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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

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CATASTROPHE THEORY AND STELLAR SYSTEMS

ЦНИИатоминформ

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Վ.Գ. ԳՈՒՐԶԱԴԻԱՆ, Ա.Ա. ԲՈԶԱՐՅԱՆ, Ս.Է. ՄԱՏԻՆՅԱՆ

ԱՐԶԱՎԻՐՐՆԵՐԻ ՏԵՍՈՒԹՅՈՒՆԸ ԵՎ ԱՍՏՂԱՅԻՆ ՀԱՄԱՎԱՐԳ

Արձավիրքների տեսության տեսանկյունից ուսումնասիրվում է գրավիտացիոն արձավիրքների երևույթը՝ կենտրոնական զանգված պարունակող աստղային համակարգում: Յուրաքանչյուր է սրված, որ ծավր տիպի արձավիրքը որոշ սեզի ունի ստորական համակարգերի դեպքում, պահպանվում է նաև այս դեպքում և չի մերժվում 'համար' -ի: Արձավիրքը սկսվում է համակարգի տասնիկ Բարթր շերտաափմանում և հարաբերական խտության ավելի ուժեղ արժեքների դեպքում:

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1. Introduction.

The problem of evolution of galaxies and star clusters is a classic one on the way of understanding the structure and evolution of the Universe. During nearly a century period of investigations many aspects of statistical mechanics of stellar systems have become clear (see, e.g. [1] : recently an approach to these problems is developed based on the conceptions of ergodic theory [2-5]).

An important and firstly unexpected result was Antonov's [6] discovery of instability of isothermal spherical system at its certain parameters. The physical analysis of this fact performed by Lynden-Bell and Wood [7] shows that stellar systems possess negative specific heat, i.e. their hot (central) regions during exchange of energy with cool (external) regions become hotter while the latter - cooler. This "gravothermal catastrophe" phenomenon has stimulated later a lot of studies concerning both the catastrophe process and its consequences, particularly the possibility of formation of a central massive object - black hole [8,9] .

In this paper we investigate the possibility of gravothermal catastrophe in a system containing central massive body. So far as our aim is to understand the stability of this system, we should not avoid those several similit

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fications adopted by Lynden-Bell and co-authors [7,8]. In particular, we also neglect such processes as evaporation of high-energy stars, formation of binary stars, tidal disruption of stars nearby the massive object, etc. (part of these effects we have taken into account in [10] while considering the evolution of these systems). This is conditioned by larger values of characteristic time scales of these processes compared with instability time scale for real systems.

We shall make use of conceptions and methods of catastrophe theory developed during the last decades [11,12]. Catastrophe theory being originated from Poincare's studies on stability of dynamical systems, is called (Thom) the theory of singularity, i.e. the investigation of transformations on maximum and minimum together with its applications. Note an interesting fact that certain ideas of catastrophe theory had been understood originally by Lindblad while considering just the dynamics of galaxies [13].

In the present work we have shown that the gravothermal catastrophe being an A_2 -fold type catastrophe for usual isothermal systems occurs for the systems with central mass as well. Moreover, in this sense the latter systems appear to be sufficiently more unstable, i.e. the catastrophe occurs at higher temperatures and lower values of the contrast of density. The last conclusion seems somewhat unexpected, so far as commonly it is assumed that the existence of a central mass should lead to a central density cusp in clusters and galaxies.

The content of the paper is as follows. In Sect. 2, several main concepts and results of catastrophe theory used later are presented. The derivation of main equations is performed in Sect. 3. The procedure of numerical analysis which appears to be not very simple is described in Sect. 4. The results of computer calculations are presented in Sect. 5 and discussed in Conclusion (Sect. 6).

2. Catastrophes in Dynamical Systems.

The detailed description of main results of catastrophe theory one can find in [11,12]. Here we have collected its several conceptions which are used further.

