Preprint ЕФИ-842(69)-85

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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

I.A.BAGDASSARYAN

NON-LOCAL GAUGE FIELD THEORY

ЦНИИатоминформ

ЕРЕВАН-1985

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NON-LOCAL GAUGE FIELD THEORY

The non-local quantum field theory, finite at a fixed elementary length ℓ and based on the non-abelian gauge group $SU(N)\otimes U(1)$, is built up. A principle possibility of calculating the form factor is shown. As an illustration the vacuum polarization operator is considered.

Yerevan Physics Institute

Yerevan 1985

Препринт ЕФИ-842(69)-85

И.А.БАГДАСАРЯН

НЕЛОКАЛЬНАЯ КАЛИБРОВОЧНАЯ ТЕОРИЯ ПОЛЯ

На основе неабелевой калибровочной группы $SU(N)\otimes U(1)$ построена конечная (при фиксированной элементарной длине ℓ) нелокальная квантовая теория поля. Показана принципиальная возможность вычисления формфактора. В качестве иллюстрации рассмотрен оператор поляризации вакуума.

Ереванский физический институт
Ереван 1985

1. Introduction

In any contemporary local quantum field theory the action usually contains only a limited number of interactions. Even Grand Unified theories, "Radical Unification", supersymmetric models and others pretending to exhaustively describe all(!?) interactions are not secure against the fact that at distances much smaller than those approved by experiment (where, in fact, the experimental information necessary for such constructions comes from), new interactions evidently unwritten in the action are possible. But as at quantization of any field theory due to inevitable virtual states of the system arbitrary space-time intervals are sounded, one is to take these interactions into account at least in some "averaged" form for exclusiveness and self-consistency of the theory. Therewith, it is possible that such "averaging" function can be determined uniquely. If so, we would deal with a universal function coupled, in a sense, with obvious manifestation of vacuum structure.

Let us consider the local gauge transformations of $SU(N)\otimes U(1)$ symmetry group

$$\left.\begin{aligned} \Psi_i'(x) &= \exp\left(i\frac{\omega(x)}{\sqrt{2N}}\right)U_{ij}(x)\Psi_j(x) \\ \Psi_i^{+\prime}(x) &= \exp\left(-\frac{i\omega(x)}{\sqrt{2N}}\right)\Psi_j^{+}(x)U_{ji}^{+}(x) \end{aligned}\right\} \tag{1.1}$$

$$
\left.\begin{aligned}
&B'_\mu(x) = B_\mu(x) + \frac{1}{g}\partial_\mu \omega(x) \\
&B^{a\prime}_\mu(x) T^a_{ij} = U_{ik}(x) B^a_\mu(x) T^a_{k\ell} U^+_{\ell j}(x) - (i/g)\partial_\mu U_{ik}(x) U^+_{kj}(x) \\
&(i = 1, \dots, N;\ a = 1, \dots, N^2 - 1)
\end{aligned}\right\} \qquad (1.2)
$$

where

$$
U_{ij}(x) = \left(\exp(i\omega^a(x) T^a)\right)_{ij} \qquad (1.3)
$$

$\omega(x)$, $\omega^a(x)$ are arbitrary functions, T^a_{ij} are generators of $SU(N)$ group, g is a constant, and $\Psi_i(x)$, $\Psi^+_i(x)$ and $B_\mu(x)$, $B^a_\mu(x)$ are spinor and gauge fields, correspondingly. Proceeding from the aforesaid these transformations are to be "averaged" because the whole theory is based on them. Let us define the "averaging" operation as follows:

$$
\langle \Psi_i(x) \rangle \overset{def}{\equiv} \psi_i(x) \qquad (1.4)
$$

$$
\langle \omega_1(x) \cdots \omega_n(x) \Psi_i(x) \rangle \overset{def}{\equiv} \int \rho(x, x', x_s)\, \omega_1(x_s) \langle \omega_2(x') \cdots \omega_n(x') \Psi_i(x') \rangle\, d^4x'\, d^4x_s \qquad (1.5)
$$

where $\psi_i(x)$ is the "averaged" field of the spinoral particle; $\omega_1(x), \dots, \omega_n(x)$ are "phase" fields entering the expressions (1.1) $\div$ (1.3); $\rho(x, x', x_s)$ is the "averaging" function, depending on some elementary length ℓ outlining the "black box" of the theory ($K\ell \geq 1$, where K is the particle momentum). As a result $\psi'_i(x)$ transforms in the following way:

$$
\psi'_i(x) = \psi_i(x) + i\int \rho(x, x', x_s)\left[\frac{1}{\sqrt{2N}}\delta_{ij}\,\omega(x_s) + T^a_{ij}\,\omega^a(x_s)\right]\psi_j(x') + \dots \qquad (1.6)
$$

[illegible] repeating space-time variables are ment to be [illegible]

"2. Non-local gauge invariance. Action.

Thus, we have the following non-local gauge transformations of the spinoral field [1-4] :

$$\left.\begin{aligned} \psi_i'(x) &= \Lambda_{ij}(x,x')\psi_j(x') \\ \psi_i^{+'}(x) &= \psi_j^{+}(x')\Lambda_{ji}^{+}(x',x) \end{aligned}\right\} \tag{2.1}$$

$$\Lambda_{ij}^{+}(x,x') \equiv \Lambda_{ji}^{*}(x',x) \tag{2.2}$$

where $\Lambda_{ij}(x,x')$ is determimed in accordance with (1.6). The requirement of invariance of the scalar product $\langle\psi_i|\psi_i\rangle \equiv \psi_i^{+}(x)\psi_i(x)$ in Gilbert space relative to (2.1) transformations reduces to the unitarity correlation

$$\Lambda_{ik}(x,x')\Lambda_{kj}^{+}(x',x'') = \delta_{ij}\,\delta(x-x'') \tag{2.3}$$

