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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

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NONLINEAR THEORY OF AMPLIFICATION IN
THE RELATIVISTIC STROPHOTRON FEL

ЦНИИатоминформ

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ՈՒԿՆԱԹԱՆ ՈՉ ԳՆԱՅԻՆ ՏԵՍՈՒԹՅՈՒՆԸ ՌԵԼՅԱՏԻՎԻՍՏԻԿ
ՍՏՐՈՓՈՏՐՈՆԱՅԻՆ ՏԻՊԻ ԱՋԱՏ ԷԼԵԿՏՐՈՆԱՅԻՆ ԼԱՋԵՐՈՒՄ

Առաջարկվում է ռելյատիվիստիկ ստրոֆոտրոնային ազատ էլեկտրոնային լազերի/ՈՒԷԼ/ ոչ գծային տեսութունը: Ենթադրվում է, որ դանդաղ շարժման փուլը բավարարում է մոմենտի հավասարմանը: Բերվում են փուլի և անշափողական ժամանակի/կամ հազեցման պարամետրի/ արտահայտությունները, և ցույց է տրվում, որ դրանք էպես ստրոֆոտրոնային ստիմուլացիայի լազերի համապատասխան արտահայտություններին: Ցույց է տրվում, որ համակարգի ռեզոնանսային համապատասխանությունը ուժեղ կախված է գտնվում էլեկտրոնի սկզբնական լայնական x_0 կոորդինատի հետ: Ուժեղ դաշտի սահմանում բերվում է ոչ գրծային ուժեղացման գործակցի անալիտիկ արտահայտություններ:

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The relativistic strophotron FEL is the system in which electrons move along the parabolic one dimensional potential trough, whose vector potential is given by

$$A_z = g x^2 / 2 \quad (1)$$

where x is one of the transverse coordinates, g is the magnetic field gradient. The wave amplified is assumed to propagate along the axis of the trough (Z -axis). The classical equations of electron motion can be reduced to the form

$$\dot{x} = -\frac{eE_0}{\varepsilon_0} \cos \varphi, \quad (2)$$

$$(1+\omega)\dot{\varphi} = \dot{\varphi}(t_0) - \frac{1}{2}\Omega^2\omega(x^2 - x_0^2) \quad (3)$$

$$\ddot{x} + \Omega^2 x = -\omega\ddot{x} - \frac{eE_0}{\varepsilon_0} \left(\frac{\dot{\varphi}}{\omega} - \dot{x}^2 \right) \cos \varphi + \frac{\Omega^2 x}{1+\omega} \left[\frac{\dot{\varphi}(t_0)}{\omega} - \frac{\Omega^2}{2}(x^2 - x_0^2) \right], \quad (4)$$

where E_0 and ω are the electric field strength and frequency of the wave, $c = 1$, $-e$ is the electron charge,

$$\varphi = \omega(t-z), \quad \omega = \frac{\varepsilon - \varepsilon_0}{\varepsilon_0} \quad (5)$$

ε is the time-dependent electron energy, the index "0" indicates the initial value of all the variables at the time $t=t_0$ when the electron enters the system,

$$\Omega = (ge/\varepsilon_0)^{1/2} \quad (6)$$

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is the frequency of strophotron oscillations [1].

The terms in brackets of Eq.(4) determine either small corrections to the frequency Ω (6) or a small anharmonicity of oscillations. Here these small corrections will be ignored, and all the mentioned terms in Eq.(4) will be dropped out. In this (harmonic) approximation the field-free ($E_0 = 0$) solution of Eq.(4) is given by $x^{(0)} = a_0 \cos[\Omega(t-t_0) + \theta_0]$, where $a_0 = (x_0^2 + \dot{x}_0^2/\Omega^2)^{1/2}$ is the field-free amplitude of strophotron oscillations, $\theta_0 = \arctg \alpha/x_0\Omega$, $x_0 = x(t=t_0)$, $\dot{x}_0 = \dot{x}$, $\alpha \equiv \alpha_x$ is the angle in the plane (X,Z) under which the electron enters the system. The first corrections to $x^{(0)}(t)$ ($\propto E_0$) have been found in Ref.[1] and in this approximation $X(t)$ can be written in the form