Usually one cannot integrate the differential equations appearing in different physical problems, i.e. it is impossible to find all the set of solutions. Originally Poincare [14] introducing the conceptions of structural and dynamical stability and critical sets, had shown that often limited volume of information is enough for qualitative description of the system. The questions concerning the behaviour of the system while changing its parameters, raised by Poincare, were investigated later by Lyapunov, Andronov, Pontryagin, Hopf, Smale. Catastrophe theory proceeds from these studies and the results of differential calculus of canonical forms of functions $f: \mathbb{R}^n \rightarrow \mathbb{R}^1$ and mappings $\mathbb{R}^n \otimes \mathbb{R}^k \rightarrow \mathbb{R}^1$ developed by Morse, Whitney and Arnold. Proceeding from these results Thom described the canonical forms of certain singularities of mappings which he called catastrophes.

In this sense the catastrophe theory can be considered as a theory studying the states of equilibrium of dynamical systems described by potential function.

Let a potential function

$$V(x; c), \quad \{x \in \mathbb{R}^n, c \in \mathbb{R}^k\},$$

is given, where x is generalized coordinate.

Parameters c which can qualitatively influence the properties of solutions $x(c)$ of equation

$$\dot{x} = -\nabla V(x; c), \quad (2.1)$$

where

$$\nabla = \left(\frac{\partial}{\partial x^i} \right),$$

are so-called control parameters.

Equilibrium or critical points of this system are determined from the equation

$$F(x, c) = \nabla V(x; c) = 0. \quad (2.2)$$

Catastrophe theory inquires into the behaviour of equilibrium states $x(c)$ of the potential functions while varying the control parameters c . Jump-like variations occurring at smooth variation of c are named catastrophes. Below we shall discuss only the cases when $n = 1$, $k = 1, 2$.

Let us consider the local properties of potential functions $V(x)$ and their families $V(x, c)$. These properties are determined by several theorems of functional analysis.

Thus, if the force acting on the point x is different from zero, i.e.

$$F(x) = -V'(x) \neq 0,$$

then from the theory on implicit function follows the possibility of smooth exchange of coordinates

$$y = y(x)$$

and

$$V \doteq y + \text{const.}$$

The sign \doteq means "equals after smooth exchange of coordinates".

When

$$F(x) = 0 \quad \text{and} \quad F'(x) \neq 0 \quad (2.3)$$

one has according to Morse's theorem:

$$V \doteq y^2. \quad (2.4)$$

The points (2.3) are ^{non-}degenerate or Morse critical points. All the rest cases reduce to them at small variations of the system. Note that the systems describing real evolutionary processes are as a rule of a common type.

The situation is quite different if we are interested in a family of parameter-dependent systems rather than in an individual one. It turns out that in this case the degenerate cases ($V' = V'' = 0$) are irremovable.

It is proved that the catastrophes of A_2 type (by Thom's classification) represented by a function

$$(n=1, k=1), \quad V(y; a) = \frac{1}{3}y^3 + ay \quad (2.5)$$

and of $A_{\pm 3}$ type

$$(n=1, k=2), \quad V(y; a, b) = \pm \frac{1}{4}y^4 + \frac{1}{2}ay^2 + by \quad (2.6)$$

are typical canonical forms. It means that at smooth plane to plane mapping every singularity turns to this catastrophe at properly small variations.

Consider the A_2 catastrophe:

$$V(x; a) = \frac{1}{3}x^3 + ax. \quad (2.7)$$

The critical points are determined by equation

$$V' = x^2 + a = 0.$$

The potential V when $a = 0$ has a non-Morse ($V' = V'' = 0$) point at $x = 0$, therefore V is a non-Morse function ($a = 0$); the point $a = 0$ is the separatrix. Critical points projected on the axis a form A_2 -catastrophe named a fold by Whitney. The separatrix shares the space of control parameters (in our case a straight line) into regions where $a < 0$

and $\alpha > 0$ and potentials have similar qualitative characteristics.

Catastrophe of A_{+3} type is given by a potential depending on two control parameters in a form

$$V(x; \alpha, \beta) = \frac{1}{4} x^4 + \frac{1}{2} \alpha x^2 + \beta x. \quad (2.8)$$

Critical points are determined from the equation

$$V' = x^3 + \alpha x + \beta = 0. \quad (2.9)$$

The separatrix shares the space into three regions - with three critical points, with one point and two points on the separatrix itself. The singularity of the plane to plane projection of control parameters (α, β) Whitney has called cusps.