For conformity with the local theory one needs also that when $\ell \to 0$

$$\Lambda_{ik}(x,x') \to \Lambda_{ik}(x)\delta(x-x') \qquad \text{(N.I.)} \tag{2.4}$$

$$\Lambda_{ik}(x)\Lambda_{kj}^{+}(x) = \delta_{ij} \qquad \text{(N.I.)} \tag{2.5}$$

where "N.I." means "No Integration". To get the covariant derivative of $\psi_i(x)$ field one must introduce the so-called bilocal field $G_{\mu,ij}(x,x')$ [1-4] transforming into

$$\begin{aligned} G'_{\mu,ij}(x,x') = {} & \Lambda_{ik}(x,x_1)\,G_{\mu,k\ell}(x_1,x_2)\,\Lambda_{\ell j}^{+}(x_2,x') - \\ & -(i/g_0)\left[\partial_\mu\Lambda_{ik}(x,x_1)+\partial_\mu^1\Lambda_{ik}(x,x_1)\right]\Lambda_{kj}^{+}(x_1,x') \end{aligned} \tag{2.6}$$

where $\partial_\mu^a \equiv \frac{\partial}{\partial x_\mu^a}$, and g_0 is the coupling constant of the bilocal field. Then, for covariant forms, composed of $\psi_i(x)$, $G_{\mu,ij}(x,x')$ fields and their derivatives, we obtain the following expressions:

$$\nabla_\mu \Psi_i(x) = \partial_\mu \Psi_i(x) - i g_0 G_{\mu,ij}(x,x') \Psi_j(x') \qquad (2.7)$$

$$G_{\mu\nu,ij}(x,x') = \partial_\nu G_{\mu,ij}(x,x') + \partial'_\nu G_{\mu,ij}(x,x') - \partial_\mu G_{\nu,ij}(x,x') - \partial'_\mu G_{\nu,ij}(x,x') + $$
$$+ i g_0 \left[G_{\mu,ik}(x,x_1) G_{\nu,kj}(x_1,x') - G_{\nu,ik}(x,x_1) G_{\mu,kj}(x_1,x') \right] \qquad (2.8)$$

The total action of the system is equal to

$$S = i \bar{\Psi}_i(x) \gamma^\mu \nabla_\mu \Psi_i(x) - m \bar{\Psi}_i(x) \Psi_i(x) - \frac{\ell^4}{4} G_{\mu\nu,ij}(x,x') G^{\mu\nu,ji}(x',x) \qquad (2.9)$$

Later on we will limit ourselves by bilocal fields of the type

$$G_{\mu,ij}(x,x') = \left[\frac{1}{\sqrt{N}} \delta_{ij} B_\mu(x_s) + \sqrt{2}\, T^\alpha_{ij} B^\alpha_\mu(x_s) \right] \rho(x,x',x_s) \qquad (2.10)$$

where $B_\mu(x_s)$ and $B^\alpha_\mu(x_s)$ are the above-mentioned local gauge fields. The requirement of invariance of the bilocal fields subspace (2.10) to infinitesimal transformations following from (2.6) ($\omega(x)$, $\omega^\alpha(x) \ll 1$) and also the principle of conformity with the local theory with regard to (2.2), (2.3) and (2.10) reduce to the following relations [4] :

$$\rho(x,x',x_s)\rho(x',x,x'_s) = \frac{1}{\ell^4} \delta(x_s - x'_s) \qquad (2.11)$$

$$\overset{*}{\rho}(x,x',x_s) = \rho(x',x,x_s) \qquad (2.12)$$

$$\left. \begin{aligned} &\rho(x,x_1,x_s)\rho(x_1,x',x'_s) - \rho(x,x_1,x'_s)\rho(x_1,x',x_s) = i f(x_s,x'_s,x_1)\rho(x,x',x_1) \\ &\rho(x,x_1,x_s)\rho(x_1,x',x'_s) + \rho(x,x_1,x'_s)\rho(x_1,x',x_s) = d(x_s,x'_s,x_1)\rho(x,x',x_1) \end{aligned} \right\} \qquad (2.13)$$

where, in accordance with (2.11), $f(x,x',x'')$ and $d(x,x',x'')$ functions are determined unambiguously as

$$\left. \begin{aligned} &f(x_s,x'_s,x''_s) = -i\ell^4 \rho(x,x',x''_s)\left[\rho(x',x_1,x_s)\rho(x_1,x,x'_s) - (x_s \rightleftarrows x'_s)\right] \\ &d(x_s,x'_s,x''_s) = \ell^4 \rho(x,x',x''_s)\left[\rho(x',x_1,x_s)\rho(x_1,x,x'_s) + (x_s \rightleftarrows x'_s)\right] \end{aligned} \right\} \qquad (2.14)$$

Moreover, one needs that $SU(N)$ group generators satisfy the equalities

$$[T^a, T^b]_{ij} = i f^{abc} T^c_{ij} \qquad (2.15a)$$

$$\{T^a, T^b\}_{ij} = \frac{1}{N}\delta^{ab}\delta_{ij} + d^{abc}T^c_{ij} \tag{2.15b}$$

where f^{abc} and d^{abc} are structure constants. It follows from the requirement of invariance of $\rho(x,x',x_s)$ function relative to the inhomogeneous Lorenz group and from (2.14) too, that

$$\left.\begin{aligned} \rho(x,x',x_s) &\equiv R(x-x_s, x'-x_s) \\ f(x_s,x'_s,x''_s) &\equiv F(x_s-x''_s, x'_s-x''_s) \\ d(x_s,x'_s,x''_s) &\equiv \mathcal{D}(x_s-x''_s, x'_s-x''_s) \end{aligned}\right\} \quad \text{(N.I.)} \tag{2.16}$$

Generally speaking, $\rho(x,x',x_s)$, $f(x_s,x'_s,x''_s)$ and $d(x_s,x'_s,x''_s)$ are generalized functions for which according to (2.4) at the limit $\ell \to 0$ we have

$$\rho(x,x',x_s) \to \delta(x-x_s)\,\delta(x'-x_s) \quad \text{(N.I.)} \tag{2.17}$$

$$f(x_s,x'_s,x''_s) \to 0 \tag{2.18}$$

$$d(x_s,x'_s,x''_s) \to 2\,\delta(x_s-x''_s)\,\delta(x'_s-x''_s) \quad \text{(N.I.)} \tag{2.19}$$

