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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

Yu.G.SHAKHNAZARYAN

ANGULAR DISTRIBUTIONS FOR THREE-JET PROCESS $e^+e^- \to q\bar{q}g$ FOR HEAVY QUARKS

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Յու.Գ.ՇԱՀՆԱԶԱՐՅԱՆ

$e^+e^- \to q\bar{q}g$ ՆԽԱՓՈՒՆՋ ՊՐՈՑԵՍԻ ԱՆԿՅՈՒՆԱՅԻՆ ԲԱՇԽՈՒՄՆԵՐԸ Ե ՆՈՐ ՔՎԱՐԿՆԵՐԻ ԴԵՊՔՈՒՄ

Սկզբնական մասնիկների բևեռացման և քվարկի զանգվածի հաշվառմամբ, քվանտային քրոմոդինամիկայի առաջին մոտավորությամբ հաշված $e^+e^- \to q\bar{q}g$ պրոցեսի կտրվածքի ամենաընդհանուր արտահայտության հիման վրա գտնված է $d^3\sigma/dTd\hat{\vec{T}}$ կտրվածքի կախվածությունը առավելագույն $\vec{T}$ իմպուլս ունեցող պարտոնի /քվարկի, հակաքվարկի կամ գլյուոնի/ բևեռային և ազիմուտային անկյուններից: T-ի մի քանի արժեքների համար կատարված թվային հաշվարկը ցույց է տալիս, որ կտրվածքի անկյունային կախվածությանը բնորոշ գործակիցները էապես կախված են քվարկի զանգվածը բնութագրող η պարամետրից:

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Yu.G. SHAKHNAZARYAN

ANGULAR DISTRIBUTIONS FOR THREE-JET PROCESS $e^+e^- \to q\bar{q}g$ FOR HEAVY QUARKS

On the basis of the most general expression for $e^+e^- \to q\bar{q}g$ process cross section calculated in the first order in α_s with regard to primary particles polarization and the mass of quark, the dependence of the cross section $d^3\sigma/dT\,d\hat{T}$ on the polar and azimuthal angles of the vector $\vec{T}$ (thrust) spatial orientation is found. Calculations carried out at some values of T show that the coefficients determining the angular dependence of the cross section considerably depend on the mass parameter η. An analysis of angular distributions depending on the values of the parameters T and η is made.

Yerevan Physics Institute

Yerevan 1986

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Ю.Г.ШАХНАЗАРЯН

УГЛОВЫЕ РАСПРЕДЕЛЕНИЯ ДЛЯ ТРЕХСТРУЙНОГО ПРОЦЕССА $e^+e^- \to q\bar{q}g$ В СЛУЧАЕ ТЯЖЕЛЫХ КВАРКОВ

На основе вычисленного в первом порядке по α_s наиболее общего выражения для сечения процесса $e^+e^- \to q\bar{q}g$ с учетом поляризаций начальных частиц и массы кварка найдена зависимость сечения $d^3\sigma/dTd\hat{\vec{T}}$ от полярного и азимутального углов пространственной ориентации вектора $\vec{T}$, характеризующего импульс наиболее энергичного партона независимо от того, является ли им кварк, антикварк или глюон. Численные расчеты, выполненные при некоторых значениях величины T, указывают на то, что коэффициенты, определяющие угловую зависимость сечения, существенным образом зависят от массового параметра η. Проведен анализ угловых распределений в зависимости от величины параметров T и η.

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In the present work the influence of the heavy quark mass on the angular distributions of three-jet events in the polarized electron-positron pair annihilation process is studied. An analogous consideration without regard to the mass of quarks was carried out in Ref. [1]. Whereas, as the results from Refs [2-6] show, taking the mass of heavy quarks into account exerts a noticeable influence on the theory predictions. The most general expression for the cross section of the process $e^+e^- \to q\bar{q}g$ with polarized initial particles and with regard to quark mass, calculated in the first order over α_s has the form:

$$d\sigma = \frac{\alpha^2 \alpha_s Q^2}{(2\pi)^2} \; \frac{1}{S} \; \frac{dx_1' \, dx_2'}{(1-x_1')(1-x_2')} \Big\{ 2(x_1^2 + x_2^2)(1 + \xi_1^{\parallel} \xi_2^{\parallel}) -$$

$$- x_1^2 \Big(1 - \frac{\eta}{2} \frac{1-x_2'}{1-x_1'}\Big) \Big[(1 + \vec{\xi}_1 \vec{\xi}_2)(1 - z_1^2) - 2(\vec{n}_1 \vec{\xi}_1^{\perp})(\vec{n}_1 \vec{\xi}_2^{\perp}) \Big] -$$

$$- x_2^2 \Big(1 - \frac{\eta}{2} \frac{1-x_1'}{1-x_2'}\Big) \Big[(1 + \vec{\xi}_1 \vec{\xi}_2)(1 - z_2^2) - 2(\vec{n}_2 \vec{\xi}_1^{\perp})(\vec{n}_2 \vec{\xi}_2^{\perp}) \Big] -$$

$$- \eta \Big(\frac{(x_1' - x_2')^2}{(1-x_1')(1-x_2')} (1 + \xi_1^{\parallel} \xi_2^{\parallel}) + x_1 x_2 \Big[(1 + \vec{\xi}_1 \vec{\xi}_2)(z_{12} - z_1 z_2) - \qquad (1)$$

$$- (\vec{n}_1 \vec{\xi}_1^{\perp})(\vec{n}_2 \vec{\xi}_2^{\perp}) - (\vec{n}_1 \vec{\xi}_2^{\perp})(\vec{n}_2 \vec{\xi}_1^{\perp}) \Big] \Big) \Big\} \, dz_1 \, d\varphi_1 \, d\varphi_2' \, .$$

