondingly for the contribution of higher continuum states and perturbative α_s corrections i.e. diagrams of Fig.1b type.

The calculation of the latter diagrams is a much more complicated computational problem than the determination of gluon condensate contribution. In Ref. [1,2,5] the perturbative corrections for two-current correlators and, in Ref. [11] , for the pseudoscalar-vector triangle diagram at $s_1 = s_2 = 0$ were calculated. The results of these works brought us to the following "empirical" conclusion: normalizing the virtual current c-quark mass at the point $p^2 = -m_c^2$ [1,2,5] the value of α_s-correction does not strongly depend on the choice of currents (pseudoscalar, vector, scalar) and for the "optimal" moments of SR it changes in the limits of ±10% from the r.h.s.(QCD side) of SR. The same accuracy for numerical results obtained below is expected.

4. Can the amplitude of $\chi_0 \to J/\Psi\gamma$ be extracted from sum rules?

First of all one should give the numerical value of parameters included in SR. It is remarkable that all of them can be independently determined by two-current SR [1,2,5]: g_Ψ, g_{χ_0} are given in (10), $m_c(p^2 = -m_c^2) = 1.28$ GeV, $\Phi = 1.35\ 10^{-3}$. As it has been noted above, even the masses of lower resonances J/Ψ, χ_0 can be calculated from SR [5] without using their experimental values. One can talk about a definite hierarchy of SR. Two-current correlators give the characteristics of lower resonances and QCD parameters which then are substituted into three-current SR for decay amplitude determination.

From here on, we shall select only those SR moments for which the continuum contribution in the above determined approximation of quark-hadron duality does not exceed 10% of the right hand(QCD) side of SR. That is why the results would weakly depend on the continuum threshold $\sqrt{s_0}$ which is chosen for definiteness to be $\sqrt{s_0} = 4.0$ GeV.

When substituting the values of SR (14) parameters it turns out that those moments into which the continuum contribution is still small ($n+k \geqslant 4$), are characterized by too large value of power correction, $c_{nk}\Phi > 40\%$ (see Fig.3). The way out of this situation is prompted in Ref.[5] where all two-current SR for charmonium levels were considered in the range of negative q^2.

Getting on deeper into the region of asymptotic freedom naturally reduces the power correction. If one does not go too far then the relative contribution of ψ' and that of continuum into l.h.s. of SR are not so large and one may choose optimal moments.

We have made an analysis of SR (7) in a general case when s_1, $s_2 < 0$ and for simplicity chosen the point $s_1=s_2= -4m_c^2$ (see Fig.2). Some details of this analysis are as follows.

In order to calculate the power corrections to SR (7) one should go back to integral expression (13) and act upon it by operator (6) at $\xi_1 = \xi_2 = 4$. In this case a numerical integration over the parameters is made, since there is no analytical answer for the moments. As was to be expected, the c^S_{nk} coefficients at $s_1=s_2= -4m_c^2$ are much smaller than at $s_1=s_2=0$ (see Fig.3), so that in the whole range of $n+k \leqslant 10$ the value $c^S_{nk}\Phi \lesssim 30\%$. At the same time, beginning with $n+k=8$, the continuum contribution turns out to be quite small (10% at $\sqrt{s_0}= 3.7$ GeV and $< 5\%$ at $\sqrt{s_0}= 4.0$ GeV) so that the moments of SR (7) in the interval $8 \leqslant n+k \leqslant 10$ do satisfy all necessary requirements.

Yet, these moments have a disadvantage. They involve too high degrees of c-quark mass m_c, a parameter which is known with accuracy of $O(\alpha_s)$ (according to Ref. [5] m_c= 1.25 GeV at $s_1=s_2=-4m_c^2$). To improve the situation, we shall not deal with SR (7) themselves but consider their combinations with the n-th moment of SR for two-current correlator $< j^S j^S >$ and the k-th moment of SR for $< j^V j^V >$. At the point $q^2=-\xi m_c^2$ these SR have the form [5]

$$g_{\chi_0}^2 = A_n^S(\xi)\left(\frac{m_{\chi_0}}{m_c}\right)^{2n}\left[1+b_n^S(\xi)\Phi+\ \ldots\ \right]\left(1+\frac{m_c^2\xi}{m_{\chi_0}^2}\right)^{n+1} \tag{16}$$

$$g_\psi^2+\left(\frac{m_\psi}{m_{\psi'}}\right)^{2k}\left(\frac{1+m_c^2\xi/m_\psi^2}{1+m_c\xi/m_{\psi'}^2}\right)^{k+1} g_{\psi'}=$$