Catastrophe of A_{-3} type having a form

$$V(x; \alpha, \beta) = -\frac{1}{4} x^4 - \frac{1}{2} \alpha x^2 - \beta x \quad (2.10)$$

is called double cusp. While transiting from cusp to double cusp the maximum and minimum change by places. Physically it leads to quite different results, so far as the cusp as distinct from double cusp always possesses at least one global minimum.

For systems depending on one control parameter and one generalized coordinate one usually has either $V' \neq 0$ or $V' = 0$, $V'' \neq 0$ at arbitrarily chosen point $(x; c)$. For arbitrary family of functions

$V(x; c)$ has a form

$$V(x; c) = \frac{1}{3} x^3 + cx. \quad (2.11)$$

Singularities of a fold and cusp type are stable, i.e. every close mapping has the same singularity at any appropriate point.

The strength of the catastrophe theory is the possibility to point out the global qualitative properties of physical systems versus n and k .

3. Derivation of Main Equations.

Consider a system of $N+1$ particles ($N \gg 1$), where N particles (stars) have equal masses m , and one - a mass M_0 , so that $M_0 \gg m$. The system is surrounded by a sphere of a radius r_e and analogously to assumptions in [7] is in a thermostat with a temperature T , i.e. is isothermal. The stars are assumed point-like and interacting with each other only gravitationally, therefore the total number of stars is conserved.

Denote via $f_m(x, v)$ and $f_0(x, v)$, where $x = (x_1, x_2, x_3)$, $v = (v_1, v_2, v_3)$, density distributions in phase space of particles with masses m and M_0 , respectively.

So far as the system has fixed volume and temperature, its equilibrium state is determined by free energy

$$F(T, v) = E - TS.$$

Antonov [6] has shown that such systems have spherical symmetry. If $M_0 \gg m$, then evidently

$$f_0(x, v) = \delta^3(x) \delta^3(v),$$

where

$$\delta^3(x) = \delta(x_1) \delta(x_2) \delta(x_3),$$

i.e. the body of a mass M_0 must be situated at the centre of the sphere. Here we neglect the role of stochastic processes in the dynamics of massive object [15].

Let us calculate the free energy F using the formula for entropy

$$S = -k \int f_m \ln f_m d^3x d^3v,$$

where k is the Boltzmann constant.

The system's energy E is

$$E = \frac{1}{2} \int m v^2 f_m d^3x d^3v - \frac{G}{2} \iint \frac{f(x, v) f(x', v')}{|r - r'|} d^3x d^3v d^3x' d^3v',$$

where

$$|r - r'| = \sqrt{\sum_{i=1}^3 (x_i - x'_i)^2},$$

$$f(x, v) = M_0 f_0(x, v) + m f_m(x, v). \quad (3.1)$$

The problem reduces to the estimation of the minimum of F when the total number of stars

$$N = \int f_m d^3x d^3v \quad (3.2)$$

is fixed.

By means of Lagrange multiplier methods one has

$$\delta(F + k\alpha N) = \delta(E - TS + k\alpha N) = 0.$$

After elementary calculations it follows that F has an extremum when

$$f_m(x, v) = A \exp\left[-\frac{m}{kT} \left(\frac{v^2}{2} - \varphi(x)\right)\right], \quad (3.3)$$

where

$$\varphi(x) = G \int \frac{f(x', v')}{|r - r'|} d^3x' d^3v' = \frac{GM_0}{r} + Gm \int \frac{f_m(x', v')}{|r - r'|} d^3x' d^3v' \quad (3.4)$$

$$= \frac{GM_0}{r} + G \int \frac{P_m(x')}{|r - r'|} d^3x',$$

$$P_m(x) = \int m f_m(x, v) d^3v = B \exp(\beta\varphi);$$

$$A = \exp[-(\alpha + 1)] = \text{const},$$

$$B = A \left(\frac{2\pi}{\beta}\right)^{3/2},$$

$$\beta = \frac{m}{kT},$$

$$r = \sqrt{\sum_{i=1}^3 x_i^2}.$$

(3.5)

From (3.4), (3.5) for $\varphi(x)$ we have

$$\Delta\varphi(x) = -4\pi GM_0 \delta^3(x) - 4\pi GB \exp(\beta\varphi(x)), \quad r < r_e;$$

$$\Delta\varphi(x) = 0, \quad r > r_e, \quad (3.6)$$

where $\varphi(x)$ is continuous at $r = r_e$ with its derivative.