Here S is the square of the reaction total energy, $\vec{\xi}_{1(2)}$ is the electron (positron) polarization vector in its own rest frame. Subdivision of the vectors $\vec{\xi}_1$ and $\vec{\xi}_2$ into longitudinal and transverse components is done relative to electron momentum direction $\vec{v}$; $\eta = 4m^2/S$ is a dimensionless parameter which helps to study the influence of the heavy quark mass; m and Q are the mass and the charge of quark (in the units of e); $\vec{n}_i$ are unit vectors along the quark, antiquark and gluon momenta (i = 1,2,3 for q , $\bar{q}$, g , respectively); $x_i = 2p_i/\sqrt{S}$ and $x_i' = 2E_i/\sqrt{S}$ characterize the dimensionless momentum and dimensionless energy of i - parton, respectively, and are related as

$$x'_{1,2} = (x^2_{1,2} + \eta)^{1/2}, \quad x'_3 = x_3 .$$

Angular variables are determined to be $z_{ij} = \cos\theta_{ij}$, θ_{ij} is the angle included between $\vec{n}_i$ and $\vec{n}_j$ vectors, $z_i = \cos\theta_i$, θ_i is the polar angle of $\vec{n}_i$ vector in the coordinate frame with its polar axis along the vector $\vec{v}$, φ_1 is the azimuthal angle between the electron polarization plane ($\vec{v}, \vec{\xi}_1$) and the plane of quark production ($\vec{v}, \vec{n}_1$) in the mentioned coordinate frame, φ_2' is the azimuth angle between ($\vec{n}_1, \vec{v}$) and ($\vec{n}_1, \vec{n}_2$) planes in the coordinate frame with its polar axis along the vector $\vec{n}_1$. In terms of the introduced variables the energy and momentum conservation laws have the form:

$$x_1' + x_2' + x_3 = 2, \qquad x_1\vec{n}_1 + x_2\vec{n}_2 + x_3\vec{n}_3 = 0 . \tag{2}$$

We shall be interested in the magnitude and spatial distributions of the vector $\vec{T}$ (thrust) which characterizes the momentum of the most energetic parton (jet) regardless if it is a quark, antiquark or a gluon (or jets formed by them). To pass on to variable $\vec{T}$ in the cross section (1) it is sufficient to consider three kinematic regions [5] which differ in the relative value of the gluon momentum:

$$\text{I. } x_1 \geqslant x_2 \geqslant x_3 , \quad \text{II. } x_1 \geqslant x_3 \geqslant x_2 , \quad \text{III. } x_3 \geqslant x_1 \geqslant x_2 . \qquad (3)$$

As the expression (1) is symmetrical relative to replacements $x_1 \rightleftarrows x_2$, three other regions obtained from (3) by these replacements give just the same contribution into the cross section as the regions (3) do.

In the regions I and II, where the quark jet is most energetic, one must take $x_1 = T$, $x_1' = T'$, $dz_1\, d\varphi_1 = d\hat{\vec{T}}$ in the cross section (1) and integrate over φ_2' at the fixed value of $z_{12} = (x_3^2 - x_1^2 - x_2^2)/2x_1x_2$, taking the dependence of z_2 on variable φ_2' into account:

$$z_2 = z_1 z_{12} + (1 - z_1^2)^{1/2} (1 - z_{12}^2)^{1/2} \cos\varphi_2' .$$

The integrated cross section is conveniently written down as:

$$\frac{d^4 \sigma_n}{dT\, d\hat{\vec{T}}\, dx_2'} = \frac{\alpha^2 \alpha_s Q^2}{2\pi} \frac{1}{s} \Big\{ A_n(T, x_2', \eta) \big[(1 + z_1^2)(1 + \xi_1'' \xi_2'') -$$

$$- (1 - z_1^2)(\vec{\xi}_1^{\perp} \vec{\xi}_2^{\perp}) + 2(\hat{\vec{T}} \vec{\xi}_1^{\perp})(\hat{\vec{T}} \vec{\xi}_2^{\perp}) \big] + B_n(T, x_2', \eta) \big[(1 - 3z_1^2)(1 + \xi_1'' \xi_2'') +$$

$$+ 3\big((1 - z_1^2)(\vec{\xi}_1^{\perp} \vec{\xi}_2^{\perp}) - 2(\hat{\vec{T}} \vec{\xi}_1^{\perp})(\hat{\vec{T}} \vec{\xi}_2^{\perp}) \big) \big] \Big\} . \qquad (4)$$

