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RELATIVE CHAOS IN STELLAR SYSTEMS

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ՀԱՐԱԲԵՐԱԿԱՆ ՔԱՈՍԸ ԱՍՏՂԱՅԻՆ ՀԱՄԱԿԱՐԳԵՐՈՒՄ

Ուսումնասիրվում են բազմաչափ դինամիկական՝ տարբեր տիպի աստղային համակարգերի վիճակագրական հատկությունները՝ գեոդեզականի արագության ուղղությամբ Րիչիի կորության հաշվարկի եղանակով: Իրականացված թվային փորձի ընթացքում ստացված են Րիչիի և սկալյար կորությունները՝ հավասար լրիվ էներգիայով համակարգերի համար: Հաշվումների արդյունքները հնարավորություն են տալիս ստանալ աստղային համակարգերի ուրվագծային դասակարգումը՝ ըստ քաոսի աստիճանի աճի:

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ОТНОСИТЕЛЬНЫЙ ХАОС В ЗВЕЗДНЫХ СИСТЕМАХ

Исследуются статистические свойства многомерных динамических систем - звездных систем разных конфигураций, путем вычисления кривизны Риччи в направлении скорости геодезической. Проведен численный эксперимент по вычислению кривизны Риччи и скалярной кривизны для систем с равной полной энергией. Результаты вычислений позволили получить схематическую классификацию звездных систем по степени возрастания хаотичности.

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RELATIVE CHAOS IN STELLAR SYSTEMS

Statistical properties of many-dimensional dynamical systems - stellar systems of different types, are investigated by means of estimation of Ricci curvature in the direction of the velocity of geodesics. Numerical experiment is performed to calculate the Ricci and scalar curvatures for systems with equal total energy. The results of calculations enable one to obtain schematic classification of stellar systems by increasing degree of chaos.

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1. Introduction.

The development of computer technique makes possible the investigation of complex dynamical systems by means of numerical methods. This direction turned out to be rather fruitful and has lead to a number of interesting and even unexpected results. Already in a pioneer study of Fermi, Pasta and Ulam [1] unexpectedly slow mixing was discovered in a one-dimensional system of nonlinearly interacting oscillators.

An important field of application of numerical analysis is N-body problem, particularly the problem of evolution of stellar systems - clusters and galaxies. Stellar systems being classical examples of collisionless systems of N gravitating bodies regardless of their apparent simplicity are considered one of most difficult for investigation of objects in the Universe.

The application of numerical methods to stellar systems firstly was reduced mainly to following in time the variation of the shape and other observable characteristics of the given system or several ones (see [2]). The main difficulty here consisted in the necessity of integration of too many equations of motion.

Simultaneously another direction of numerical investigations began to develop which included the understanding of qualitative-statistical and other properties of Hamiltonian dynamical systems, in particular stellar systems.

The main feature of this direction is the use of new methods of numerical analysis including criteria of stochasticity and regularity as well as ways of representation of the final information. Thus in their well-known paper Henon and Heiles [3] used Poincare section method for the analysis of two-dimensional system with cubic potential (models of spiral galaxies). Chirikov [4]. Benettin, Contopoulos and co-authors [5,6] developed and applied for specific systems strong methods based on the calculation of Lyapunov characteristic numbers and Krylov-Kolmogorov-Sinai (KS) entropy.

In the present paper we use a method of investigation of statistical properties of many-dimensional systems based on the calculation of so-called Ricci curvature. The idea of the method is that as is well known, motion equations of a Hamiltonian system can be reduced to equations of geodesics of certain Riemannian manifold. As described in Sect. 2, the value of Ricci curvature of this manifold can give a definite information on statistical properties of the Hamiltonian system. This paper is the development of papers [7-9] (see also [10]), where the stellar systems as geodesic flows were investigated; in [7] was also proposed a method of numerical analysis using two-dimensional curvature.

The problem formulated and investigated in the paper by the method mentioned is as follows: to determine the relative degree of instability (chaos) of several many-dimensional dynamical systems, namely stellar systems with different spatial structure, distribution of stellar velocity vectors, angular moment, etc. The main purpose here consists in a possibility to get a sort of quantitative estimation of degree of chaos of N-dimensional system. Definition of "relative chaos" given in Sect. 2 and the corresponding quantitative estimations are based just on the calculation of Ricci curvature.

The reason of resorting to a numerical method not used earlier is determined by the fact that in general the methods mentioned above are not effective for many-dimensional systems. Thus, while calculating Lyapunov numbers, difficulties can appear not only with the securing of the existence of corresponding limits, but also with the interpretation of those numbers themselves.