According to Antonov's theorem the system has spherical symmetry at an equilibrium state, therefore Eq.(3.6) can be replaced by

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\varphi}{dr} \right) = -4\pi GM_0 \delta(r) - 4\pi GB \exp(\beta\varphi(r)), \quad r < r_e;$$

$$\varphi(r) = \frac{GM_0}{r} + \frac{GM}{r}, \quad r > r_e; \quad (M = Nm), \quad (3.7)$$

where $\varphi(r), \frac{d\varphi}{dr}$ are continuous at $r = r_e$.

If

$$\varphi(r) = \frac{GM_0}{r} + \psi(r),$$

then $\psi(r)$ is the solution of the following equation:

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\psi}{dr} \right) = -4\pi GB \exp\left(\beta\psi + \frac{GM_0}{r}\beta\right), \quad r < r_e;$$

$$\psi(r) = \frac{GM}{r}, \quad r > r_e, \quad (3.8)$$

(3.8)

where $\psi(r)$, $\frac{d\psi}{dr}$ are continuous at $r=r_e$.

Introducing the following notations ($\psi(0)$ is assumed finite)

$$\begin{aligned} v &= \beta(\psi - \psi_0), \\ R &= [4\pi G \beta B \exp(\beta\psi(0))]^{1/2} r = (4\pi G \rho_0 \beta)^{1/2} r, \quad (3.9) \\ z &= (4\pi G \rho_0 \beta)^{1/2} r_e, \\ \mu &= M_0/M, \quad \tilde{\beta} = GM\beta/r_e, \end{aligned}$$

Eq.(3.8) can be rewritten in a form

$$\frac{d^2 v}{dR^2} + \frac{z}{R} \frac{dv}{dR} + \exp\left[v + \mu \tilde{\beta} \frac{z}{R}\right] = 0, \quad (3.10)$$

$$v(0) = \left. \frac{dv}{dR} \right|_{R=0} = 0.$$

So far as at $r > r_e$

$$\psi(r) = \frac{GM}{r} \quad \text{and} \quad \frac{d\psi}{dr} = -\frac{GM}{r^2},$$

we have from Eq. (3.9)

$$\frac{d\psi}{dr} = \frac{1}{\beta} \frac{z}{r_e} \frac{dv(R)}{dR}.$$

From the continuity of the solution at the boundary $r=r_e$ ($R=z$), it follows that

$$\left. \frac{d\psi(r)}{dr} \right|_{r=r_e} = -\frac{GM}{r_e^2} = \frac{1}{\beta} \frac{dv(R)}{dR} \Big|_{R=z} = \frac{1}{\beta} \frac{z}{r_e} v'(z),$$

whence

$$\tilde{\beta} = \frac{GM\beta}{r_e} = -z v'(z). \quad (3.11)$$

Finally, using Eq.(3.11) one can present Eq.(3.10) in a form

$$\begin{aligned} \frac{d^2 v}{dR^2} + \frac{z}{R} \frac{dv}{dR} + \exp\left[v - \mu \frac{z^2 v'(z)}{R}\right] &= 0, \\ v(0) = \left. \frac{dv}{dR} \right|_{R=0} &= 0, \quad 0 \leq R \leq z. \end{aligned} \quad (3.12)$$

4. Calculation Procedure.

As distinct from elementary theory of catastrophe (some results are briefly described in Sect. 2), in our problem one should minimize a functional depending on a function and parameter rather than a function of a variable and parameter. That functional - free energy F depends on distribution function f and $\tilde{\beta}$: $F(f(\cdot), \tilde{\beta})$. As it is seen from (3.3), $f(\cdot)$ is determined via $v(\cdot)$. However we are interested only in spherically symmetric systems which can be described not by whole function $v(\cdot)$, but only by its value on the boundary z . Thus our functional transforms into a simple function which depends on $u(z) = -v(z)$ and $\tilde{\beta}$:

$$F(f(\cdot), \tilde{\beta}) \rightarrow F(v(\cdot), \tilde{\beta}) \rightarrow F(u, \tilde{\beta}).$$

Consider u as a generalized coordinate for spherically symmetric system. From Eq.(3.11) we have $v'(z) < 0$, i.e. $v(z)$ is a monotonously decreasing function of z . So long as $v(0) = 0$, then $v(z) < 0$ for all $z > 0$. Thus, $v(z)$ is a negative monotonously decreasing function of z at $z > 0$.