$$x(t) = a(t) \cos[\Omega(t-t_0) + \theta(t)], \quad (7)$$

where $a(t)$ and $\theta(t)$ are some slowly varying functions (amplitude and phase)

$$|\dot{a}/a|, |\dot{\theta}| \ll \Omega. \quad (8)$$

We will assume that in the general case the expression (7) for $X(t)$ and the condition (8) hold for any values of E_0 . This assumption about slow variations of $a(t)$ and $\theta(t)$ is justified if

$$|w| \ll 1, \quad a\Omega \ll 1 \quad (9)$$

i.e. if the total change of electron energy is small in comparison with \mathcal{E}_0 and if the amplitude of oscillations "a" is small in comparison with their period along the z-axis $\lambda_0 = 2\pi/\Omega$. The latter condition means in particular that $|\alpha| \ll 1$ and $d_e \Omega \ll 1$, where d_e is the diameter of the beam in the (x,z)

plane ($|x_0| \lesssim d_e$). We will assume also that $\gamma \equiv \mathcal{E}_0/m \gg 1$. Under these conditions Eq.2 give $|\dot{\psi}/\omega| \ll 1$. Besides, it is possible to show that the change of amplitude $\delta a \equiv a(t) - a_0$ is small in comparison with a_0 , $|\delta a| \ll a_0$. The condition $\Omega a \ll 1$ (9) justifies also the mentioned above possibility to neglect all the terms in the square brackets of Eq.4.

The relative emitted energy W contains both fast and slow varying parts, W_{fast} and W_{sl} . The equations of motion (4) can be averaged over the period of fast oscillations $2\pi\Omega^{-1}$ to give equation for the slowly varying functions W_{sl} , θ and δa only.

A procedure of separation of fast and slow motions does work well enough only near resonances.

The strophotron FEL is characterized by it's resonance frequency

$$\omega_{res} = \frac{\Omega}{1/2\gamma^2 + a_0^2\Omega^2/4}. \quad (10)$$

Amplification can occur near odd harmonics of ω_{res} , $\omega \approx (2s+1)\omega_{res}$, $s=0,1,2,\dots$ [1,2]. We will consider the case $s \gg 1$ assuming the detuning from the (2s+1)-st resonance

$$\Delta_s = \omega \left(\frac{1}{2\gamma^2} + \frac{a_0^2\Omega^2}{4} \right) - (2s+1)\Omega \quad (11)$$

is small $|\Delta_s| \ll \Omega$.

Under these conditions the equations for W_{sl} , $\theta(t)$ and $\delta a(t)$ can be found from Eqs (2), (3) and (4)

$$\dot{W}_{sl} = - \frac{eE_0}{2c_0} \frac{a_0}{\Omega} \left[J_s(\xi) - J_{s+1}(\xi) \right] \sin \psi \quad (12)$$

$$\dot{\theta} = - \frac{\Omega}{2} W_{sl}, \quad (13)$$

$$\delta \dot{a} = -\frac{eE_0}{4\gamma^2 \epsilon_0 \Omega} [J_s(\bar{z}) - J_{s+1}(\bar{z})] \sin \Psi, \quad (14)$$

where ω_{s1} is the slow varying part of ω , $\bar{z} = a_0^2 \Omega \omega / 8$,

$$\Psi \equiv \omega t_0 + \Delta_s(t-t_0) - (2s+1)\theta + \bar{z} \sin 2\theta - \frac{\Omega^2 \omega a_0}{2} \int_{t_0}^t \delta a dt - \omega \left(\frac{1}{2\gamma^2} + \frac{a_0^2 \Omega^2}{4} \right) \int_{t_0}^t \omega_{s1} dt \quad (15)$$

is the phase of the slow (resonance) motion of electron. The phase Ψ is determined not only by the detuning Δ_s and the phase θ of the transverse electron coordinate $X(t)$ (7) but also by the change of amplitude of $X(t)$ (7) $\delta a(t)$ and by the change of electron energy $\omega(t)$.