Results of computer experiments enabled us to get schematical classification of stellar systems of different types by degree of chaoticity. Particularly it turned out that the existence of a central mass in the system leads to its sufficient instability; it is remarkable that this conclusion does not depend on system's energy, angular moment, stellar velocity distribution, etc. Analogously disk systems turned out to be less chaotic than spherically symmetrical ones, etc.

In the total, the results of analysis demonstrate that calculation of Ricci curvature in the direction of the geodesic's velocity can be an effective method for investigation of chaos in many-dimensional dynamical systems.

The outline of the paper is as follows. In Sect. 2 the formalism of Ricci curvature is developed, the definition of relative chaos (instability) is given together with necessary formulae. Results of numerical experiments are presented in Sect. 3 and are analysed in Sect. 4. Discussion of the main conclusions both from the methodic and physical viewpoints is carried out in Sect. 5. Some alternative definitions of chaos and relations between them are discussed in Appendix.

2. The Ricci Curvature.

Consider N-body system with Hamiltonian

$$H(p,q) = \sum_{k=1}^{3N} \frac{p_k^2}{2} + V(q), \qquad (2.1)$$

$$p = (p_1, \dots, p_k),$$

$$q = (q_1, \dots, q_k),$$

evolution of which is determined by equations

$$\frac{dq}{dt} = \frac{\partial H}{\partial p},$$

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q}. \tag{2.2}$$

We are interested in the behavior of close geodesics. Using variational principle of Maupertuis, let us represent the motion of the Hamiltonian system as geodesical flow in configurational space

$$M = \left\{ q_1, \dots, q_{3N};\ E - V(q_1, \dots, q_{3N}) > 0 \right\}$$

with Riemannian metric

$$ds^2 = \left[E - V(q_1, \dots, q_{3N})\right] \sum_{k=1}^{3N} (dq_k)^2 \tag{2.3}$$

or

$$ds^2 = \sum_{i,k} g_{ik}\, dq_i\, dq_k,$$

where

$$g_{ik} = \left[E - V(q)\right] \delta_{ik}.$$

Then Hamiltonian equations (2.2) coincide with equations of geodesics on M of a metric (2.3):

$$u^i_{;j}\, u^j = 0, \tag{2.4}$$

where the normalized velocity is

$$u^i = \frac{dq^i}{ds}, \quad \|u\|^2 = g_{ik} u^i u^k = 1.$$

Denote by n^i vector on a geodesics $\gamma(s)$ which determines the divergence of close geodesics. Equation for $\|n\|^2$ has a form [11.12]

$$\frac{d^2 \|n\|^2}{ds^2} = -2 K_{u,n}(s) \|n\|^2 + 2 \| n^i_{;j} u^j \|^2, \tag{2.5a}$$

where

$$K_{u,n}(s) \equiv K_{u,n}(\gamma(s)) = \frac{R_{ijkl} u^i n^j u^k n^l}{\|n\|^2 \|u\|^2 - (u^i n_i)^2} \tag{2.5b}$$

is two-dimensional curvature in the direction σ given by vectors u and v, R_{ijkl} is the Riemann tensor.

It is easy to see that Eq.(2.5a) can be rewritten in a form

$$\frac{d^2 z}{ds^2} = \left(-K_{u,n}(s) + \| \hat{n}_{;j} u^j \|^2 \right) z, \tag{2.6}$$

where

$$n^i = z \hat{n}^i, \quad \|\hat{n}\| = 1.$$

The stability of considered systems is related to the behavior of close geodesics, i.e. to the character of solutions of Eq.(2.6). Thus, if for every u and n

$$K_{u,n}(s) < k < 0, \tag{2.7}$$

then close geodesics diverge no slower than exponentially, the system possesses maximally strong instability and is proved to be Anosov U-system [12]. At non-negative values of two-dimensional curvature $K_{u,n}(s)$ the properties of many-dimensional system in general are not determined (cf. [9]).

The condition (2.7) for physical systems usually is not fulfilled. Particularly, as is shown in [7,8], for a system of N gravitating bodies $K_{u,n}(s)$ is sign-indefinite. Just for this reason we put forward the question on investigation of <u>average deviation of geodesics</u>. For that we average Eq.(2.6) by all directions n. So far as $K_{u,n}(s)$ does not depend on the choice of u and n defining the plane σ and

$$K_{u,u}(s) = 0 ,$$

then

$$\langle K_{u,n}(s) \rangle = \frac{\sum_{a=1}^{3N} K(\hat{n}_a, u(s))}{3N} , \tag{2.8}$$

where $\hat{n}_a$ are normalized vectors orthogonal to u and to each other.

The Ricci curvature $r_u(s)$ of manifold M in the direction of u is equal to [13]

$$r_u(s) = \frac{R_{ij} u^i u^j}{\|u\|^2} , \tag{2.9}$$

where $R_{ij} = R^k_{ikj}$ is Ricci tensor on M.