Substituting (2.10) into (2.8) with regard to (2.11) (2.13), (2.15) and (2.16) we get for the action in (2.9) the following expression (adopting, for conformity with the local theory, that $g_0 = \frac{g}{\sqrt{2}}$):

$$S = i\bar{\Psi}_i(x)\gamma^\mu\partial_\mu\Psi_i(x) - m\bar{\Psi}_i(x)\Psi_i(x) + g\bar{\Psi}_i(x)\gamma^\mu T^a_{ij}\rho(x,x',x_s)B^a_\mu(x_s)\Psi_j(x') +$$
$$+\frac{g}{\sqrt{2N}}\bar{\Psi}_i(x)\gamma^\mu\rho(x,x',x_s)B_\mu(x_s)\Psi_i(x') - \frac{1}{4}\left[N\cdot\left(F_{\mu\nu}(x)\right)^2 + \frac{1}{2}\cdot\left(F^a_{\mu\nu}(x)\right)^2\right] \tag{2.20}$$

where

$$F_{\mu\nu}(x) = \frac{1}{\sqrt{N}}\Phi_{\mu\nu}(x) - \frac{g}{N\sqrt{2}}f(x_s,x'_s,x)\left[B_\mu(x_s)B_\nu(x'_s) + B^a_\mu(x_s)B^a_\nu(x'_s)\right] \tag{2.21}$$

$$F^a_{\mu\nu}(x)=\sqrt{2}\,\phi^a_{\mu\nu}(x)-\frac{g}{\sqrt{2}}f^{abc}d(x_s,x'_s,x)B^b_\mu(x_s)B^c_\nu(x'_s)-$$
$$-\frac{g}{\sqrt{2}}f(x_s,x'_s,x)\Big[\sqrt{\tfrac{2}{N}}\big(B^a_\mu(x_s)B_\nu(x'_s)+B_\mu(x_s)B^a_\nu(x'_s)\big)+d^{abc}B^b_\mu(x_s)B^c_\nu(x'_s)\Big] \tag{2.22}$$
$$\phi_{\mu\nu}(x)\equiv\partial_\nu B_\mu(x)-\partial_\mu B_\nu(x)$$
$$\phi^a_{\mu\nu}(x)\equiv\partial_\nu B^a_\mu(x)-\partial_\mu B^a_\nu(x)$$

3. **Quantization. Diagrammatics.**

Let us determine the producing functional of the system of $\psi_i(x)$, $\bar{\psi}_i(x)$, $B_\mu(x)$ and $B^a_\mu(x)$ fields in terms of the following continual integral ($\hbar = c = 1$):

$$Z[J,J^a,\eta,\bar{\eta}]=\int \mathcal{D}B\,\mathcal{D}B^a\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\times G(B,B^a)\times$$
$$\times\exp\{i[S+J_\mu(x)B^\mu(x)+J^a_\mu(x)B^{a\mu}(x)+\bar{\psi}_i(x)\eta_i(x)+\bar{\eta}_i(x)\psi_i(x)]\} \tag{3.1}$$

where $\bar{\eta}_i(x)$, $\eta_i(x)$, $J_\mu(x)$ and $J^a_\mu(x)$ are the currents of the corresponding fields, and the functional $G(B,B^a)$ excludes the integration over gauge-equivalent fields; S is the action determined according to (2.20) - (2.22). In the Lorenz gauge ($\partial_\mu B^{a\mu}(x)=0$, $\partial_\mu B^\mu(x)=0$) in conformity with the infinitesimal transformations following from (2.6)

$$B'_\mu(x)=B_\mu(x)+\frac{1}{g}\partial_\mu\omega(x)+\frac{1}{\sqrt{2N}}f(x_s,x'_s,x)\big[B_\mu(x_s)\omega(x'_s)+B^a_\mu(x_s)\omega^a(x'_s)\big] \tag{3.2}$$
$$B^{a\prime}_\mu(x)=B^a_\mu(x)+\frac{1}{2}f^{abc}d(x_s,x'_s,x)B^b_\mu(x_s)\omega^c(x'_s)+\frac{1}{g}\partial_\mu\omega^a(x)+$$
$$+f(x_s,x'_s,x)\Big[\frac{1}{\sqrt{2N}}\big(B^a_\mu(x_s)\omega(x'_s)+B_\mu(x_s)\omega^a(x'_s)\big)+\frac{1}{2}d^{abc}B^b_\mu(x_s)\omega^c(x'_s)\Big] \tag{3.3}$$

we have

$$G(B,B^a)=\det M\cdot\prod_x\delta(\partial^\mu B_\mu(x))\prod_{y,a}\delta(\partial^\mu B^a_\mu(y)) \tag{3.4}$$

where matrix M is equal to

$$M \equiv M_{AB}(x,y) = \begin{pmatrix} M_{00}(x,y) & M_{0a}(x,y) \\ M_{a0}(x,y) & M_{ac}(x,y) \end{pmatrix} \tag{3.5}$$

and $M_{00}(x,y) = \partial^2 \delta(x-y) + \frac{g}{\sqrt{2N}} \partial^x_\mu f(x,x_s,y) B^\mu(x_s)$

$$M_{0a}(x,y) = M_{a0}(x,y) = \frac{g}{\sqrt{2N}} \partial^x_\mu f(x,x_s,y) B^{a\mu}(x_s)$$

$$M_{ac}(x,y) = \partial^2 \delta(x-y)\delta^{ac} + \frac{g}{2} f^{abc} \partial^x_\mu d(x,x_s,y) B^{b\mu}(x_s) +$$

$$+ \frac{g}{\sqrt{2N}} \partial^x_\mu f(x,x_s,y) B^\mu(x_s)\delta^{ac} + \frac{g}{2} d^{abc} \partial^x_\mu f(x,x_s,y) B^{b\mu}(x_s)$$