Combining Eqs (12), (13) and (14) we find equation for the phase Ψ (15), which can be written as the usual pendulum equation [3].

$$\frac{d^2 \Psi}{d\mu^2} = \sin \Psi \quad (16)$$

with the initial conditions

$$\Psi(\mu=0) = \omega t_0, \quad \frac{d\Psi}{d\mu}(\mu=0) = \frac{\Delta_s}{\Delta_m}, \quad (17)$$

where the dimensionless time (or the saturation parameter) μ is given by

$$\mu = (t-t_0) \left\{ \frac{eE_0 \omega \Omega a_0}{4\epsilon_0} \left(\frac{1}{\gamma^2} + \frac{\Omega^2 a_0^2}{4} \right) [J_s(\bar{z}) - J_{s+1}(\bar{z})] \right\}^{1/2}, \quad (18)$$

$\Delta_s = \epsilon - \epsilon_{res}$, ϵ_{res} is that value of the initial electron energy at which $\Delta_s = 0$ (10),

$$\Delta_m = \frac{4\epsilon}{\omega} \tilde{\Delta}_m / \left(-\frac{3}{\gamma^2} + \frac{\alpha^2 - X_0^2 \Omega^2}{4} \right), \quad (19)$$

$$\tilde{\Delta}_m = \mu/t.$$

The emitted energy $\Delta \bar{\epsilon} = \epsilon_0 \omega_{s1}$ is given either by Eq (12) or by the following equivalent equation

$$\Delta \bar{\epsilon} = \frac{2\epsilon}{\omega \left(\frac{1}{\gamma^2} + \frac{a_0^2 \Omega^2}{4} \right)} \left(\Delta_s - \tilde{\Delta}_m \frac{d\Psi}{d\mu} \right). \quad (20)$$

In the weak-field approximation Eq.(16) solved by the iteration method to give results coinciding with the result of Refs. [1,2]

In the strong-field asymptotics $\mu \gg 1$, $\tilde{\Delta}_m \gg |\Delta_s|$ the phase Ψ obeying the pendulum equation and averaged over its initial value $\Psi_0 = \Psi(t=t_0)$ and the energy $\Delta \bar{\epsilon}$ emitted by a single electron are known analytically [3].

The strong-field characteristic detuning $\tilde{\Delta}_m$ determines the order of magnitude of the detuning Δ_s at which the emitted energy $\Delta \bar{\epsilon}(\Delta_s)$ achieves its maximum. The maximum emitted energy $\Delta \bar{\epsilon}_{max}$ in the strong-field limit has the order of magnitude

$$\Delta \bar{\epsilon}_{max} \sim \Delta \bar{\epsilon}(\Delta_s \sim \tilde{\Delta}_m). \quad (21)$$

Hence as a whole the function $\Delta \bar{\epsilon}(\Delta_s)$ in the strong-field asymptotics $\mu \gg 1$ can be written as

$$\Delta \bar{\epsilon} = \frac{2\epsilon \tilde{\Delta}_m}{\omega \left(\frac{1}{\gamma^2} + \frac{a_0^2 \Omega^2}{4} \right)} \left\{ 1 - \frac{2}{\sqrt{\pi} \mu} \sin \left(\mu + \frac{\pi}{4} \right) \right\} F \left(\frac{\Delta_s}{\tilde{\Delta}_m} \right), \quad (22)$$

where $F(\xi)$ is some dimensionless function, $F(\xi) = -F(-\xi)$, $F(0) = 0$, having its maximum $F_{max} \sim 1$ at $\xi \sim 1$. In the region $\xi \ll 1$, $F(\xi) \sim \xi$. For any $|\xi| \geq 1$, F can not be found and written down analytically and requires a numerical calculation [4].

Substituting $\omega_{s1} = (2s+1) \omega_{res}$ (10) into Eq.(21) we obtain an estimate of efficiency of RS FEL:

$$\eta = \frac{\Delta \bar{\epsilon}_{max}}{\epsilon} \sim \frac{\mu}{N_0(2s+1)}, \quad (23)$$

where $N_0 = \Omega t / 2\pi = L/\lambda$ is the number of strophotron oscillations along the length of the system L . When $S=0$ the estimate (23) coincides with the results well known in the usual undulator FEL [3]. When S becomes large, η (23) decreases.