One can prove that [13]

$$r_u(s) = \sum_{a=1}^{3N-1} K_{n_a, u}(s) .$$

Then averaging Eq.(2.6) by means of (2.8) and (2.9) we have

$$\frac{d^2 z}{ds^2} = \left(- \langle K_{u,n}(s) \rangle + \langle \| \hat{n}^i_{;j} u^j \|^2 \rangle \right) z$$

or

$$\frac{d^2 z}{ds^2} = \left(\frac{1}{3N} r_u(s) + \langle \| \hat{n}^i_{;j} u^j \|^2 \rangle \right) z .$$

Now it follows that

$$\frac{d^2 z}{ds^2} + \frac{1}{3N} r_u(s) z \geq 0 . \tag{2.10}$$

Considering inequality (2.10) in an interval $0 \leq s \leq s_*$ we can write

$$\frac{d^2 z}{ds^2} + r z \geq 0 , \tag{2.11}$$

where

$$r = \frac{1}{3N} \inf_{0 \leq s \leq s_*} [r_u(s)] .$$

We are interested in the stability of geodesics $\gamma(s)$ within the interval $0 \leq s \leq s_*$.

We shall say that a given geodesics γ_1 with velocity u_1 is <u>less stable</u> with respect to geodesics γ_2 with velocity u_2 within the interval $0 \leq s \leq s_*$, if

$$r_1 < r_2 \tag{2.12a}$$

and

$$r_1 < 0 . \tag{2.12b}$$

This definition is grounded on the following considerations. According to (2.11)

$$\frac{d^2 z}{ds^2} \geq - r z , \tag{2.13}$$

if $z(s)$ is assumed to be an arbitrary smooth function (within the interval $0 \leq s \leq s_*$) satisfying the condition (2.13) with "equal probability", then it is clear that in function z at the condition (2.12) the larger d^2z/ds^2 the smaller r . The larger positive value of d^2z/ds^2 corresponds to higher increasing rate of z :

$$z \geq e^{\sqrt{-r}\,s}$$

at

$$z > 0, \quad \left.\frac{dz}{ds}\right|_{s=0} \geq 0 .$$

This means that at smaller negative values of r the system is <u>unstable with more probability</u>; such systems we shall call <u>more chaotic</u> ones.

Let us calculate now Ricci curvature $r_u(s)$ for a system of N gravitating bodies.

In this case the metric (2.3) has a form

$$g_{ik} = W\delta_{ik},$$

where

$$W = E - V(q) = E + \sum_{\substack{a=2N \\ b=1,\dots,a-1}} \frac{1}{r_{ab}},$$

$$r_{ab}^2 = (r_{ab}^{(1)})^2 + (r_{ab}^{(2)})^2 + (r_{ab}^{(3)})^2, \qquad (2.14)$$

$$r_{ab}^i = r_a^i - r_b^i .$$

One can easily derive the formula for Ricci tensor for metric (2.14) (cf. [7]):

$$R_{ik} = -\frac{(3N-2)}{2}\frac{W_{ik}}{W} - \frac{\Delta W}{2W^2} g_{ik} + \frac{3}{4}(3N-2)\frac{W_i W_k}{W^2} - \frac{(3N-4)}{4}\frac{|\nabla W|^2}{W^3} g_{ik} \qquad (2.15)$$

where

$$W_i = \frac{\partial W}{\partial q_i},$$

$$W_{ik} = \frac{\partial^2 W}{\partial q_i \partial q_k},$$

$$\Delta W = \sum_i W_{ii} \ ,$$

$$|\nabla W|^2 = \sum_i \left(\frac{\partial W}{\partial q_{r_i}} \right)^2 .$$

Transiting from generalized coordinates q_{r_i} to real ones of a -th particle $r_a^i (i = x, y, z)$ for derivatives of W one has

$$\frac{\partial W}{\partial r_a^i} = - \sum_{\substack{c=2,\dots,N \\ c \neq a}} \frac{r_{ac}^i}{r_{ac}^3} \ , \qquad (2.16)$$

$$\frac{\partial^2 W}{\partial r_b^i \partial r_a^j} = \frac{1}{r_{ab}^3} \delta_{ij} - \frac{3 r_{ab}^i r_{ab}^j}{r_{ab}^5} \ , \quad a \neq b, \qquad (2.17)$$

$$\frac{\partial^2 W}{\partial r_b^i \partial r_a^j} = \sum_{\substack{c=1,\dots,N \\ a \neq c}} \left[\frac{1}{r_{ac}^3} - \frac{3 r_{ac}^i r_{ac}^j}{r_{ac}^5} \right] , \quad a = b. \qquad (2.18)$$

So far as we are considering collisionless systems, i.e.