$$=A_k^V(\xi)\left(\frac{m_\psi}{m_c}\right)^{2k}\left(1+m_c^2\xi/m_\psi^2\right)^{k+1}\left[1+b_k^V(\xi)\Phi+\cdots\right] \tag{17}$$

where the continuum contribution is involved in the coefficients A_n^S, A_k^V. The optimal moments of these SR at $\xi=4$ are in the interval $4\leqslant n,k\leqslant 8$ (continuum contribution is < 10%, power correction in the r.h.s. is < 30%). As it has been argued above the perturbative correction for these moments is less than 10%.

Let us devide the (nk)-th moment of SR (7) at $\xi_1=\xi_2=\xi$ by the product of (16) and (17). The final expression

$$A(\chi_0\to J/\Psi\gamma)+\frac{g_{\psi'}}{g_\psi}\left(\frac{m_\psi}{m_{\psi'}}\right)^{2k}\left(\frac{1+m_c^2\xi/m_\psi^2}{1+m_c^2\xi/m_{\psi'}^2}\right)^{k+1}A(\psi'\to\chi_0\gamma)=$$

$$=\frac{3}{\pi^2}g_\psi g_{\chi_0}\frac{m_{\chi_0}}{m_c}\frac{A_{nk}}{A_n^S A_k^V}\left[1+\frac{g_{\psi'}^2 m_\psi^{2k}(1+m_c^2\xi/m_\psi^2)^{k+1}}{g_\psi^2 m_{\psi'}^{2k}(1+m_c^2\xi/m_{\psi'}^2)^{k+1}}\right]\cdot$$

$$\cdot\left[1+\Phi(c_{nk}^S-b_n^S-b_k^V)+\ \ldots\ \right]. \tag{18}$$

is valid for those (nk), n and k for which corrections to the moments of SR (7), (16) and (17), respectively, are small. One may assume that the perturbative α_s -corrections are, at least, partially cancelled in the r.h.s. of (18). Six moments of SR (18): (nk)= (44), (54), (64), (45), (55), (46) which can be considered as linear equations for amplitudes $\mathcal{A} \equiv \mathcal{A}(\chi_0 \to J/\psi\gamma)$ and $\mathcal{A}' \equiv \mathcal{A}(\psi' \to \chi_0\gamma)$, satisfy the above determined conditions of $8 \leqslant n+k \leqslant 10$, $4 \leqslant n,k \leqslant 8$. After the numerical values were substituted in these equations, they turned out to be almost degenerate. The coefficients at A' and the r.h.s. of (18) occured to be the same within $\sim$20% i.e. within the limits of the expected accuracy of the obtained SR ($\pm$ 10% - contribution of perturbative corrections, $\pm$ 10% - the ambiguity of continuum saturation). The least squared average over six moments is

$$\mathcal{A} \pm 0{,}24\mathcal{A}' = 5{,}1 \tag{19}$$

The $\pm$ sign reveals the uncertainty of $g_\psi \mathcal{A}$ and $g_{\psi'} \mathcal{A}'$ products relative sign.

So, the SR predict a definite, stable over a few moments combination of amplitudes A and A'. At the same time, the radiative transition $\psi' \to \chi_0\gamma$ enters this combination with appreciable weight. The impossibility to extract the main amplitude $\chi_0 \to J/\psi\gamma$ is a dirrect consequence of relatively large nonperturbative effects for a triangle amplitude containing c-quarks in P-wave.

If one substitutes in Eq. (19) the experimental value

$|A'|=3.3 \pm 0.6$ determined, according to (11), from experimentally measured width $\Gamma(\Psi' \to \chi_0 \gamma) = 17 \pm 6$ keV [10,16], then the width

$$\Gamma(\chi_0 \to J/\Psi\gamma) = 205 \pm 60 \text{ keV} \tag{20}$$

where the sign +(-) corresponds to the choice of relative sign -(+) in (19). The experimental value of this width, measured by CRYSTAL BALL group [10], is: $\Gamma(\chi_0 \to J/\Psi\gamma) = 97 \pm 38$ keV. So, it is not excluded that with $\chi_0 \to J/\Psi\gamma$ decay as well as with $J/\Psi \to \eta_c\gamma$ one we run into the same situation: the width extracted from QCD SR *is larger than the experimental one. New measurements of* $\chi_0 \to J/\Psi\gamma$ would allow to make a more distinct conclusion. It is a pity that in the framework of SR method one can not fix the sign in (20). At the same time, there exist, at least, two arguments in favour of a negative sign in (19) i.e. in favour of taking the upper limit of the interval (20):

1) in charmonium nonrelativistic model products $g_\Psi A$ and $g_{\Psi'} A'$ are the products of wave functions $J/\Psi(1^3S_1)$ and $\Psi'(2^3S_1)$ in the origin and the dipole overlapping integrals $1P \to 1S$ and $2S \to 1P$, respectively. In Ref.[17] these products have been proved to have opposite signs regardless the type of "confinement" potential.