Here its dependence on the most energetic parton (for the regions I and II - that of a quark) angular variables is picked out and the coefficients A_n and B_n are functions of energy variables only:

$$A_n(T, x_2', \eta) = \frac{T}{T'(1-T')(1-x_2')} \Big\{ T'^2 + x_2'^2 + \eta \Big[1 - x_3 - \frac{1}{2}\Big(1 + \frac{\eta}{2}\Big) \frac{x_3^2}{(1-T')(1-x_2')} \Big] \Big\}$$

$$B_n(T,x_2',\eta)=\frac{4(1-T')(1-x_2')(1-x_3)-\eta x_3^2}{2TT'(1-T')(1-x_2')}\left[1-\frac{\eta(1-2T'+\eta)}{2(1-T')(1-x_2')}\right], \quad (5)$$

where $x_3 = 2-T'-x_2'$. Note that if the expression (4) is integrated over angles, the structure with B_n will vanish.

The presented cross section (4) refers both to the regions $n=\mathrm{I}$ and $n=\mathrm{II}$. The difference between them, which is due to the fact that antiquark or gluon has a larger momentum, arises as a consequence of integration over the remained variable x_2' which in different regions varies differently [5] .

Let us write the integrated over x_2' cross section (4), normalized to $\sigma_{\mu\mu}$, in the following form:

$$\frac{1}{\sigma_{\mu\mu}}\frac{d^3\sigma_n}{dT d\hat{\vec{T}}}=\frac{3\alpha_s G^2}{(2\pi)^2}\Big\{A_n(T,\eta)\big[(1+z^2)(1+\xi_1^{\parallel}\xi_2^{\parallel})-$$

$$-(1-z^2)(\vec{\xi}_1^{\perp}\vec{\xi}_2^{\perp})+2(\hat{\vec{T}}\vec{\xi}_1^{\perp})(\hat{\vec{T}}\vec{\xi}_2^{\perp})\big]+B_n(T,\eta)\big[(1-3z^2)(1+\xi_1^{\parallel}\xi_2^{\parallel})+ \quad (6)$$

$$+3\big((1-z^2)(\vec{\xi}_1^{\perp}\vec{\xi}_2^{\perp})-2(\hat{\vec{T}}\vec{\xi}_1^{\perp})(\hat{\vec{T}}\vec{\xi}_2^{\perp})\big)\big]\Big\},$$

where a factor of 2 is added (with regard to the contributior of the regions obtained from (3) by replacements $x_1 \rightleftarrows x_2$) and the coefficients $A_n(T,\eta)$ and $B_n(T,\eta)$ are obtained by integrating the corresponding coefficients in (4). Here and further on z indicates the cosine of the angle between the most energetic parton momentum and that of electron.

In the region I , for the values of T in the range

$$-\frac{2}{3}+\frac{4}{3}\left(1-\frac{3}{4}\eta\right)^{1/2}\leqslant T\leqslant 1-\frac{3}{4}\eta\left(1+\frac{1}{4}\eta^2\right), \quad (7)$$

for which the x_2' varies in the limits

$$\frac{(2-T')^2+\eta}{2(2-T')} \leqslant x_2' \leqslant T',$$

the coefficients included in (6) have the form:

$$A_I(T,\eta)=\frac{T}{T'(1-T')}\Big\{\Big[1+T^2+\eta(T'-\tfrac{1}{2}\eta)\ln\frac{T'(2-T')-\eta}{2(1-T')(2-T')}+$$

$$+\frac{4(1-T')^2-T^2}{8(2-T')}\Big[6+T'+\frac{\eta}{2-T'}\Big(9-4T'+(2+\eta)\frac{2-T'}{1-T'}\Big[1+\frac{2(1-T')(2-T')}{T'(2-T')-\eta}\Big]\Big)\Big]\Big\}, \quad (8)$$

$$B_I(T,\eta)=\frac{1}{TT'}\Big\{\frac{\eta}{2(1-T')}\Big[1-3T^2-4\eta(1-T'+\frac{\eta}{4})\Big]\ln\frac{2(1-T')(2-T')}{T'(2-T')-\eta}-$$

$$-\frac{1}{2(2-T')}\Big[4(1-T')^2-T^2\Big]\Big(\frac{5}{2}T'-1+\frac{\eta}{8(1-T')}\Big[2-7T'-\frac{4-\eta}{2-T'}+$$

$$+2\eta\Big(4+\frac{1-2T'+\eta}{1-T'}\Big(1+\frac{2(1-T')(2-T')}{T'(2-T')-\eta}\Big)\Big)\Big]\Big)\Big\}. \quad (9)$$