$$(r_{ab}, r_{ac}) \neq 0 \ ,$$

we have

$$\Delta W = 0 .$$

Then formula for Ricci curvature has a form ($\|u\| = 1$)

$$r_u(s) = - \frac{(3N-2)}{2} \frac{W_{ik} u^i u^k}{W} + \frac{3}{4} (3N-2) \frac{(W_i u^i)^2}{W^2} - \frac{(3N-4)}{4} \frac{|\nabla W|^2}{W^3} , \qquad (2.19)$$

and the scalar curvature $R = R_{ik} g^{ik}$ is equal to [7]

$$R = -\frac{3N(3N-1)}{W^3}\left(\frac{1}{4} - \frac{1}{2N}\right)(\nabla W)^2 . \qquad (2.20)$$

3. The Numerical Experiment.

Numerical experiments were performed for systems consisting of N=25 particles. We have used the following scheme for constructing the needed configurations.

In the capacity of initial figure we have taken a cube (C_0, L_0, M_0) with particles of equal mass (m = 1) and given velocity distribution situated at its apices (Fig.1). E.g. it is not difficult to see that the velocity distribution shown in Fig.1.1 corresponds to zero angular moment of system C_0 . Then, e.g. initial cube C_0 was transformed to cubes $C_i(\ell_x, \ell_y, \ell_z, \alpha_i)$ having sides multiplied by ℓ_x, ℓ_y, ℓ_z and velocities - by α_i . Creating enclosed structures from transformed cubes $C_i(S_i, L_i, M_i)$ we get a sequence of different configurations. Constants α_i are chosen so that to have systems with equal total energy.

Investigated systems are as follows.

1. Spherically symmetric system C_1 without moment M in a form (Fig.2.1)

$$C_1 \begin{pmatrix} 1 & 1 & 1 & \alpha_1 \\ 2 & 2 & 2 & \alpha_1 \\ 3 & 3 & 3 & \alpha_1 \end{pmatrix} .$$

2. Spherically symmetric system with a central mass ($M = 0$, here and everywhere below) (Fig.2.2)

$$C_2 \begin{pmatrix} 0.1 & 0.1 & 0.1 & \alpha_2 \\ 2 & 2 & 2 & \alpha_2 \\ 3 & 3 & 3 & \alpha_2 \end{pmatrix}.$$

3. Spherical nonhomogeneous system with a central mass (Fig.2.3)

$$C_3 \begin{pmatrix} 0.1 & 0.1 & 0.1 & \alpha_3 \\ 2.9 & 2.9 & 2.9 & \alpha_3 \\ 3 & 3 & 3 & \alpha_3 \end{pmatrix}.$$

4. Disk system (Fig.2.4)

$$C_4 \begin{pmatrix} 1 & 1 & 0.1 & \alpha_4 \\ 2 & 2 & 0.1 & \alpha_4 \\ 3 & 3 & 0.1 & \alpha_4 \end{pmatrix}.$$

5. Disk system with a central mass (Fig.2.5)

$$C_5 \begin{pmatrix} 0.1 & 0.1 & 0.1 & \alpha_5 \\ 2 & 2 & 0.1 & \alpha_5 \\ 3 & 3 & 0.1 & \alpha_5 \end{pmatrix}.$$

6. Disk system with a bulge (Fig.2.6)

$$C_6 \begin{pmatrix} 1 & 1 & 1 & \alpha_6 \\ 2 & 2 & 0.1 & \alpha_6 \\ 3 & 3 & 0.1 & \alpha_6 \end{pmatrix}.$$

7. **Ring system with a central mass** (Fig.2.7)

$$C_7 \begin{pmatrix} 0.1 & 0.1 & 0.1 & \alpha_7 \\ 2.9 & 2.9 & 0.1 & \alpha_7 \\ 3 & 3 & 0.1 & \alpha_7 \end{pmatrix}.$$

8. **Homogeneous bar** (Fig.2.8)

$$C_8 \begin{pmatrix} 1 & 1 & 1 & \alpha_8 \\ 1 & 1 & 3 & \alpha_8 \\ 1 & 1 & 6 & \alpha_8 \end{pmatrix}.$$

Numerical results for Ricci curvature r_1 found for these systems at different values of total energy E are listed in Table 1 and Fig.3.1. Simultaneously the scalar curvature was calculated too (see Table 1).

Analogously the systems S_i, L_i and a combination $\{S_i, C_i, S_i\}$ with three other arbitrarily chosen velocity distributions (Figs 1.3, 1.2; Tables 3a. 3b. 3c, respectively) were investigated (rotational moment was again equal to zero). Evidently the aim was to exclude the probable dependence of results on velocity distribution. Results found for some configurations C_i ($i = 1,2,3$) are presented in Figs 3.2, 3.3.