2) the carried out in Ref.[18] analysis of SR (7) in zeroth approximation over α_s at $s_1 = s_2 = 0$ $(n,k > 0)$ and $s_1 = s_2 = -\infty$ $(n,k < 0)$ using local quark-hadron duality (it was assumed that the contribution of amplitudes A and A$'$ is dual to the quark loop in the interval $4m_c^2 \leqslant s_1, s_2 \leqslant (3.7 \text{ GeV})^2$) also predicts a minus sign for $g_{\Psi'} A' / g_\Psi A$.

5. The $\chi_0 \to 2\gamma$ decay.

The obtained above SR (7) for amplitude $\Delta_{\mu\nu}$ permit to calculate at once the two-photon decay width $\chi_0 \to 2\gamma$. For this, it is necessary to take $s_2 = p^2 = 0$ in Eq. (1) from the very beginning, and consider one-variable SR for triangle amplitudes with two real photon vertices ($q^2 = p^2 = 0$) and a virtual scalar vertex ($(q + p)^2 = s_1$). In zeroth approximation over α_s these SR were already obtained in Ref. [1]. Here we shall improve them by adding the gluon power correction.

The final form of SR at $s_1 = 0$ is

$$\mathcal{A}(\chi_0 \to 2\gamma) + \dots = \frac{3}{\pi^2 g_{\chi_0}} \left(\frac{m_{\chi_0}}{m_c}\right)^{2n+1} \frac{(n!)^2}{(2n+4)!} (3n+4) \cdot$$

$$\cdot \left(1 + c_n^s \phi + \dots\right), \qquad (21)$$

$$c_n^s = -\frac{(n+1)(n+2)}{(2n+5)(3n+4)} (9n^3 + 43n^2 + 64n + 24)$$

where the amplitude $\mathcal{A}(\chi_0 \to 2\gamma)$ determines the decay with

$$\Gamma(\chi_0 \to 2\gamma) = \frac{1}{4} \pi \alpha^2 Q_c^4 m_{\chi_0} |\mathcal{A}(\chi_0 \to 2\gamma)|^2 \qquad (22)$$

Here, as in the case with radiative transitions, the point $s_1 = 0$ is not convenient for amplitude extraction. One should go to region $s_1 < 0$ i.e. come back to more general expressions (7), (13) at $\xi_2 = 0, k = 0$. Repeating the procedure described in the preceding section, including combination with SR (17) we have extracted at $s_1 = -4m_c^2$ the interval of optimal momenta $5 \leqslant n \leqslant 8$. The conditions of small power correction (< 30%) and continuum contribution (< 10%) are at the same time valid for these moments. The predicted amplitude

$$\mathcal{A}(\chi_0 \to 2\gamma) = 0{,}33 \pm 0{,}02$$

according to (22) corresponds to the width

$$\Gamma(\chi_0 \to 2\gamma) = 3{,}0 \pm 0{,}4\, keV$$

Only the upper limit is experimentally measured up to now [16]:

$$\Gamma(\chi_0 \to 2\gamma) < 27\, keV$$

The decay $\chi_0 \to 2\gamma$ is closely related with the lepton decay $\chi_0 \to e^+e^-\gamma$ in the nonresonance region of lepton pair masses $m_{e^+e^-} < m_\psi$. The phase space volume of this decay, in fact, is cut off already at $m_{e^+e^-} \sim 1$ GeV. At the first glance it seems that during the calculation of $\chi_0 \to e^+e^-\gamma$ width one must take the form factor over virtual photon into account. The above obtained SR in the general case of $s_2 \neq 0$ permit accurately determine this form factor as a function of $m^2_{e^+e^-} = s_2$ at small $s_2 > 0$ (up to $\sqrt{s_2} \sim 1$ GeV the asymptotic freedom condition $s_2 \ll 4m_c^2$ is well satisfied):

$$A(\chi_0 \to e^+e^-\gamma) \simeq A(\chi_0 \to 2\gamma)\left[1 + 0{,}1\frac{m^2_{e^+e^-}}{m_c^2} + \cdots\right] \quad (23)$$