One of the main conclusions of our previous papers [1,2] consisted in a prediction of a very strong inhomogeneous broadening connected with a distribution of electrons over the beam's cross section. The result has been derived earlier [1,2] in the weak-field approximation. But, as a matter of fact, this feature of the RS FEL is general enough to hold good for any field strength E_0 . The reason of this inhomogeneous broadening is in a strong dependence of the resonance frequency ω_{res} (10) or the initial transverse electron coordinate X_0 (through the amplitude A_0). To take into account this inhomogeneous broadening in the strong field limit we must average the emitted energy $\Delta \bar{\epsilon}$ (22) over X_0 .

The gain is proportional to the averaged resonance emitted energy $\langle \Delta \bar{\epsilon} \rangle_{res}$ and has a form

$$G = \frac{8\pi N_0}{E_0^2} \langle \Delta \bar{\epsilon} \rangle_{res} = \frac{8(\ln 2)^{1/2} \left[\frac{e\alpha^3}{\epsilon\omega} (J_s(z) - J_{s+1}(z)) \right]^{3/4}}{d_e \Omega E_0^{5/4} \left(\frac{1}{8} + \frac{\alpha^2}{4} \right)} \quad (24)$$

$$\cdot \tilde{F}_{res}(\omega) \left\{ 1 - \frac{2}{\sqrt{\pi}\mu} \sin\left(\omega + \frac{\pi}{4}\right) \right\},$$

where N_0 is the electron number density, $\tilde{F}_{res}(\omega)$ is the form-factor of the resonance curve.

As a function of E_0 the nonlinear gain (24) falls down as $E_0^{-5/4}$ (in contrast with the law $E_0^{-3/2}$ in the usual undulator FEL [3]).

Discussion

Taking $\omega = \omega_{max} = 3\gamma^3 \alpha \Omega$, $S = S_{min}(\omega = \omega_{max}) \approx \frac{3}{8} (\alpha\gamma)^3 [1]$ for the saturation parameter μ (18) we obtain

$$\mu = \left(\frac{\sqrt{3}}{8\pi} \frac{eE_0 \alpha^2 \Omega}{mc} \right)^{1/2} t. \quad (25)$$

A saturation occurs when $\mu > 2$ or $E_0 \approx E_{0,sat}$ where the saturation field strength is given by

$$E_{0,sat} = \frac{32\pi mc}{\sqrt{3} e \Omega (\alpha t)^3}. \quad (26)$$

Substituting here $g = 10^4 \text{ oe/cm}$, $L = 3\text{m}$, $\alpha = 0.7$ we find $E_0 \sim 10^3 \text{ V/cm}$.

This estimate shows that the gain saturates at rather moderate field E_0 and hence the efficiency of the RS FEL is not too high. This result is not surprising because we have considered here an amplification at very high harmonics ($S \sim 10^2$). There is an evident way to increase the efficiency constructing a tapered RS FEL, i.e. a system with $g = g(z)$. We hope to consider this improvement in details later.

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References

1. Fedorov M.V., Oganesyanyan K.B. "Classical Theory of Emission and Amplification at High Harmonics in the Relativistic Strophotron FEL". IEEE J. of Quant. Electr. 1985, QE-21, N.7, pp 1059-1069.
2. Заренский Д.Ф., Нерсесов Э.А., Оганесян К.Б., Федоров М.В. Лазер на свободных электронах, движущихся в полях с поперечным градиентом. Квантовая электроника, 1986, т.13, № 4.
3. Федоров М.В. Взаимодействие электронов с электромагнитным полем в лазерах на свободных электронах. УФН, 1981, т.135, с.213-234.
Fedorov M.V. Free-Electron Lasers: Amplification, Multi-photon Transitions, Saturation and Geometry IEEE J. of Quant. Electr. QE-17, N.8, 1981, pp 1359-1363.
4. Louisell W.H., Lam J.F., Copeland D.A., Colson W.B. Exact Classical Electron Dynamic Approach for a FEL Amplifier. Phys.Rev., A19, 1979, pp. 288-300.

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НЕЛИНЕЙНАЯ ТЕОРИЯ УСИЛЕНИЯ В ЛАЗЕРЕ НА СВОБОДНЫХ ЭЛЕКТРОНАХ
ТИПА РЕЛЯТИВИСТСКОГО СТРОФОТРОНА

(на английском языке, перевод авторов)

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