Systems M_i (Fig.1.4) having the same spatial structure as C_i, S_i, L_i but possessing also rotational moment were considered too. The results are listed in Table 3.

4. Relative Instability of Systems.

In the previous section the results of calculations of Ricci curvature in direction of velocity u for several systems were presented. Let us try now in accord to definition given in Sect. 2 to apply the criterion (2.12) in order to study these systems as to relative chaos (instability).

First analyse systems C_i without rotational moment and with moment (Tables 1 and 2). As is seen from Table 1 and Fig.3.1. for every value of energy one has

$$r_2^c < r_3^c < r_5^c < r_1^c < 0 < r_4^c , \qquad (8.1)$$

i.e. the system with r_2^c is the most unstable one with respect to others. It means that if one considers trajectories of close systems C_2 and C_2' and, say, C_1' and C_1, then on the average the formers will diverge with higher rate than the latters; C_1 corresponds to spherically symmetric systems, C_2 - to analogous systems with central mass added. Inequalities

$$r_2^c < r_1^c < 0, \qquad (8.2)$$
$$r_2^M < r_1^M < 0 ,$$

following from Tables 1-3 show that systems with a central mass (at one and the same energy) are more unstable than those without mass. This conclusion is true also for systems with different values of rotational moment. Moreover, as is seen from Fig.3, this fact depends neither on initial velocity distribution of particles nor on system's total energy.

From Table 1 it is also seen that disk system without rotational moment M is more stable than the rest ones. Adding of a central mass makes it more unstable as it is the case with spherical systems.

Existence of rotational moment according to Table 2 leads in general to decrease of stability. Indeed, as is seen from comparison of Tables 1 and 2, for disk configurations without moment Ricci curvature is positive, while it becomes negative for systems possessing moment.

Scalar curvature $\mathcal{R}$ being negative in all cases as it follows also from Eq.(2.20), numerically correlates with values of Ricci curvature, namely the smaller $\mathcal{R}$ the smaller r_u .

5. Conclusion.

In the present paper it is formulated and investigated the problem of quantitative determination of relative chaos in different many-dimensional dynamical systems. With this aim, a formalism of calculating of Ricci curvature in direction of velocity of geodesics is suggested and the definition of relative chaos is given. As was mentioned in Introduction, the necessity of using a new value for investigation of statistical properties of dynamical systems is justified by non-effectiveness of existing numerical methods (calculation of Lyapunov numbers, etc.) for many-dimensional problems. It is worth noting that in view of absence of common terminology, there exist different understandings of "chaos" based on different definitions, parameters (physical entropy, metric entropy, Lyapunov characteristic numbers, two-dimensional curvature, etc.). Discussion of some definitions is performed in Appendix.

By this suggested method we have numerically investigated on relative chaos (instability) systems consisting of N=25 gravitating particles of different configurations (models of stellar systems). Ricci r_u and scalar $\mathcal{R}$ curvatures were calculated on a computer. Equation of energy enables one to compare the systems with each other. We have considered spherically symmetric, disk, nonhomogeneous systems, systems containing central mass and possessing rotational moment, etc. The aim was the understanding of degree of chaos in real stellar systems existing in the Universe: globular clusters, elliptic, lenticular, spiral, ring galaxies, bars as well as galaxies and clusters containing central massive point objects, etc.

Computer calculations showed that as a rule systems with massive centre are more chaotic than analogous ones without central mass. This conclusion does not depend on the system's energy. spatial inhomogeneity. velocity distribution of particles and finally on the existence of rotational moment. In other words, the "switching" of regular field $1/r$ to the self-consistent one $\sum \frac{1}{r_{ab}}$ makes the system more chaotic. This result is in accordance with that of the paper [14] , where it is shown that an ellipsoidal stellar system with central mass becomes spherical by time, as effectively as larger the central mass. The crucial role of a central mass in global properties of stellar systems is shown also in [15] , where the problem of gravothermal catastrophe was discussed from the concept of catastrophe theory. It was shown that systems with central mass are more unstable with respect to this catastrophe. The contribution of central mass in the evolution of those systems using the methods of theory of dynamical systems is investigated in particular in [16] .

It further turned out that disk systems are more stable (regular) than sphericals, systems with rotational moment are more stable than without it, and so on.

Considered systems are schematically classified by increasing of degree of chaos in Fig.4.

Results of computer experiments have demonstrated that the calculation of Ricci curvature in direction of geodesical velocity can be an effective method of numerical investigation of statistical properties of many-dimensional dynamical systems. The exposed correlation between Ricci and scalar curvatures has shown the possibility of obtaining analogous information for certain systems by means of scalar curvature.