To within a constant multiplier

$$\det M = \exp Sp \ln (1 + \Box^{-1}(M - \Box))$$
$$\Box = \partial^\mu \partial_\mu \tag{3.6}$$

Let us write relations (2.11), (2.12) and (2.14), necessary for subsequent consideration, in momentum representation (the Euclidean momentum space is ment)

$$\int |R_E(k, q-k)|^2 \frac{d^4k}{(2\pi)^4} = \frac{1}{\ell^4} \tag{3.7}$$

$$R_E^*(k_1,k_2) = R_E(k_2,k_1) \tag{3.8}$$

$$F_E(k_1,k_2) = i\ell^4 \int \frac{d^4p}{(2\pi)^4} \left[R_E(k_2+p, k_1-p) R_E(k_1-p,p) R_E(p,-k_2-p) - (1 \rightleftarrows 2) \right] \tag{3.9}$$

$$D_E(k_1,k_2) = -\ell^4 \int \frac{d^4p}{(2\pi)^4} \left[R_E(k_2+p, k_1-p) R_E(k_1-p,p) R_E(p,-k_2-p) + (1 \rightleftarrows 2) \right] \tag{3.10}$$

where $R(k_1,k_2)$, $F(k_1,k_2)$ and $D(k_1,k_2)$ are Fourier-transformed images of functions in (2.16), for which in accordance with (2.17) ÷ (2.19) we have at the limit $\ell \to 0$

$$R(k_1,k_2) \to 1, \quad F(k_1,k_2) \to 0, \quad D(k_1,k_2) \to 2 \tag{3.11}$$

since

$$\square^{-1}(x,y)=-\mathcal{D}_0(x-y)=\int\frac{d^4k}{(2\pi)^4}\,\frac{e^{-ik(x-y)}}{k^2+i\varepsilon}$$

and as according to (3.9)

$$F(0,k)=-F(k,0)\equiv 0 \tag{3.12}$$

we get

$$Sp\left(\square^{-1}(M-\square)\right)=\int\frac{d^4q}{(2\pi)^4}\,i\,\frac{g}{\sqrt{2N}}\,q_\mu\mathcal{D}_0(q)F(q,0)B_\mu(0)\left(1+\delta_{aa}\right)=0 \tag{3.13}$$

Introducing fictitious anticommutating fields ("ghosts") $\overline{C^A}(x)$, $C^A(x)$, where $A=(0,a)$, the determinant in (3.4) can be written in terms of the continual integral

$$\det M=\int\mathcal{D}\bar{C}\,\mathcal{D}C\,\exp\left[i\,\overline{C^A}(x)M_{AB}(x,y)C^B(y)\right] \tag{3.14}$$

It follows from (3.13) that in diagrammatics "ghosts" come only by loops. As the function $F(k_1,k_2)$ is small both, at low ($k_E l\ll 1$) and at high ($k_E l\gg 1$) momenta and when (3.12) takes place, and also in consequence of antisymmetry of $F(k_1,k_2)$ function

$$\left.\begin{aligned}F(k,-k)&\equiv 0\\ F(k,k)&\equiv 0\end{aligned}\right\} \tag{3.15}$$

one may in zero approximation (over l) neglect the contribution of terms containing $f(x_s,x_s',x_\varepsilon'')$ function in the action (2.20). In this case the producing functional (3.1) can be written as (with an accuracy of an inessential constant factor)

$$Z\left[J,J^\alpha,\eta,\bar{\eta},\xi,\bar{\xi}\right]=\int\mathcal{D}B\,\mathcal{D}B^\alpha\mathcal{D}\bar{\Psi}\,\mathcal{D}\Psi\,\mathcal{D}\bar{C}\,\mathcal{D}C\cdot\exp(iS_{eff}) \tag{3.16}$$

where

$$S_{eff}\doteq S_0+S_I \tag{3.17}$$

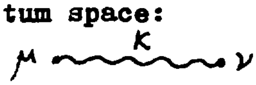
$$S_0=i\overline{\Psi}_i(x)\gamma^\mu\partial_\mu\Psi_i(x)-m\overline{\Psi}_i(x)\Psi_i(x)+\frac{1}{2}B_\mu(x)\left(g^{\mu\nu}\partial^2-\partial^\mu\partial^\nu+\frac{1}{\alpha}\partial^\mu\partial^\nu\right)B_\nu(x)+$$
$$+\frac{1}{2}B^a_\mu(x)\left(g^{\mu\nu}\partial^2-\partial^\mu\partial^\nu+\frac{1}{\beta}\partial^\mu\partial^\nu\right)B^a_\nu(x)+\overline{C^0(x)}\partial^2C^0(x)+\overline{C^a(x)}\partial^2C^a(x)+$$
$$+J_\mu(x)B^\mu(x)+J^a_\mu(x)B^{a\mu}(x)+\overline{\Psi_i(x)}\eta_i(x)+\overline{\eta}_i(x)\Psi_i(x)+\overline{C^a(x)}\xi^a(x)+\overline{\xi^a(x)}C^a(x)$$

(3.18)

$$S_I=g\overline{\Psi}_i(x)\gamma^\mu T^a_{ij}\rho(x,x',x_s)B^a_\mu(x_s)\Psi_j(x')+\frac{g}{\sqrt{2N}}\overline{\Psi}_i(x)\gamma^\mu\rho(x,x',x_s)B_\mu(x_s)\Psi_i(x')+$$
$$+\frac{g}{2}f^{abc}d(x,x_s,x'_s)\partial_\nu B^a_\mu(x)B^{b\mu}(x_s)B^{c\nu}(x'_s)-$$
$$-\frac{g^2}{16}f^{abc}f^{ab'c'}d(x_s,x'_s,x)d(y_s,y'_s,x)B^b_\mu(x_s)B^c_\nu(x'_s)B^{b'\mu}(y_s)B^{c'\nu}(y'_s)+$$
$$+\frac{g}{2}f^{abe}\overline{C^a(x)}\partial^x_\mu d(x,x_s,y)B^{b\mu}(x_s)C^e(y)$$