As a result the approximation $A(\chi_0 \to e^+e^-\gamma) \simeq A(\chi_0 \to 2\gamma)$ at $m_{e^+e^-} < 1$ GeV is valid with good accuracy. In this approximation the width

$$\Gamma(\chi_0 \to e^+e^-\gamma;\ m_{e^+e^-} < 1 GeV) \simeq \frac{4\alpha}{3\pi}\, I\, \Gamma(\chi_0 \to 2\gamma)$$

where the numerical factor $I = 5.3$ arises from the integration over the phase space volume. Hence, the "nonresonance conversion" factor of χ_0-meson is equal to

$$\Gamma(\chi_0 \to e^+e^-\gamma) / \Gamma(\chi_0 \to 2\gamma) \simeq 1/60$$

Correction to this value, determined from (23) is no more than 0.1%.

In conclusion, an essential difference between the $\chi_0 \to 2\gamma$ decay and the analogous pseudoscalar charmonium decay $\eta_c \to 2\gamma$ should be mentioned. As it was noted in Ref. [13] the latter decay is connected with the radiative decay $J/\Psi \to \eta_c \gamma$ by a simple vector dominance relation, the QCD SR predicting a small contribution of higher transitions ($\Psi' \to \eta_c \gamma$ etc.) into this relation. It is easy to be convinced that for scalar charmonium situation is just the inverse. Indeed, the vector dominance relation is reproduced if, using the coincidence of the right hand (QCD) sides of SR (14) at $s_2 = 0$ ($k = 0$) and SR (21) one equates their left hand (physical) sides:

$$A(\chi_0 \to 2\gamma) = g_\psi A(\chi_0 \to J/\Psi\gamma) + g_{\psi'} A(\Psi' \to \chi_0 \gamma) + \ldots \qquad (24)$$

The dots denote the contribution of continuum states. The fact, that in SR (14) at $k \gg 1$ the contribution of $\Psi' \to \chi_0 \gamma$ is still significant, unambiguously means that this contribution (and may be the contribution of higher states Ψ'' , $D\bar{D}$ etc. as well) is much more significant in (24). So, from the QCD point of view one must expect a strong violation of $\chi_0 \to J/\Psi\gamma$ dominance in the relation (24).

The authors would like to thank A. Ts. Amatuni and S.G. Matinyan for their stimulating interest in this work.

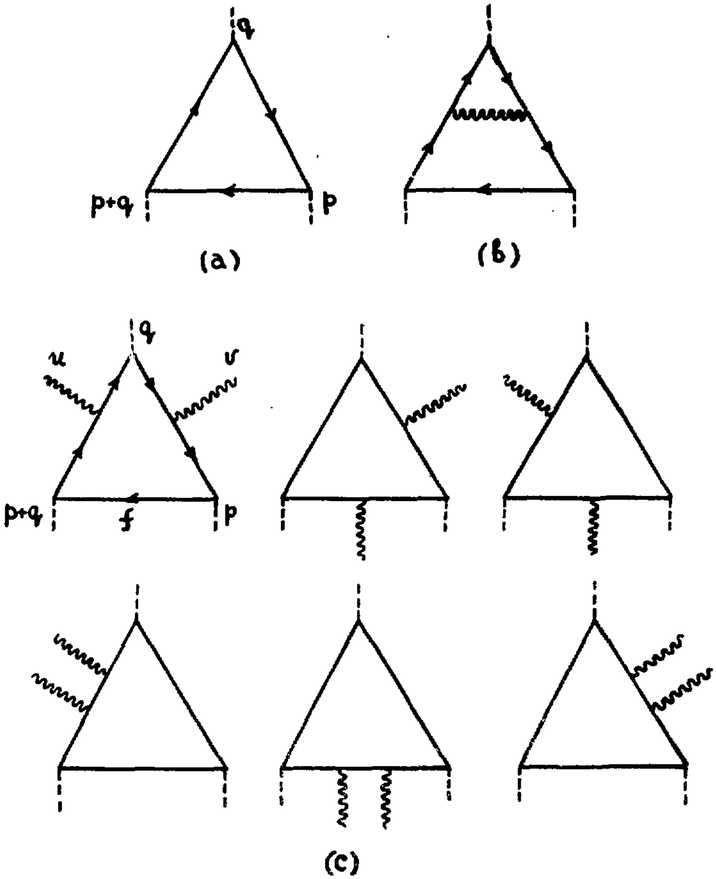

Fig. 1 QCD diagrams corresponding to the zeroth (a) and the first (b,c) power of three-current amplitude (1) over α_s. The vertices j^s, j^{em}, j^V carry external momenta p+q, q,p, respectively.