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APPENDIX

Conceptions of chaos and regularity widely used at present are understood not always equally. It is related both with absence of commonly adopted terminology and specific methods and aims of investigations. In view of the fact that the same system is regarded chaotic by one definition and non-chaotic (regular) by the other, we shall briefly discuss this aspect of the problem.

The following definitions are most conventional.

1. An invariant phase region of a dynamical system with positive measure is considered chaotic if KS-entropy $h > 0$ and regular if $h = 0$ [17]. It is known that a typical Hamiltonian system is a system with shared phase space, i.e. the latter contains both regular and chaotic (stochastic) regions.

2. A system is considered chaotic if there is mixing, and regular if it does not possess this property [18].

It is proved that the system with $h > 0$ is decomposable into no more than countable sets of ergodic components [19]. It means that systems chaotic by definition 1 can be regular by definition 2. And vice versa, there exist mixing systems with $h = 0$, i.e. chaos by definition 2 corresponds to regularity by definition 1. It should be noted that both these properties are often impossible to check.

To our opinion, the main demerit of these definitions is not the mentioned above, but the fact that those criteria reckoning the systems to one or another type (positiveness of KS-entropy, mixing) depend on the system's properties on infinity by time t. In particular, according to Pesin theorem [19] KS-entropy is equal to

$$h = \int_M \sum_{\chi_i > 0} \chi_i(x)\, d\mu(x),$$

where $\chi_i(x)$ are Lyapunov characteristic numbers, and depends on properties of the system on time infinity.

Because of these reasons, of particular importance is the possibility of definition of chaos reflecting local (in time) properties of systems. An example of such a characteristic can be two-dimensional curvature $K_{u_1 u_2}(s)$ (2.5b), by which it is determined in general KS-entropy [19]. However mentioned above difficulties connected with the application of two-dimensional curvature for many-dimensional physical systems made us turn to investigation of Ricci curvature.

Table 1.

r^c \ E	-50	-20	0	20	50
r_1	-3.51-03	-7.53-04	-4.08-04	-2.55-04	-1.48-04
R_1	-1.80+00	-1.59-01	-6.21-02	-3.03-02	-1.32-02
r_2	-3.09-01	-1.94-01	-1.48-01	-1.16-01	-8.30-02
R_2	-2.89+01	-1.76+01	-1.32+01	-1.01+01	-7.10+00
r_3	-2.28-01	-1.53-01	-1.21-01	-9.75-02	-7.26-02
R_3	-1.99+01	-1.31+01	-1.02+01	-8.08+00	-5.90+00
r_4	1.86-01	1.46-01	1.22-01	1.02-01	7.99-02
R_4	-1.90+01	-8.70+00	-5.68+00	-3.91+00	-2.40+00
r_5	-9.44-02	-6.13-02	-4.70-02	-3.66-02	2.58-02
R_5	-1.38+01	-9.40+00	-7.47+00	-6.04+00	-4.50+00
r_6	1.22-01	1.20-01	1.01-01	8.46-02	6.49-02
R_6	-2.82+01	-1.05+01	-6.28+00	-4.06+00	-2.32+00
r_7	-7.07-02	-4.95-02	-3.95-02	-3.18-02	-2.33-02
R_7	-9.96+02	-7.41+00	-6.17+00	-5.19+00	-4.08+00
r_8	-3.26-04	-4.35-04	-2.83-04	-1.95-04	-1.22-04
R_8	-1.96+00	-2.02-01	-8.12-02	-4.05-02	-1.80-02

Table 2a

r^s \ E	-50	-20	0	20	50
r_1	-2.50-02	-2.22-03	-8.63-04	-4.22-04	-1.84-04
r_2	-4.01-01	-2.45-01	-1.84-01	-1.42-01	-9.88-02
r_3	-2.77-01	-1.82-01	-1.42-01	-1.12-01	-8.20-02
r_4	1 38+00	-1.08+00	-8.00-01	-6.17-01	-4.40-01
r_5	-4.15-01	-3.04-01	-2.52-01	-2.12-01	-1.68-01

Table 2b

r^{scs} \ E	-50	-20	0	20	50
r_1	-2.98-02	-3.27-03	-1.44-03	-7.83-04	-3.94-04
r_2	-4.65-01	-2.92-01	-2.22-01	-1.74-01	-1.25-01
r_3	-3.22-01	-2.17-01	-1.71-01	-1.38-01	-1.03-01
r_4	-1.17+00	-6.91-01	-5.19-01	-4.05-01	-2.92-01
r_5	-3.68-01	-2.70-01	-2.24-01	-1.89-01	-1.50-01