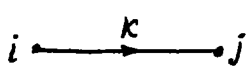
(3.19)

$\xi^a(x)$ and $\overline{\xi^a(x)}$ are currents, corresponding to the "ghosts" $\overline{C^a(x)}$ and $C^a(x)$. The currents of $\overline{C^0(x)}$ and $C^0(x)$ fields are not included in (3.18) because in the given approximation these "ghosts" are absent in the interaction (3.19). Integrating (3.16) we get the following elements of diagrammatics in momentum space:

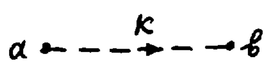


$$i\mathcal{D}^\alpha_{\mu\nu}(K)=-\frac{i}{K^2}\left(g_{\mu\nu}-\frac{K_\mu K_\nu}{K^2}(1-\alpha)\right) \qquad (3.20)$$

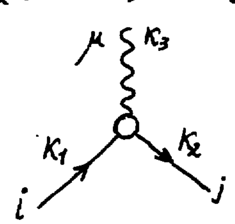
$$i\mathcal{D}^\beta_{\mu\nu}(K)\delta^{ab}=-\frac{i}{K^2}\left(g_{\mu\nu}-\frac{K_\mu K_\nu}{K^2}(1-\beta)\right)\delta^{ab} \qquad (3.21)$$



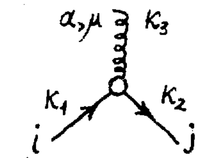
$$iS_0(K)\delta_{ij}=\frac{i}{\hat{K}-m}\delta_{ij} \qquad (3.22)$$



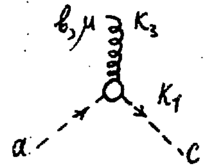
$$i\mathcal{D}_0(K)\delta_{ab}=\frac{i}{K^2}\delta_{ab} \qquad (3.23)$$



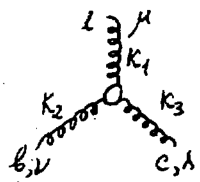
$$\frac{ig}{\sqrt{2N}}\gamma^\mu R(K_2,-K_1)\delta_{ij} \qquad (3.24)$$



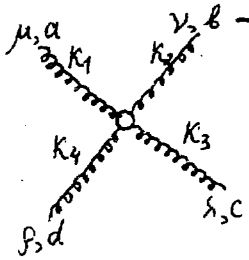
$$ig\gamma^\mu T^a_{ji} R(k_2,-k_1) \tag{3.25}$$



$$\frac{g}{2} f^{abc} k_{1\mu} \mathcal{D}(k_1,k_3) \tag{3.26}$$



$$-\frac{g}{2} f^{abc}\Big[\mathcal{D}(k_1,k_2)(k_1-k_2)_\lambda g_{\mu\nu} + \mathcal{D}(k_2,k_3)(k_2-k_3)_\mu g_{\nu\lambda} + \mathcal{D}(k_3,k_1)(k_3-k_1)_\nu g_{\mu\lambda}\Big] \tag{3.27}$$



$$-\frac{ig^2}{4}\Big[\mathcal{D}(k_1,k_2)\mathcal{D}(k_3,k_4) f^{abe}f^{cde}(g_{\mu\lambda}g_{\nu\rho}-g_{\lambda\nu}g_{\mu\rho}) + \mathcal{D}(k_1,k_3)\mathcal{D}(k_2,k_4) f^{ace}f^{bde}(g_{\mu\nu}g_{\lambda\rho}-g_{\lambda\nu}g_{\mu\rho}) + \mathcal{D}(k_1,k_4)\mathcal{D}(k_2,k_3) f^{ade}f^{bce}(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho})\Big] \tag{3.28}$$

4. On solution of fundamental equations of the theory.

There is an unknown function $R(k_1,k_2)$ in diagrammatics elements (the function $\mathcal{D}(k_1,k_2)$ is determined according to (3.10)). We trace one of the possible ways to find $R(k_1,k_2)$, so we consider a system of equations

$$\lim_{\to 0} \rho_\delta(x,x',x_5)\rho_\delta(x',x,x_5') = \frac{1}{g^4}\delta(x_5-x_5') \tag{4.1}$$

[illegible] $= 0$ (4.2)

[illegible] equivalent [illegible] (3.[illegible]), where

$\rho_\delta(x,x',x_s)$ is a representation of the generalized $\rho(x,x',x_s)$ function. The momentum representation (Euclidian momentum space) is

$$\lim_{\delta\to 0}\int |R_\delta(k,q-k)|^2 \frac{d^4k}{(2\pi)^4} = \frac{1}{\ell^4} \tag{4.3}$$

$$\lim_{\delta\to 0}\Big[R_\delta(k_1,p)R_\delta(-p,k_2)- \\ -\ell^4\int\frac{d^4q}{(2\pi)^4}R_\delta(p-k_2+q,-k_1-p-q)R_\delta(-k_1-p-q,q)R_\delta(q,-p+k_2-q)R_\delta(k_1,k_2)\Big]=0 \tag{4.4}$$