For other values

$$1-\frac{3}{4}\eta\left(1+\frac{1}{4}\eta^2\right)\leqslant T\leqslant(1-\eta)^{1/2} \quad (10)$$

in the region I x_2' varies in the range of

$$2-T'-\frac{2(1-T')}{2-T-T'}\leqslant x_2'\leqslant T',$$

and for the corresponding coefficients in (6) one obtains

$$A_I(T,\eta)=\frac{T}{T'(1-T')}\Big\{\Big[1+T^2+\eta(T'-\tfrac{1}{2}\eta)\Big]\ln\frac{T+T'}{2-T-T'}-$$

$$-\frac{2(1-T')(T+T'-1)}{2-T-T'}\Big[\frac{3-2T-T'}{2-T-T'}+\eta\Big(1+\frac{1+\frac{1}{2}\eta}{(1-T')(T+T')}\Big)\Big]\Big\}, \quad (11)$$

$$B_I(T,\eta)=\frac{1}{TT'}\Big\{\frac{\eta}{2(1-T')}\Big[1-3T^2-4\eta(1-T'+\tfrac{1}{4}\eta)\Big]\ln\frac{2-T-T'}{T+T'}+$$

$$+\frac{T+T'-1}{2-T-T'}\Big[(1-T)(2+T-T')-4(1-T')^2-\eta(1+2T'-2\eta)+\eta^2\frac{1-2T'+\eta}{(1-T')(T+T')}\Big]\Big\}. \quad (12)$$

It is not difficult to see that at the lower limit of the variation range (7) of T, the coefficients (8) and (9) vanish, while at the boundary point $T \simeq 1-(3\eta/4)(1+\eta^2/4)$, which separates the ranges (7) and (10), with the same accuracy its value is given [5], the expressions (8) and (9) turn, accordingly, into (11) and (12).

In the region II, at values

$$-\frac{2}{3}+\frac{4}{3}\left(1-\frac{3}{4}\eta\right)^{1/2} \leqslant T \leqslant \frac{2(1-\sqrt{\eta})}{2-\sqrt{\eta}} \qquad (13)$$

the cross section (4) must be integrated over x_2' in the range

$$2-T-T' \leqslant x_2' \leqslant \frac{(2-T')^2+\eta}{2(2-T')},$$

while at values

$$\frac{2(1-\sqrt{\eta})}{2-\sqrt{\eta}} \leqslant T \leqslant 1-\frac{3}{4}\eta\left(1+\frac{1}{4}\eta^2\right) \qquad (14)$$

– in the range

$$2-T'-\frac{2(1-T')}{2-T-T'} \leqslant x_2' \leqslant \frac{(2-T')^2+\eta}{2(2-T')}.$$

After integration one obtains the following expressions for the coefficients in (6):

for the values of T in the range (13)

$$\begin{aligned} A_{II}(T,\eta) = \frac{T}{T'(1-T')}\Big\{\Big[1+T^2+\eta\left(T'-\frac{1}{2}\eta\right)\Big]\ln\frac{2(2-T')(T+T'-1)}{T'(2-T')-\eta} - \\ -\frac{(2-T)(T+2T'-2)}{8(2-T')}\Big[10-2T-3T'+\frac{\eta}{2-T'}\Big(9-4T'+ \\ +(2+\eta)\frac{2-T'}{1-T'}\Big[1+\frac{2(1-T')^2(2-T')}{(T+T'-1)(2T'-T'^2-\eta)}\Big]\Big)\Big]\Big\}, \end{aligned} \qquad (15)$$

$$B_{II}(T,\eta) = \frac{1}{TT'}\Big\{\frac{\eta}{2(1-T')}\Big[1-3T^2-4\eta\left(1-T'+\frac{1}{4}\eta\right)\ln\frac{T'(2-T')-\eta}{2(2-T')(T+T'-1)}+$$

$$+\frac{(2-T)(T+2T'-2)}{2(2-T')}\Big(1-T+\frac{1}{2}T'+\frac{\eta}{8(1-T')}\Big[6-11T'-2T-\frac{4-\eta}{2-T'}+$$

$$+2\eta\Big(4+\frac{1-2T'+\eta}{1-T'}\Big(1+\frac{2(1-T')^2(2-T')}{(T+T'-1)(2T'-T'^2-\eta)}\Big)\Big)\Big]\Big)\Big\}\;; \tag{16}$$

for the values of T in the range (14)

$$A_{II}(T,\eta)=\frac{T}{T'(1-T')}\Big\{\big[1+T^2+\eta(T'-\tfrac{1}{2}\eta)\big]\ln\frac{2(1-T')(2-T')(T+T')}{(2-T-T')(2T'-T'^2-\eta)}-$$

$$-\frac{T[4(1-T')-T(2-T-T')]}{8(2-T')(2-T-T')}\Big[10-3T'-\frac{4(1-T')}{2-T-T'}+\frac{\eta}{2-T'}\Big(\ldots$$

$$(2+\eta)\frac{2-T'}{1-T'}\Big[1+\frac{2(1-T')(2-T')(2-T-T')}{(T+T')(2T'-T'^2-\eta)}\Big]\Big)\Big]\Big\}\,, \tag{17}$$