We have to solve Equation (3.12) in order to answer the following question: at what temperature of hard thermostat (for fixed u) the system is in an equilibrium state, i.e. $\delta F = 0$, when the number of stars is constant.

In other words we are interested in parametric curve $\tilde{\beta} = \tilde{\beta}(z)$, $u = u(z)$ on the plane $(\tilde{\beta}, u)$ with a parameter z . Knowing this curve one can answer the above question. So far as $V(z)$ is a monotone function, a single value of V corresponds to fixed z , and therefore the function $\tilde{\beta} = \tilde{\beta}(v)$ is defined uniquely.

In order to find the curve $\tilde{\beta} = \tilde{\beta}(z)$, $v = v(z)$, one has to solve Eq.(3.12) for given z . However, as is seen from Eq.(3.12), it is impossible to integrate it directly, because the value of $v'(z)$ is needed as well. Meanwhile we solve (3.12) to find just the $v'(z)$ ($\tilde{\beta} = -z v'(z)$).

In order to escape from this principal difficulty we shall act as follows. Instead of Eq.(3.12) let us solve the following equation:

$$\frac{d^2 v}{dR^2} + \frac{z}{R} \frac{dv}{dR} + \exp\left[v + \mu \frac{z^2 \lambda}{R}\right] = 0, \quad (4.1)$$

$$v(0) = v'(0) = 0, \quad 0 \leq R \leq z,$$

where λ is a positive constant, μ is fixed. Solving it for given z and λ we have

$$v(R) = g(R; z, \lambda), \quad (4.2)$$

$$v'(R) = \frac{\partial g(R; z, \lambda)}{\partial R},$$

and on the boundary $R = z$

$$v(z) = g(z; z, \lambda),$$

$$v'(z) = \left. \frac{\partial g(R; z, \lambda)}{\partial R} \right|_{R=z} \equiv h(z). \quad (4.3)$$

The necessary condition for (4.2) to be a solution of Eq.(3.12) is $v'(z) = \lambda > 0$ (therefore we consider positive λ only).

For λ we have the following equation

$$h(z, \lambda) + \lambda = 0. \quad (4.4)$$

Determining $\lambda = \lambda(z)$ from this equation one can obtain

$$\tilde{\beta}(z) = -z v'(z) = -z h(z, \lambda(z)),$$

$$v(z) = g(z, \lambda(z)).$$

Thus the problem reduces to solving Eqs (4.1) and (4.4), which is possible only numerically. The calculations show that these equations or Eq.(3.12) are mathematically incorrect [16], i.e. the solution $v(R)$ strongly depends on initial data.

Using the known properties of $v(z)$ (monotony, negativity) and definiteness of β with respect to v , according to the methods of solutions of incorrect equations it is possible to obtain correct points. The corresponding solutions are stable with respect to the increasing of accuracy of computer simulations.

5. Catastrophes in Stellar Systems.

First, let us discuss the curve obtained by Lynden-Bell and Wood [7] for $\mu = 0$ (Fig.1a). The potential function $F(u, \tilde{\beta})$ has qualitatively different behaviour for different values of $\tilde{\beta}$ (temperatures of thermostat).

Consider now the behaviour of the function F from the point of view of catastrophe theory. That function depends on a variable u and single parameter $\tilde{\beta}$. On $(\tilde{\beta}, -v)$ plane the curve $\tilde{\beta}(-v)$ is a projection of extremal points of function F . Projecting the curve $\tilde{\beta} = \tilde{\beta}(-v)$ onto the axis $\tilde{\beta}$, one has a catastrophe of a fold type (2.5). From Lynden-Bell's and Wood's results it follows that infinitely large number of separatrices

exist (Fig.1b).