It is seen from (4.3) that a whole function with asymptotic behaviour no slower than $\frac{1}{k^{2+\varepsilon}}$, where $\varepsilon > 0$ and $k\to\infty$, for any variable can stand for $R_\delta(k_1,k_2)$. Therefore, we look for the solution of equations (4.3) and (4.4) in the form

$$R_\delta(k_1,k_2)=\frac{1}{(2i)^3}\int\limits_{-\sigma_\xi+i\infty}^{-\sigma_\xi-i\infty}d\xi\int\limits_{-\sigma_\zeta+i\infty}^{-\sigma_\zeta-i\infty}d\zeta\int\limits_{-\sigma_\eta+i\infty}^{-\sigma_\eta-i\infty}d\eta\,\frac{v(\xi+\delta,\zeta+\delta,\eta+\delta)}{\sin\pi\xi\cdot\sin\pi\zeta\cdot\sin\pi\eta}(k_1^2\ell^2)^{\xi+\delta}(k_2^2\ell^2)^{\zeta+\delta}[(k_1+k_2)^2\ell^2]^{\eta+\delta}$$

$$0<\delta\ll 1 \tag{4.5}$$

where $\frac{1}{2}<\sigma_\xi,\sigma_\zeta<1 ; 0<\sigma_\eta<1 : v^*(\xi,\zeta,\eta)=v(\zeta^*,\xi^*,\eta^*)$ and $\sigma_\xi=\sigma_\eta$ owing to (3.8) and, moreover, $v(0,0,0)=1$ according to (3.11). Substituting (4.5) into (4.3) and (4.4), doing all integrations and passing to the limit $\delta\to 0$ we get a system of non-linear algebraic equations, generally speaking, for complex variables $v(n,m,k)$, $\frac{\partial}{\partial n}v(n,m,k)$ and $\frac{\partial}{\partial n}v(m,n,k)$ (where $\frac{\partial}{\partial n}v(n,a,\ell)= =\frac{\partial v}{\partial \xi}(\xi,a,\ell)\Big|_{\xi=n}$):

$$\sum\sum\frac{s!(s+1)!}{(s-N-2)!(s-N-1)!}v(n,m,k)\frac{\partial v}{\partial N}(N-s-m,s-n,\ldots-k)=16\pi^2\delta_{\ldots}\delta_{\ldots} \tag{4.6}$$

[illegible] (4.7)

$$Q_2(A,B,C)=0 \qquad (4.8)$$

$$(A,B,C,\mathcal{D},E,G,H=0,1,2,\ldots;N=-2,-1,0,\ldots;\tau=0,1,2,\ldots)$$

where

$$Q_{1,2}(\alpha,\beta,\gamma)=\frac{1}{16\pi^2}\sum_{M=0}^{\infty}(-1)^{M+1}\sum_{n,m,k=0}^{\infty}\sum_{S=0}^{M}\sum_{l=0}^{S}\frac{1}{l!(S-l)!}\sum_{\substack{a=M-l\\ b=M-S+l}}^{\infty}(-1)^S\times$$

$$\times\frac{a!b!(a+b+S+1-M)!}{(a-M+l)!(b-M+S-l)!(a+b+1-M)!(M+1)!(M-S)!}\times$$

$$\times\mathcal{V}(n,b-m,\gamma-M+S)\mathcal{V}(k,a-n,\beta-S+l)\times\begin{cases}-\dfrac{\partial\mathcal{V}}{\partial M}(m,M-2-a-b-k,\alpha-l)\\ \mathcal{V}(m,M-2-a-b-k,\alpha-l)\end{cases} \qquad (4.9)$$

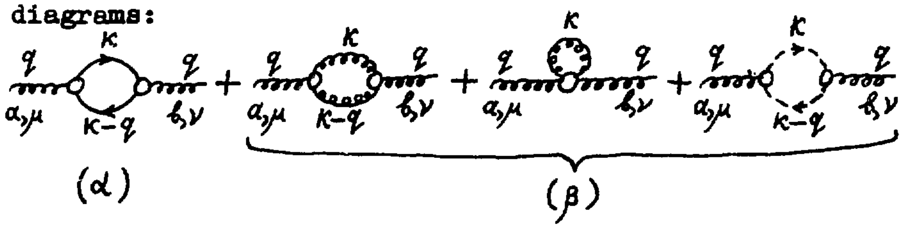
Therewith, Eq. (4.8) ensures the regularity of $\mathcal{D}(k_1,k_2)$ function. The analysis of equations (4.3) and (4.4) and also that of self-energy diagrams show that the $\mathcal{V}(\xi,\zeta,\eta)$ function must behave as $1/[\Gamma(1+\xi)\Gamma(1+\zeta)\Gamma^2(1+\eta)]$, where $\Gamma(x)$ is the gamma function and, moreover, near the intiger values of α and β variables

$$\sum_{n,m,k=0}^{\infty}\mathcal{V}(n,m,k)\mathcal{V}(\alpha-m,\beta-n,\gamma-k)\sim(\alpha+\beta+1)/[\Gamma(\alpha+1)\Gamma(\beta+1)] \qquad (4.10)$$

Therefore, with regard to the implied convergency of series in (4.6) - (4.9) one can "cut off" the system limiting oneself to a large but finite number of equations and just to the same number of variables. The exactness of numerical solution of this "cut off" system can be improved by enlarging the number of variables and, consequently that of equations. The single-valued determination of $\mathcal{V}(\xi,\zeta,\eta)$ function (of course to within an error, by integer-valued $\mathcal{V}(n,m,k)$, $\frac{\partial \mathcal{V}}{\partial n}(n,m,k)$, and $\frac{\partial \mathcal{V}}{\partial n}(m,n,k)$ is bound up with asymptotic behaviour of the latter ones [5] , partly included in equations themselves and in (4.10) and is in need of more thorough study.