Table 2c

r^L \ E	-50	-20	0	20	50
r_1	-2.40-02	-2.15-03	-8.40-04	-4.13-04	-1.81-04
r_2	-4.01-01	-2.45-01	-1.84-01	-1.41-01	-9.88-02
r_3	-2.73-01	-1.80-01	-1.40-01	-1.11-01	-8.10-02
r_4	-1.54+00	-8.76-01	-6.47-01	-4.97-01	-3.54-01
r_5	-3.16-01	-2.27-01	-1.87-01	-1.56-01	-1.22-01

Table 3

r^M \ E	-50	-20	0	20	50
r_1	-4.66-03	-9.83-04	-5.31-04	-3.32-04	-1.92-04
r_2	-3.24-01	-2.04-01	-1.57-01	-1.23-01	-8.90-02
r_3	-2.39-01	-1.61-01	-1.28-01	-1.03-01	-7.73-02
r_4	-2.65-01	-1.21-01	-7.92-02	-5.45-02	-3.36-02
r_5	-1.65-01	-1.16-01	-9.42-02	-7.76-02	-5.94-02

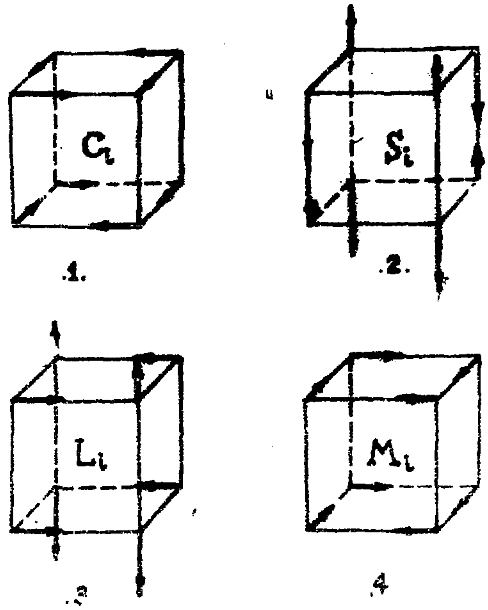


Fig.1. Initial cubes with vectors of particle's velocity shown;
C_i, S_i, L_i without rotational moment. M_i - with moment.

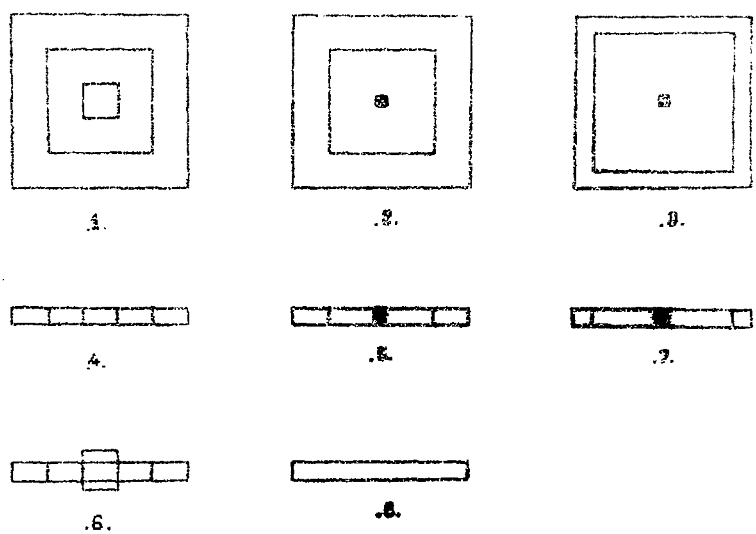
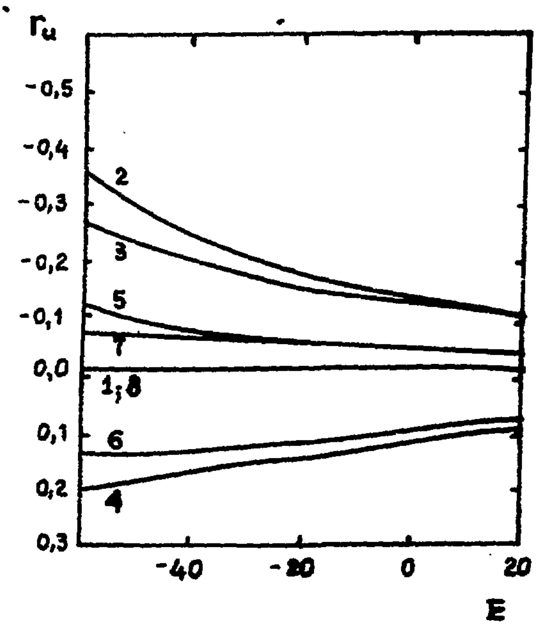
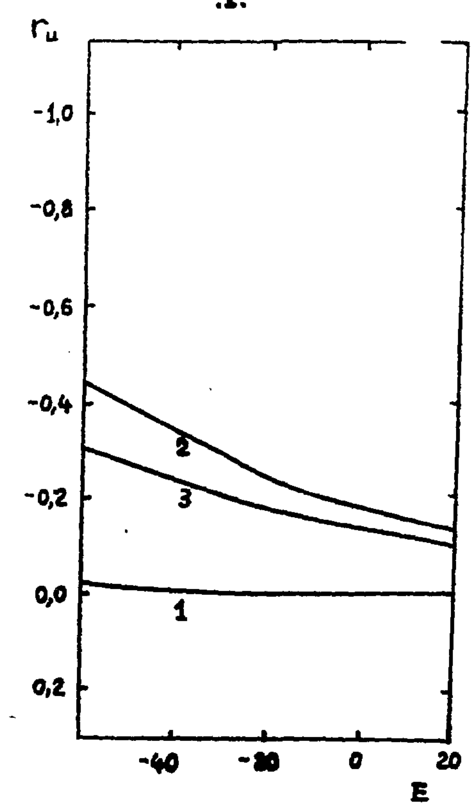