$$B_{II}(T,\eta)=\frac{1}{TT'}\Big\{\frac{\eta}{2(1-T')}\big[1-3T^2-4\eta(1-T'+\tfrac{1}{4}\eta)\big]\ln\frac{(2-T-T')(2T'-T'^2-\eta)}{2(1-T')(2-T')(T+T')}+$$

$$+\frac{T[4(1-T')-T(2-T-T')]}{2(2-T')(2-T-T')}\Big(1+\frac{1}{2}T'-\frac{2(1-T')}{2-T-T'}+\frac{\eta}{8(1-T')}\Big[6-11T'-\frac{4(1-T')}{2-T-T'}-$$

$$-\frac{4-\eta}{2-T'}+2\eta\Big(4+\frac{1-2T'+\eta}{1-T'}\Big(1+\frac{2(1-T')(2-T')(2-T-T')}{(T+T')(2T'-T'^2-\eta)}\Big)\Big)\Big]\Big)\Big\}\,. \tag{18}$$

It can be checked that at the lower limit of (13) the coefficients (15) and (16) vanish, while at the point $T=2(1-\sqrt{\eta})/(2-\sqrt{\eta})$ they are transformed into (17) and (18), respectively.

Let us now pass on to the region $\overline{\text{III}}$ where the gluon jet is most energetic. To obtain analogous formulae, it is convenient to start from a cross section expressed via gluon and, say, quark variables. It can be easily obtained from (1) making use of the conservation laws (2) and of the phase volume in the form

$$d\Phi = dx_1'\,dx_3\,dz_3\,d\varphi_3\,d\varphi_1'' ,$$

where φ_3 is the azimuthal angle of gluon emission in the coordinate frame with its polar axis along the vector $\vec{v}$, and

φ_1'' is the azimuthal angle between $(\vec{n}_3, \vec{v})$ and $(\vec{n}_3, \vec{n}_1)$ planes in the coordinate frame with its polar axis along the vector $\vec{n}_3$.

Now, by taking $x_3 = T$, $dz_3 d\varphi_3 = d\hat{\vec{T}}$ and integrating over φ_1'' one obtains

$$\frac{d^4\sigma_{III}}{dT d\hat{\vec{T}} dx_1'} = \frac{\alpha^2\alpha_s Q^2}{2\pi} \frac{1}{s}\Big\{A_{III}(T,x_1',\eta)\big[(1+z_3^2)(1+\xi_1''\xi_2'') - (1-z_3^2)(\vec{\xi}_1^{\perp}\vec{\xi}_2^{\perp}) + 2(\hat{\vec{T}}\vec{\xi}_1^{\perp})(\hat{\vec{T}}\vec{\xi}_2^{\perp})\big] + B_{III}(T,x_1',\eta)\big[(1-3z_3^2)(1+\xi_1''\xi_2'') + 3\big((1-z_3^2)(\vec{\xi}_1^{\perp}\vec{\xi}_2^{\perp}) - 2(\hat{\vec{T}}\vec{\xi}_1^{\perp})(\hat{\vec{T}}\vec{\xi}_2^{\perp})\big)\big]\Big\}, \qquad (19)$$

where

$$A_{III}(T,x_1',\eta) = \frac{1}{(1-x_1')(1-x_2')}\Big\{x_1'^2 + x_2'^2 + \eta\big[1-T-\tfrac{1}{2}(1+\tfrac{1}{2}\eta)\frac{T^2}{(1-x_1')(1-x_2')}\big]\Big\},$$

$$B_{III}(T,x_1',\eta) = \frac{1}{T^2(1-x_1')(1-x_2')}\big[4(1-x_1')(1-x_2')(1-T) - \eta T^2\big],$$

$$x_2' = 2 - T - x_1'$$

By integrating the latter cross section over the variable x_1' in the range found in Ref.[5] , in the region III too one will come to the expression (6) where

$$A_{III}(T,\eta) = \frac{1}{T}\big[2(1-T) + T^2 - \eta(T+\tfrac{1}{2}\eta)\big]\ln\frac{T+T'-1}{1-T'} - (T+2T'-2)\Big[1 + \frac{\eta(2+\eta)}{4(1-T')(T+T'-1)}\Big], \qquad (20)$$

$$B_{III}(T,\eta) = \frac{1}{T^2}\Big[2(1-T)(T+2T'-2) - \eta T\ln\frac{T+T'-1}{1-T'}\Big] \qquad (21)$$

for values

$$-\frac{2}{3} + \frac{4}{3}\Big(1-\frac{3}{4}\eta\Big)^{1/2} \leq T \leq \frac{2(1-\sqrt{\eta})}{2-\sqrt{\eta}} \qquad (22)$$

and

$$A_{III}(T,\eta)=\frac{1}{T}\left[2(1-T)+T^2-\eta(T+\frac{1}{2}\eta)\right]\ln\frac{1+(1-\frac{\eta}{1-T})^{1/2}}{1-(1-\frac{\eta}{1-T})^{1/2}}-$$

$$-(1-\frac{\eta}{1-T})^{1/2}\left[T+(2+\eta)\frac{1-T}{T}\right], \qquad (23)$$

$$B_{III}(T,\eta)=\frac{1}{T}\left[2(1-T)(1-\frac{\eta}{1-T})^{1/2}-\eta\ln\frac{1+(1-\frac{\eta}{1-T})^{1/2}}{1-(1-\frac{\eta}{1-T})^{1/2}}\right] \qquad (24)$$

for values

$$\frac{2(1-\sqrt{\eta})}{2-\sqrt{\eta}}\leqslant T\leqslant 1-\eta . \qquad (25)$$