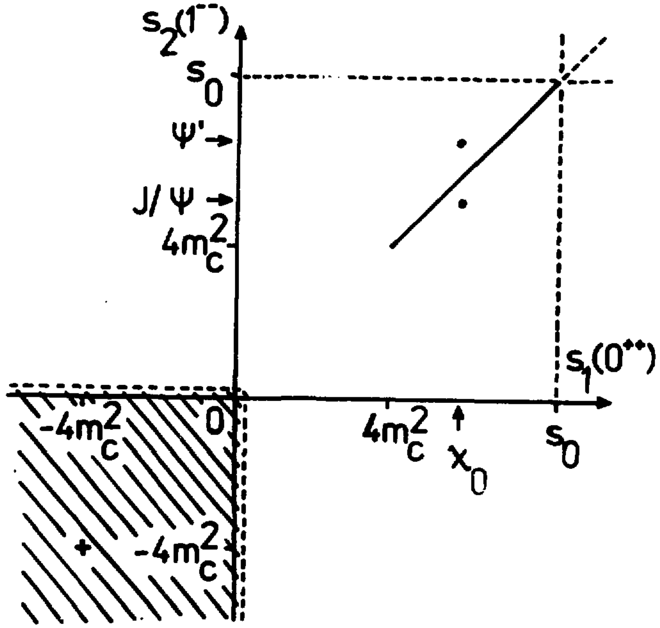

Fig.2 The plane of kinematical invariants s_1, s_2 of triangle amplitude. The dots show the dislocation of resonance contributions corresponding to the main transitions $\chi_0 \to J/\Psi\gamma$, $\Psi' \to \chi_0\gamma$. The diagonal $s_1 = s_2 > 4m_c^2$ is the integration region in the double-dispersion integral (3), corresponding to the contribution of bare c-quark loop. The solid line (dotted line) indicates a part of this diagonal which is dual to resonance (continuum) contribution. The region of asymptotic freedom $s_1, s_2 \ll 4m_c^2$ is shaded. The point $s_1 = s_2 = -4m_c^2$, in which the SR (7) were considered, is indicated by a cross.