Using the conjugation theorem Katz had investigated the stability points in the sequence of folds on equilibrium trajectory [21,22,17,18]. Particularly he had shown that the system with $T, V = \text{const}$ with increasing u loses stability in the vicinity of the first fold (β_{\max}, u_{\max}), and restabilization does not subsequently take place. Thus, at $\beta > \beta_{\max}$ and/or $u > u_{\max}$ there cannot occur any equilibrium state.

If assuming simultaneously that the system preserves the spherical symmetry, one can see that it will collapse catastrophically. This is the so-called "gravothermal catastrophe". It is remarkable that here the term "catastrophe" can be understood both in the sense of catastrophe theory (Sect. 2) and in the physical sense.

What will happen with this catastrophe if the system contains central point mass?

As our calculations have shown, when $\mu \leq 10^{-3}$ the curve hardly differs from that of $\mu = 0$ (Fig.1a); however it begins to change significantly as μ increases.

Series of curves corresponding to different values of μ is shown in Fig.2. One can see that the fold catastrophe occurs at every mass value, so that β_{\max} decreases for higher μ (Fig.3). It means that the system with central mass is less stable, i.e. to have it stable one should heat the thermostat to higher temperatures (for equal stellar masses, etc.).

From Fig.4 and the results of [18,21,22] it follows that systems with central mass are stable only when their contrast of densities ^{*} is sufficiently less than the corresponding values of systems without central mass. In other words, the considered systems must be significantly more homogeneous.

^{*} More precisely, u is $\ln \frac{\rho_0}{\rho} + \mu \hat{\beta}$, rather than $\ln \frac{\rho_0}{\rho}$.

In spite of the fact that our problem depends on two parameters (β, μ), catastrophe of a "cusp" type does not occur, as it could be for two-parametric potential (see Sect. 2).

6. Conclusion.

In the present paper we investigated the stability of stellar systems containing central massive point-like body. During the study we used methods and some results of the catastrophe theory.

It is shown that the gravothermal catastrophe being an A_2 -"fold" type catastrophe not only occurs for systems with central mass, but becomes substantially more "destroying". First, the catastrophe occurs at higher temperatures than for usual isothermal sphere. Second, which is more important, it begins at sufficiently lesser values of density contrast, i.e. at equilibrium states these systems are to be more homogeneous. At first sight, this result seems to be somewhat unexpected, so far as according to widely adopted ideas a massive object situated at the centre of a stellar system must cause an additional density cusp ^{*} (see, e.g. [20]). Just this effect is the basis of hypotheses on existence of massive black holes in some globular clusters (e.g. M 15) and galaxies (e.g. M 87), demonstrating additional central luminosity spikes.

Meanwhile our result is physically quite natural. Indeed, if the catastrophe in a system without a central mass occurs, say, at ρ_0^* , and the existence of a mass M_0 leads to an effective increase of central density in a physically small volume $V (r \leq r_0; r_0 \ll R)$: $\rho_0^M + \frac{M_0}{V}$, then the system becomes unstable when $\rho_0^M + \frac{M_0}{V} \sim \rho_0^*$, i.e. $\rho_0^M < \rho_0^*$. Note that the larger is M_0 , the stronger is this inequality. In other words,

^{*} This is connected with accumulation of stars on finite orbits around the central object.

the stability of the system with a central mass requires compensation of certain part of central density ("negative density cusp" *).

It is worth to emphasize once more that the fold catastrophe does not turn to a cusp when a new control parameter (the central mass) appears, as it could in general take place.

In conclusion we note that the investigation of the problem in its simplest form in the manner of Antonov, Lynden-Bell and Wood, is conditioned by a belief, which is realized relatively recently (see [19]) and is neglected in a number of studies: the accounting of as many as possible different effects not always makes the model more realistic.

The present investigation is stimulated by the splendid book of V.I. Arnold [11].

The authors are sincerely grateful to V.A. Ambartsumian, V.A. Antonov, G.A. Gurzadyan, R.L. Mkrtchyan and A.G. Sedrakyan for the fruitful discussions, and to A.M. Fridman who kindly called our attention to the paper of Lindblad.