5. Polarization operator.

As an example we consider the self-energy operator of $B^{a}_{\mu}(x)$ gauge fields in the second order of the perturbation theory which is described in this approximation by the following diagrams:



(5.1)

In accordance with the afore-cited Feynman rules this operator is equal to (taking into account that $\mathcal{D}(k,q-k)=\mathcal{D}(q,-k)=\mathcal{D}(k,-q)$)

$$\mathcal{P}^{ab}_{\mu\nu}(q) = \mathcal{P}^{(\alpha)ab}_{\mu\nu}(q) + \mathcal{P}^{(\beta)ab}_{\mu\nu}(q) \tag{5.2}$$

$$\mathcal{P}^{(\alpha)ab}_{\mu\nu}(q) = -\frac{g^2}{2}\delta^{ab}\int\frac{d^4k}{(2\pi)^4}|R(k,q-k)|^2 Sp\left(\gamma_\nu S_0(k)\gamma_\mu S_0(k-q)\right) \tag{5.3}$$

$$\mathcal{P}^{(\beta)ab}_{\mu\nu}(q) = g^2 N\delta^{ab}\int\frac{d^4k}{(2\pi)^4}\left(\mathcal{D}(k,-q)\right)^2\left(\sum_{i=1}^{3} J_{i\mu\nu}\right) \tag{5.4}$$

where

$$\begin{aligned} J_{1\mu\nu} = \frac{1}{8}\{ & 2(q+k)_\rho(q+k)_{\rho'}\mathcal{D}_{\rho\rho'}(k-q)\mathcal{D}_{\mu\nu}(k)+ \\ & +2(q+k)_{\rho'}(k-2q)_\lambda\mathcal{D}_{\lambda\nu}(k)\mathcal{D}_{\mu\rho'}(k-q)+ \\ & +2(q+k)_{\rho'}(q-2k)_\mu\mathcal{D}_{\rho\nu}(k)\mathcal{D}_{\rho\rho'}(k-q)+ \\ & +2(q-2k)_\nu(k+q)_\rho\mathcal{D}_{\mu\rho'}(k)\mathcal{D}_{\rho\rho'}(k-q)+ \\ & +(q-2k)_\nu(q-2k)_\mu\mathcal{D}_{\rho\rho'}(k)\mathcal{D}_{\rho\rho'}(k-q)\} \end{aligned}$$

$$J_{2\mu\nu}=-\frac{1}{8}\mathcal{D}_0(K)\mathcal{D}_0(K-q)[K_\mu(K-q)_\nu+K_\nu(K-q)_\mu]$$

$$J_{3\mu\nu}=\frac{1}{4}\mathcal{D}_{\rho\lambda}(K)(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\rho}g_{\nu\lambda})$$

$$\mathcal{D}_{\mu\nu}(K)=-\frac{1}{K^2+i\varepsilon}\left(g_{\mu\nu}-\frac{K_\mu K_\nu}{K^2}\right) \qquad \text{(transverse gauge, } \beta=0\text{)}$$

Summing up on the number of families of fermions in (5.3) is meant. Using the expressions (3.10), (4.5) and (4.8), the functions $|R(K,q-K)|^2$ and $(\mathcal{D}(K,-q))^2$ can be represented as

$$|R_E(K_E,q_E-K_E)|^2=\sum_{n_1,n_2,n_3=0}\frac{(-m^2\ell^2)^{n_1+n_2}}{n_1!n_2!}(-q_E^2\ell^2)^{n_3}\tau_{n_1n_2n_3}\left(-(K_E^2+m^2)\ell^2,-[(K_E-q_E)^2+m^2]\ell^2\right) \tag{5.5}$$

$$(\mathcal{D}_E(K_E,-q_E))^2=\sum_{n=0}^{\infty}(-q_E^2\ell^2)^n d_n\left(-K_E^2\ell^2,-(K_E-q_E)^2\ell^2\right) \tag{5.6}$$

with

$$\tau_{n_1n_2n_3}(y,z)=\frac{1}{(2i)^2}\int_{-a_\xi+i\infty}^{-a_\xi-i\infty}\frac{d\xi}{\sin\pi\xi}\int_{-a_\eta+i\infty}^{-a_\eta-i\infty}\frac{d\eta}{\sin\pi(\eta-\xi)}W(\xi+n_1,\eta-\xi+n_2,n_3)\frac{\Gamma(\xi+n_1+1)\Gamma(\eta-\xi+n_2+1)}{\Gamma(\xi+1)\Gamma(\eta-\xi+1)}y^\xi z^{\eta-\xi} \tag{5.7}$$

$$d_n(y,z)=\frac{1}{(2i)^2}\int_{-b_\xi+i\infty}^{-b_\xi-i\infty}\frac{d\xi}{\sin\pi\xi}\int_{-b_\eta+i\infty}^{-b_\eta-i\infty}\frac{d\eta}{\sin\pi(\eta-\xi)}\sum_{\mu,\nu,\lambda=0}^{\infty}\theta(\mu,\nu,\lambda)\theta(\xi-\mu,n-\nu,\eta-\xi-\lambda)y^\xi z^{\eta-\xi} \tag{5.8}$$

$$(1<a_\xi,b_\xi<2\,;\,a_\eta,b_\eta>2)$$

$$W(\alpha,\beta,\gamma)=\sum_{\mu,\nu,\lambda=0}^{\infty}v(\mu,\nu,\lambda)v(\beta-\nu,\alpha-\mu,\gamma-\lambda) \tag{5.9}$$

$$\theta(\alpha,\beta,\gamma)=Q_1(\alpha,\beta,\gamma)+Q_1(\beta,\alpha,\gamma) \tag{5.10}$$

where m is the mass of fermion , and $Q_1(\alpha,\beta,\gamma)$ is determine from expression (4.9). Substituting (5.5) and (5.6) into (5.3) and (5.4), correspondingly, after integrations one obtains

$$\mathcal{P}^{(d)ab}_{\mu\nu}(q) = -\frac{ig^2}{4\pi^2}\delta^{ab}\sum_{n_1,n_2,n_3=0,\,n=-1}^{\infty}\frac{(-1)^{n_1+n_2}}{n_1!\,n_2!\,\Gamma(n+1)}(m^2\ell^2)^{n_1+n_2+n}(q^2\ell^2)^{n_3}\times$$

$$\times\sum_{\kappa=0}^{\infty}\Big\{W_{n_1n_2n_3}(\kappa,n-\kappa)\Phi_{\mu\nu}(\kappa,n,q)\big(\ln m^2\ell^2-\psi(1+n)\big)+\frac{\partial}{\partial n}\big(W_{n_1n_2n_3}(\kappa,n-\kappa)\Phi_{\mu\nu}(\kappa,n,q)\big)\Big\} \qquad (5.11)$$