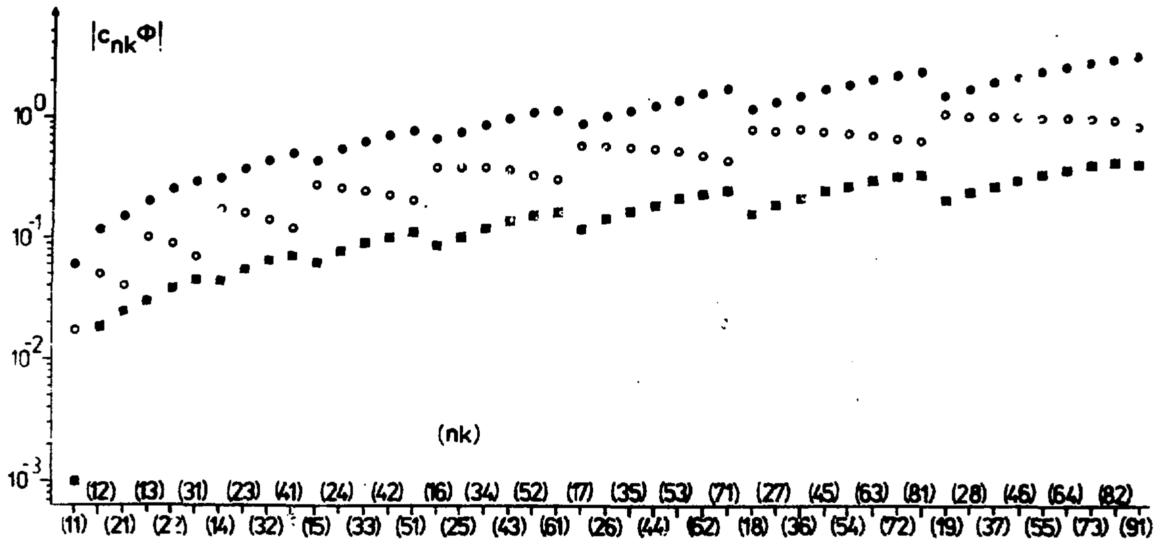

Fig.3 The absolute value of gluon power correction $|C^S_{nk}\,\Phi|$ to r.h.s. of SR (7): at $s_1 = s_2 = 0$ (●) and at $s_1 = s_2 = -4m_c^2$ (■) for (nk) moments ($n + k \leqslant 10$). The gluon condensate parameter $\Phi = 1.35 \; 10^{-3}$. Analogous corrections $|C^P_{nk}\,\Phi|$ to SR for $J/\psi \to \eta_c \gamma$ decay at $s_1 = s_2 = 0$ (o) are represented for comparison [8] .

REFERENCES

1. Novikov V.A. et al, Charmonium and Gluons. Phys. Rep., 1978, v. 41, p. 1-133.
2. Shifman M.A. et al, QCD and Resonance Physics. Nucl. Phys., 1979, v. B147, p. 385-448.
3. Reinders L.J. et al, QCD Spectroscopy and Couplings with Sum Rules: an Overview. Preprint CERN-TH-3767, 1983.
4. Ioffe B.L. Status of QCD. Proc. of XXII International Conference on High Energy Physics, Leipzig, 1984, v.2, p.176-201.
5. Reinders L.J. et al, QCD Sum Rules for Heavy Quark Systems. Nucl. Phys., 1981, v. B186, p. 109-121.
6. Khodjamirian A.Yu. Dispersion Sum Rules for the Amplitudes of Radiative Transitions in Quarkonium. Phys. Lett., 1980, v. 90B, p. 460-464.
7. Reinders L.J. et al, Decays of Heavy Quark Systems: Effects of Gluon Condensate. Phys. Lett., 1982, v.113B, p. 411-414.
8. Khodjamirian A.Yu. On the Calculation of $J/\psi \to \eta_c \gamma$ Width in QCD. Yad. Fiz. 1984, v.39, p. 970-976 (in Russian).
9. Beilin V.A., Radyushkin A.V. Analysis of the Decay $J/\psi \to \eta_c \gamma$ by the Method of QCD Sum Rules. Yad. Fiz., 1984, v.39, p. 1270-1274 (in Russian).
10. Gaiser J.E. Charmonium Spectroscopy from Inclusive Photons in J/ψ and ψ' Decays. Preprint SLAC-PUP-2887, 1982.
11. Beilin V.A., Radyushkin A.V. Quantum Chromodynamics Sum Rules and $J/\psi \to \eta_c \gamma$ Decay. Preprint JINR P2-84-557, Dubna, 1984 (in Russian).

12. Baglin C. et al, Formation of Charmonium States in Antiproton-Proton Annihilation. Preprint CERN-EP-84-145, 1984.

13. Shifman M.A. Theory of Heavy Quark-Antiquark States.Proc. of the 1981 International Symposium on Lepton and Photon Interactions at High Energies.Ed. W. Pfeil, Bonn,1981, p. 242-278.

14. Novikov V.A. et al, Are All Hadrons Alike? Preprint ITEP-42, 1981.

15. Smilga A.V. Calculation of Degree Corrections in Fixed--Point Gauge. Yad. Fiz., 1982, v.35, p. 473-484 (in Russian).

16. Particle Data Group. Particle Properties. Rev. Mod. Phys., 1984, v.56, p.2.

17. Martin A. The Sign of the Electric Dipole Matrix Element 2S → 1P. Preprint CERN-TH-2684, 1979.

18. Khodjamirian A.Yu. Radiative Transitions in Quarkonium and Quantum Chromodynamics. Preprint EPI-427-34, Yerevan,1980.

The manuscript was received 4 September 1985

Л.С.ДУЛЬЯН, А.Г.ОГАНЕСЯН, А.Ю.ХОДЖАМИРЯН

РАДИАЦИОННЫЕ РАСПАДЫ P - УРОВНЕЙ ЧАРМОНИЯ В КХД

(на английском языке, перевод Г.А.Папяна)

Редактор Л.П.Мукаян
Технический редактор А.С.Абрамян

Подписано в печать I9/XII-85г. ВФ-09202 Формат 60x84/I6
Офсетная печать. Уч.изд.л. I,5 Тираж 299 экз. Ц. 22к.
Зак.тип. № 60I Индекс 3624

Отпечатано в Ереванском физическом институте
Ереван 36, Маркаряна 2

индекс 3624

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