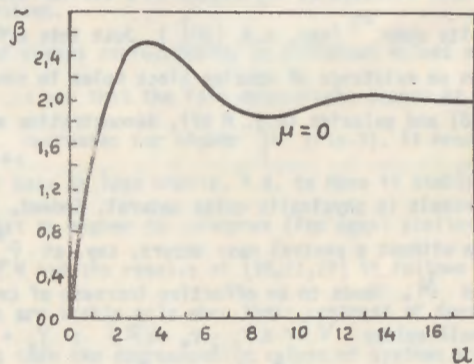
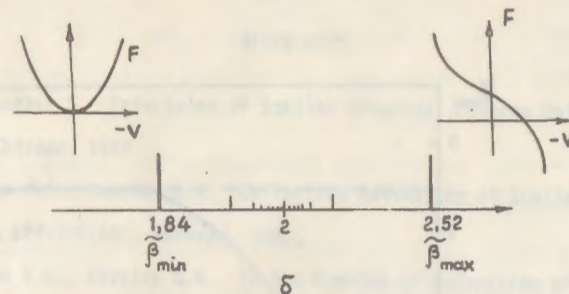


Fig.1. a) Dependence of temperature parameter $\tilde{\beta}$ on relative density $u = \ln \frac{\rho_0}{\rho}$ at $\mu = 0$;

*) For this remark we are indebted to V.A. Ambartsumian.



b) Sequence of separatrices corresponding to fold catastrophe.

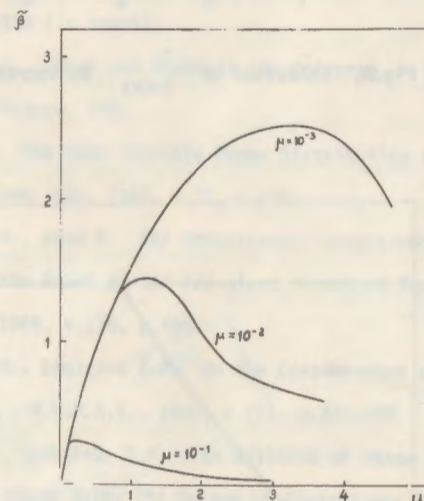


Fig.2. Dependence of $\tilde{\beta}$ on u at different values of the central mass parameter μ .

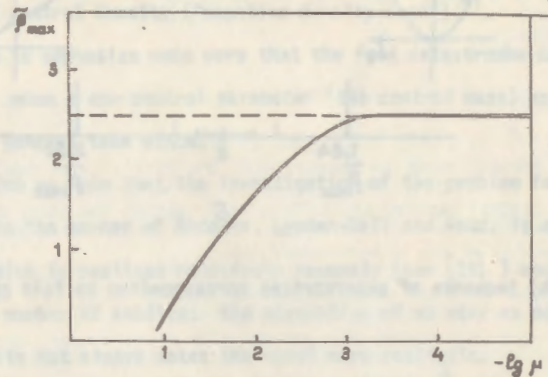


Fig.3. Behaviour of $\tilde{\beta}_{max}$ at increasing μ

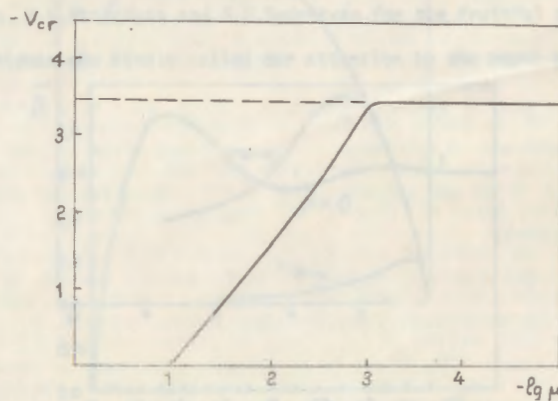
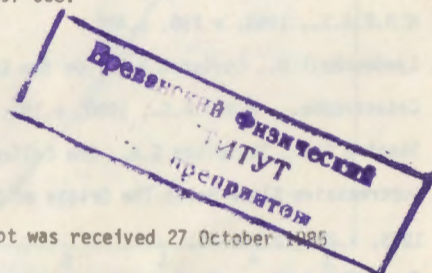


Fig.4. Behaviour of $-V_{cr}$ corresponding to values $\tilde{\beta}_{max}$ at increasing μ

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