It is easy to see that at the lower limit of the variation range (22) of T the coefficients (20) and (21) vanish, and at the upper limit of the range (25) the coefficients (23) and (24) do. At the point $T=2(1-\sqrt{\eta})/(2-\sqrt{\eta})$ they turn accordingly into each other.

Let us write, at last, the momentum magnitude and angular variables distributions of a parton, having the largest momentum in a three-jet process $e^+e^-\to q\bar{q}g$, regardless if it is a quark, antiquark or a gluon:

$$\frac{1}{\sigma_{\mu\mu}}\frac{d^3\sigma}{dTd\hat{\vec{T}}}=\frac{3\alpha_s Q^2}{(2\pi)^2}\Big\{A(T,\eta)\left[(1+z^2)(1+\xi_1^{\parallel}\xi_2^{\parallel})-\right.$$

$$\left.-(1-z^2)(\vec{\xi}_1^{\perp}\vec{\xi}_2^{\perp})+2(\hat{\vec{T}}\vec{\xi}_1^{\perp})(\hat{\vec{T}}\vec{\xi}_2^{\perp})\right]+B(T,\eta)\left[(1-3z^2)(1+\xi_1^{\parallel}\xi_2^{\parallel})+\right.$$

$$\left.+3((1-z^2)(\vec{\xi}_1^{\perp}\vec{\xi}_2^{\perp})-2(\hat{\vec{T}}\vec{\xi}_1^{\perp})(\hat{\vec{T}}\vec{\xi}_2^{\perp}))\right]\Big\}. \qquad (26)$$

Functions $A(T,\eta)$ and $B(T,\eta)$ in three different variation ranges of T have the form:

$$\mathcal{A}(T,\eta) = \mathcal{A}_{I+II}(T,\eta) + \mathcal{A}_{III}^{(20)}(T,\eta),$$

$$B(T,\eta) = B_{I+II}(T,\eta) + B_{III}^{(21)}(T,\eta) \tag{27}$$

for values

$$-\frac{2}{3} + \frac{4}{3}\left(1 - \frac{3}{4}\eta\right)^{1/2} \leqslant T \leqslant \frac{2(1-\sqrt{\eta})}{2-\sqrt{\eta}} \quad ,$$

where

$$\mathcal{A}_{I+II}(T,\eta) \equiv \mathcal{A}_{I}^{(8)}(T,\eta) + \mathcal{A}_{II}^{(15)}(T,\eta) = \frac{T}{T'(1-T')}\Big\{\Big[1+T^2+\eta\Big(T'-\frac{\eta}{2}\Big)\Big]\ln\frac{T+T'-1}{1-T'} +$$

$$+\big[2(1-T')-T\big]\Big[2-\frac{T}{2}+\eta\Big(1+\frac{(2+\eta)T}{4(1-T')(T+T'-1)}\Big)\Big]\Big\} , \tag{28}$$

$$B_{I+II}(T,\eta) \equiv B_{I}^{(9)}(T,\eta) + B_{II}^{(16)}(T,\eta) = \frac{1}{TT'}\Big\{\frac{\eta}{2(1-T')}\Big[1-3T^2-4\eta\Big(1-T'+\frac{\eta}{4}\Big)\Big]\ln\frac{1-T'}{T+T'-1} +$$

$$+(T+2T'-2)\Big[2T'-T-\frac{\eta}{4(1-T')}(4T'+T-4\eta) + \frac{\eta^2 T(1-2T'+\eta)}{4(1-T')^2(T+T'-1)}\Big]\Big\} ; \tag{29}$$

$$\mathcal{A}(T,\eta) = \mathcal{A}_{I}^{(11)}(T,\eta) + \mathcal{A}_{III}^{(23)}(T,\eta),$$

$$B(T,\eta) = B_{I}^{(12)}(T,\eta) + B_{III}^{(24)}(T,\eta) \tag{30}$$

for values

$$\frac{2(1-\sqrt{\eta})}{2-\sqrt{\eta}} \leqslant T \leqslant 1-\eta ;$$

$$\mathcal{A}(T,\eta) = \mathcal{A}_{I}^{(11)}(T,\eta) , \quad B(T,\eta) = B_{I}^{(12)}(T,\eta) \tag{31}$$

for values

$$1-\eta \leqslant T \leqslant (1-\eta)^{1/2}.$$

The figures in brackets by the functions $\mathcal{A}_n$ and B_n show the number of the formula they are assigned to. One can see that at the limits of ranges the expressions (27), (30) and (31) turn one into another.