where
$$\Phi_{\mu\nu}(\xi,\eta,q) = \frac{1}{\Gamma(1-\xi)\Gamma(1-\eta+\xi)}\int_0^1 dt\cdot t^{-\eta+\xi}(1-t)^{-\xi}\left(1-\frac{q^2}{m^2}t(1-t)\right)^{\eta}\times$$
$$\times\left[t(1-t)(q_\mu q_\nu-q^2g_{\mu\nu})-\frac{\eta}{2(1+\eta)}(m^2-q^2t(1-t))g_{\mu\nu}\right] \qquad (5.12)$$

and
$$\mathcal{P}^{(\beta)ab}_{\mu\nu}(q) = -\frac{ig^2}{16\pi^2}N\delta^{ab}\sum_{n,L=0}^{\infty}\frac{1}{L!}(q^2\ell^2)^{n+L}(-1)^L\times$$
$$\times\sum_{\kappa=0}^{\infty}\Big\{\theta_n(\kappa,n-\kappa)\Big(\sum_{i=1}^{3}F_{i\mu\nu}(\kappa,L,q)\Big)\big(\ln(-q^2\ell^2)-\psi(1+L)\big)+$$
$$+\frac{\partial}{\partial L}\Big(\theta_n(\kappa,L-\kappa)\Big(\sum_{i=1}^{3}F_{i\mu\nu}(\kappa,L,q)\Big)\Big)\Big\}$$

(5.13)

where
$$F_{i\mu\nu}(\xi,\eta,q_E) = \frac{1}{\Gamma(2-\xi)\Gamma(2-\eta+\xi)}\int_0^1 dt\cdot t^{\xi-1}(1-t)^{\eta-\xi-1}\frac{1}{(q_E^2)^2}\eta(\eta-1)M_{i\mu\nu}$$

$$M_{i\mu\nu} = I_{i\mu\nu}\Big/\left(\frac{\pi^2\Gamma(2-\eta)}{\Gamma(4-\eta)}\left[q_E^2t(1-t)\right]^{\eta-2}\right)$$

$$I_{i\mu\nu} = \int\frac{J_{i\mu\nu}(K_E^2)^2[(K_E-q_E)^2]^2}{[(K_E-q_E)^2t+K_E^2(1-t)]^{4-\eta}}d^4K_E \qquad (5.14)$$

$$\psi(x)\equiv\frac{d\ln\Gamma(x)}{dx}$$

and according to (5.7) and (5.8)

$$
\left.\begin{aligned}
W_{n_1 n_2 n_3}(\alpha,\beta) &\equiv W(\alpha+n_1,\beta+n_2,n_3)\,\frac{\Gamma(\alpha+n_1+1)\Gamma(\beta+n_2+1)}{\Gamma(\alpha+1)\Gamma(\beta+1)}\\
\theta_n(\alpha,\beta) &\equiv \sum_{\mu,\nu,\lambda=0}^{\infty}\theta(\mu,\nu,\lambda)\theta(\alpha-\mu,n-\nu,\beta-\lambda)
\end{aligned}\right\}
\qquad (5.15)
$$

Hence, in the approximation $m^2\ell^2 \ll |q^2|\ell^2 \ll 1$, for the transverse part of the polarization operator we have

$$
\mathcal{P}^{tr\,ab}_{\mu\nu}(q) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\,\Pi^{ab}(Q^2) \qquad (5.16)
$$

where

$$
\Pi^{ab}(Q^2) = -\frac{ig^2}{16\pi^2}\delta^{ab}\left\{\left(N\cdot\frac{13}{6} - n_f\cdot\frac{2}{3}\right)\ln Q^2\ell^2 + O(1)\right\},\ Q^2 = -q^2
$$

and n_f is the number of families of fermions. This result agrees with the analogous expression in the local non-abelian gauge theory (in the limit $\ell \to 0$).

6. Conclusion

Thus, the afore-cited non-local theory satisfies the correspondence principle. Moreover, firstly, in contrast to the existing non-local quantum field theories [5,6], mainly the necessary restrictions on analytical and asymptotical properties of form factors underlie the theory and are not introduced from outside by requirements of finiteness, unitarity, causality and so on, made to S-matrix; secondly, due to the presence in all vertices at least of one of the functions $R(k_1,k_2)$, $F(k_1,k_2)$ or $D(k_1,k_2)$ with asymptotic behaviour $\frac{1}{K_E^{2+\varepsilon}}$, where $\varepsilon > 0$ at $K_E \to \infty$, all diagrams of the perturbative series are finite; thirdly, on the basis of the united principle of non-local gauge invariance, the symmetry groups $SU(N)$ and $U(1)$ are naturally unified, since it follows from (3.2) and (3.3) that even in the case with only one $SU(N)$ group a part of $U(1)$ type gauge transformations are generated.

The unique determination of the Mellin image of $R(K_1,K_2)$ function in Eq. (4.5) is connected with two-way limitation on the asymptotical behaviour of the function $R(K_1,K_2)$ at high Euclidean momenta (in the meantime, the theory gives only limitation from below), and also with analytical properties of $\mathcal{V}(\xi,\zeta,\eta)$ function and requires a further research.

The author expresses his thanks to prof. Matinyan S.G., Asatryan G.M., Savvidy G.K., Prochorenko E.B. for valuable critical remarks and also to Mamijanian E.A. and Mnatsakanyan E.A. for assistance.

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The manuscript was received 18 September 1985

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НЕЛОКАЛЬНАЯ КАЛИБРОВОЧНАЯ ТЕОРИЯ ПОЛЯ

(на английском языке, перевод Г.А.Папяна)

Редактор Л.П.Мукаян

Технический редактор А.С.Абрамян

Подписано в печать 25/XII-85г. ВФ-09201 Формат 60x84/16
Офсетная печать.Уч.изд.л 1,5 Тираж 299 экз. Ц. 15 к.
Зак.тип. № 620 Индекс 3624

Отпечатано в Ереванском физическом институте
Ереван 36, Маркаряна 2

индекс 3624

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