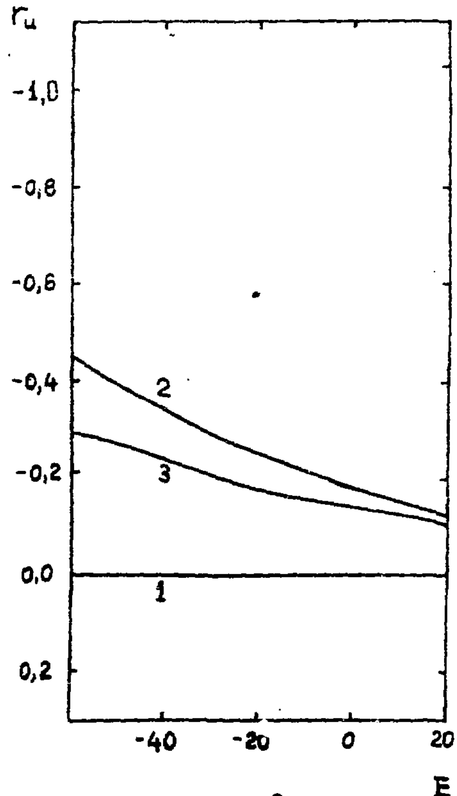
Fig.2. Cross sections of configurations constructed from initial cubes.


rᵤ
-0,5
-0,4
-0,3
-0,2
-0,1
0,0
0,1
0,2
0,3
2
3
5
7
1;8
6
4
-40
-20
0
20
E

.1.


rᵤ
-1,0
-0,8
-0,6
-0,4
-0,2
0,0
0,2
2
3
1
-40
-20
0
20
E

.2.



. 3.

Fig.3. Dependence of Ricci curvature r_u on total energy of the systems constructed from three cubes: 1. $\{C_i, C_i, C_i\}$ 2. $\{S_i, C_i, S_i\}$, 3. $\{L_i, L_i, L_i\}$.

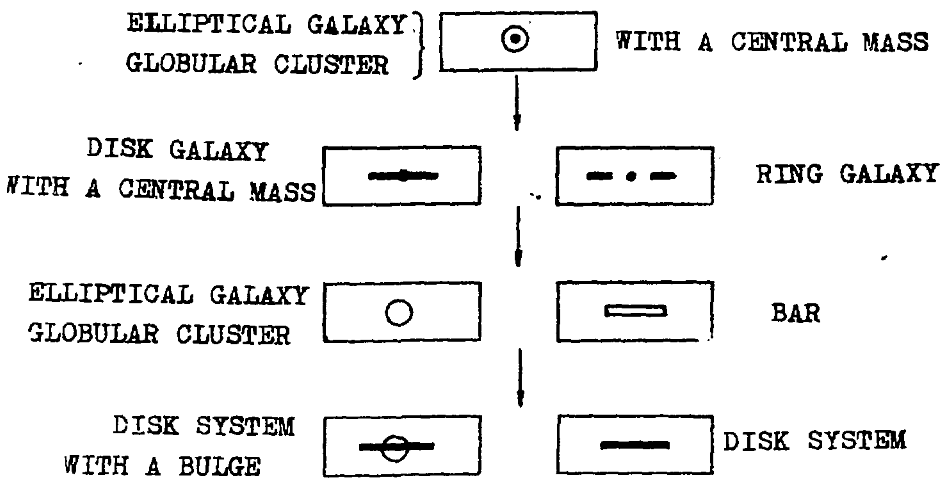


Fig.4. Schematic classification of considered models of galaxies and clusters by degree of chaos (arrows indicate direction of decreasing).

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(на английском языке, перевод, З.Н.Асланян, В.Г.Гурзадяна)

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