Let us find the dependence of functions $A(T,\eta)$ and $B(T,\eta)$ on η at the fixed values of T. To do this, one is to determine at a given T the variation ranges of η in the three intervals written out above where the expressions (27), (30) and (31) determine the functions $A(T,\eta)$ and $B(T,\eta)$. The corresponding variation ranges of η are:

$$\frac{3}{4}(2+T)\left(\frac{2}{3}-T\right) \leqslant \eta \leqslant \left[\frac{2(1-T)}{2-T}\right]^2 \qquad \text{at } T \leqslant \frac{2}{3}$$

or

$$0 \leqslant \eta \leqslant \left[\frac{2(1-T)}{2-T}\right]^2 \qquad \text{at } T \geqslant \frac{2}{3},$$

$$\left[\frac{2(1-T)}{2-T}\right]^2 \leqslant \eta \leqslant 1-T$$

and

$$1-T \leqslant \eta \leqslant 1-T^2 .$$

In Figs 1 and 2 the curves of dependence of the functions $A(T,\eta)$ and $B(T,\eta)$ on the variable η in its whole admissible variation range at some values of the parameter T are plotted. Note, that they are plotted in the logarithmic scale, and the dependence of the mentioned functions on the mass parameter is rather significant and one cannot neglect it.

To have an idea about how the account of the mass of heavy quarks affects on the angular distributions in the process we are interested in, let us consider the case with transverse antiparallel polarized initial particles, when the cross section (26) transforms into:

$$\frac{1}{\sigma_{\mu\mu}}\frac{d^3\sigma}{dT\,d\hat{T}} = \frac{3\alpha_s Q^2}{2\pi^2}\left[A(T,\eta)(1-\sin^2\theta\cos^2\varphi) - B(T,\eta)(1-3\sin^2\theta\cos^2\varphi)\right], \quad (32)$$

where the azimuthal angle φ is considered from the plane of electron polarization.

In Fig.3 the dependence of angular distribution, assigned by the function

$$F(T,\eta,\theta,\varphi) \equiv \left(\frac{3\alpha_s Q^2}{4\pi^2}\sigma_{\mu\mu}\right)^{-1} \frac{d^3\sigma}{dT\,d\hat{T}} = 2\Big\{\left[A(T,\eta) - B(T,\eta)\right] -$$

$$- \left[A(T,\eta) - 3B(T,\eta)\right] \sin^2\theta \cos^2\varphi\Big\} ,$$

on the azimuthal angle at the polar angle θ =0, 30, 60 and 90° is plotted at some values of η from the admissible variation range for any of the three values of T = 0.5, 0.7, 0.9. Note, that as the combination $\sin^2\theta\cos^2\varphi$ is invariant relative to substitutions $\theta \to \pi/2 - \varphi$ and $\varphi \to \pi/2 - \theta$, the presented curves describe also a dependence on the polar angle θ at the corresponding values of the azimuthal angle φ = 90, 60, 30° and 0 at the opposite direction of the polar angle growth.

Let us follow the behaviour of the angular distributions with the varying mass parameter η . At T = 0.5 only large η are realized, and for some of them the curves of angular distributions are plotted in Fig.3a (note, that the events with $T < 2/3$ can be due to heavy quarks only, since for such T the parameter η is other than 0). The absolute value of the cross sections increases with η and their dependence on the angles becomes more appreciable. In the case of $T \geqslant 2/3$ the value of the mass parameter may also be η =0. It is well seen in Fig.3b,c how the absolute value of the cross sections and the angular distributions with regard to the mass of quarks, vary in comparison with the massless case. However, while at T = 0.7 the angular dependence becomes more distinct as η increases, then, at T =0.9 the opposite is true, though at the same values for η the absolute size of the cross sections in this case is noticeably larger and the angular dependence is stronger than when $T = 0.7$ (note, that in Fig.3c a logarithmic scale is used). This is due to the fact that at small values of η the coefficients $A(T,\eta)$ and $B(T,\eta)$ behave differently when $T = 0.7$ and $T = 0.9$.

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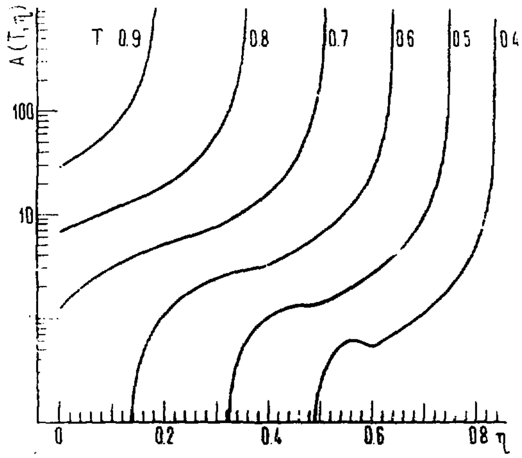


Fig.1. Dependence of $A(T,\eta)$ on the mass parameter η at some values of T.

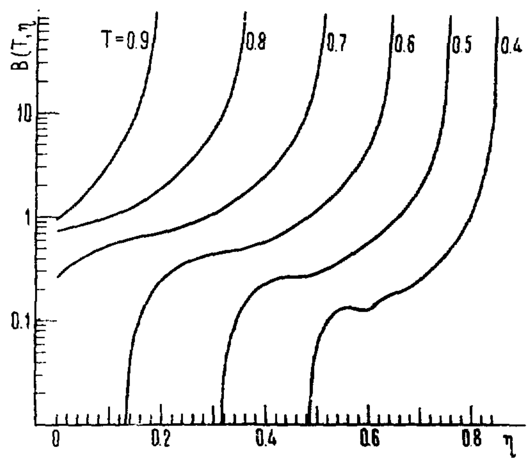
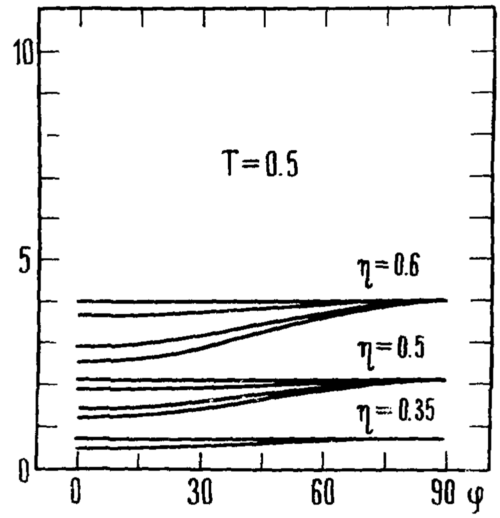
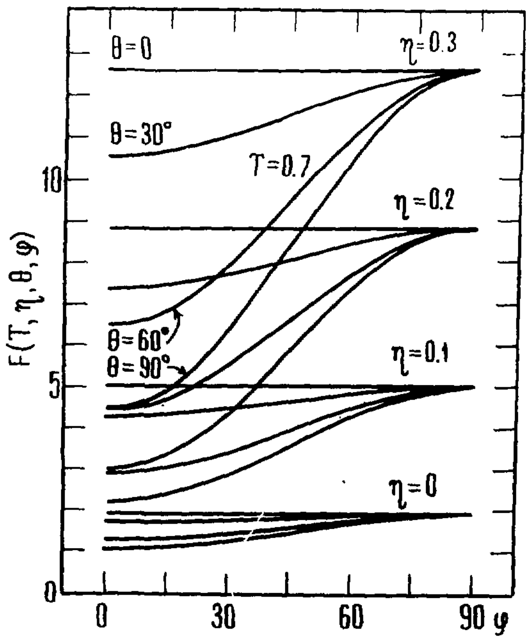


Fig.2. Dependence of $B(T,\eta)$ on the mass parameter η at some values of T.



10
T=0.5
5
η=0.6
η=0.5
η=0.35
0
0
30
60
90 φ



θ=0
η=0.3
θ=30°
T=0.7
10
η=0.2
F(T, η, θ, φ)
θ=60°
θ=90°
η=0.1
5
η=0
0
0
30
60
90 φ

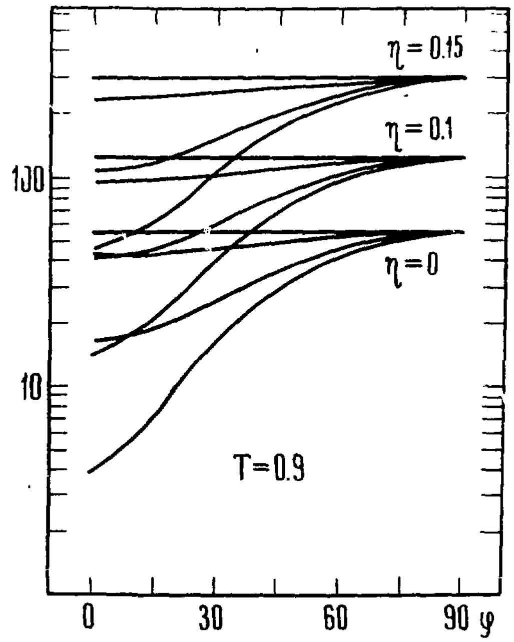


Fig.3. Dependence of $F(T,\eta,\theta,\varphi)$ (differential cross section $d^3\sigma/dTd\hat{\vec{T}}$ of the process $e^+e^-\to q\bar{q}g$, normalized to the value $(3\alpha_s Q^2\sigma_{\mu\mu}/4\pi^2)$) on the azimuthal angle φ when the polar angle θ = 0, 30, 60 and 90^o and at some values of the mass parameter η from the admissible, for the considered values of T = 0.5 (a), T =0.7 (b) and T = 0.9 (c), ranges. The curves at each η, corresponding to four values of θ, are plotted from top to bottom. At T = 0.5 and η = 0.35, due to small difference only the curves corresponding to θ = 0 and 90^o are plotted.

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$e^+e^-\to q\bar{q}g$ В СЛУЧАЕ ТЯЖЕЛЫХ КВАРКОВ
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