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POYNTINGS VECTOR FIELD LINES OF SYNCHROTRON RADIATION

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ՓՈՅՆԹԻՆԳԻ ՍԻՆՔՐՈՏՐՈՆԱՅԻՆ ՃԱՌԱԳԱՅԹՄԱՆ ՎԵԿՏՈՐՆԵՐԻ ԴԱՇՏԻ ԳԾԵՐԸ

Արտածված են Փոյթինգի վեկտորների դաշտի գծերի ընդհանուր հավասարումները կետային լիցքի կամայական շարժումի դեպքում: Մանրամասն քննարկվում են շրջանագծով հավասարաչափ շարժման դեպքը: Քննարկվել է ուղղագիծ հավասարաչափ շարժմանն անցման դեպքը: Ցույց է տրվում, որ սինքրոտրոնային ճառագայթման համար, ի տարբերություն հավասարաչափ ուղղագիծ շարժմանը, Փոյնթինգի վեկտորների գծերը չեն առաջացնում լիցքը պարունակող փակ, սահմանափակ մակերևույթներ:

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POYNTINGS VECTOR FIELD LINES OF SYNCHROTRON RADIATION

General equations for the Poynting vectors field lines of an arbitrary moving point charge are presented. The case of uniform circular motion is considered in detail and the transition to the uniform rectilinear motion is discussed. It is shown that unlike the uniform rectilinear motion, in case of synchrotron radiation the field lines of Pointing vectors do not form charge-containing closed surfaces.

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ЛИНИИ ПОЛЯ ВЕКТОРОВ ПОЙНТИНГА СИНХРОТРОННОГО ИЗЛУЧЕНИЯ

Выписаны общие уравнения линий поля векторов Пойнтинга произвольно движущегося точечного заряда. Подробно рассмотрен случай равномерного движения по окружности. Прослежен переход к равномерному прямолинейному движению. Показано, что для синхротронного излучения, в отличие от равномерного прямолинейного движения, линии векторов Пойнтинга не образуют замкнутых ограниченных поверхностей, содержащих заряд.

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The constant interest in the investigation of the Lienard-Wiechert field of one particle by by non-spectral methods [1-3] is stimulated by, at least, three factors: the urge towards the graphical representation of complicated radiation fields of relativistic particles, attempts for the application of such methods in the accelerator physics [1,2] and for obtaining high field intensities [3,4] , as well as the hopes, that such an analysis would be useful for wider range of problems. Macroscopic phenomena, as a rule, are satisfactorily described by averaging large numbers of interacting point charges. If one can confine to the linear approximation when there works an efficient method of Fourier expansion of fields, then one can consider only one spectral harmonic. If the interaction is nonlinear, no "disengagement" of harmonics takes place. The averaging must be done over more accurate microscopic processes and necessarily exhaustive information is required the spatial picture of the charge field in a wide range including the subwave zone.

The lines of electric $\vec{E}$ and magnetic $\vec{H}$ fields turn to be useful *for they carry information* about the radiation and Coulomb term [5,6]. The field topology, i.e. the connection between sections of lines is distorted in dependent of the distance between the observation point and the source, if the Coulomb term is removed. Also a relation is established between a certain time-space volume, occupied by the field, and the given field-forming section of the trajectory.

In this work an attempt is made to complement the electric and magnetic field lines by integral curves of field lines of Poynting vectors which are orthogonal to both, $\vec{E}$ and $\vec{H}$. Several equations of an arbitrary motion are obtained. The case with synchrotron radiation is considered in detail. The passage to uniform rectilinear motion is traced. It is shown, that in case of synchrotron radiation the field lines of Poynting vectors do not form charge-containing enclosed surfaces.

General Equations

The radius vector $\vec{R}_s$ of Poynting vector field lines of Lienard-Wiechert field, analogous to electric field lines [5,6], is sought for in the form

$$\vec{R}_s = \vec{r}_0(D) + D\vec{n}, \qquad (1)$$

where $\vec{r}_0(D)$ is the radius vector of an arbitrary moving charge $\vec{n}$ is the unit vector directed from the point $\vec{r}_0(D)$ to the observation point. $D = c(t-t')$ where t is the moment of observation, t' is the time lag determining the radius of the sphere of light signals emitted at t' instant of time, and observed at t instant.

If we differentiate (1) with respect to D we find the tangent vector on which the condition of proportionality to the Poynting vector $\vec{S} = c[\vec{E}\times\vec{H}]/4\pi$ is imposed ($\vec{E}$ and

$\vec{H} = [\vec{n} \times E]$ are the electric and magnetic fields of the charge):

$$\frac{d\vec{R}_s}{dD} = (\vec{n} - \vec{\beta}) + D\frac{d\vec{n}}{dD} = \alpha[\vec{E} \times [\vec{n} \times E]], \qquad (2)$$

where $\vec{\beta}c$ is the charge velocity during the time lag. The normalizing factor is found from the condition $\vec{n}^2 = 1$ whence

$$\alpha = \frac{1 - \vec{\beta}\vec{n}}{[\vec{n} \times \vec{E}]^2}. \qquad (3)$$

Hence, the equation, determining the unknown function $\vec{n}(D)$, has the form:

$$D\frac{d\vec{n}}{dD} = \frac{[\vec{n} \times [[\vec{E} \times [\vec{n} \times \vec{E}]] \times (\vec{n} - \vec{\beta})]]}{[\vec{n} \times \vec{E}]^2}. \qquad (4)$$

$\vec{E} = \gamma^{-2}(\vec{n} - \vec{\beta}) + D[\vec{n} \times [(\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}}]]$ is put for $\vec{E}$, where $\dot{\vec{\beta}} = d\vec{\beta}/dt'$, $\gamma = (1 - \beta^2)^{-1/2}$ is the Lorentz factor of the particle. The right hand side of the Eq.(4) is singular on the neutral line of the magnetic field, where $[\vec{n} \times \vec{E}] = 0$.

For the uniforming and rectilinearly moving charge the Eq.(4) is reduced to

$$D\frac{d\vec{n}}{dD} = \frac{1 + \vec{\beta}^2 - 2\vec{\beta}\vec{n}}{\vec{\beta}^2 - (\vec{\beta}\vec{n})^2}[\vec{\beta} - \vec{n}(\vec{\beta}\vec{n})],. \qquad (5)$$

the complementary integral of which has the form

$$D(1 - \vec{\beta}^2 - 2(\vec{\beta}\vec{n}))^{1/2} = \alpha \qquad (6)$$

where α is the constant of integration having the dimension of length. The equation (6) describes circles of radius α centered at the point $\vec{r}(0)$ which cross the trajectory of motion. The obtained result is obvious - it is known [7] that the magnetic field lines are the circles, perpendicular to the trajectories; and the electric field lines are straight lines traversing the charge at the observation moment t.

However, the approach developed above allows to find the Poyntings vector field lines for more general cases of motion than the uniform and rectilinear one, when the equation of retarding is solved in an explicit form.

Poynting Vector Field Lines
of Synchrotron Radiation

Let us restrict ourselves to the analysis of Poyntings vector field lines in the orbit plane of a uniforming and circularly moving charge.

It is convenient to decompose the unit vector $\vec{n}$ into the sattelite unit vectors $\vec{e}_1$, $\vec{e}_2$ directions of the normal to the trajectory and velocity at the point $\vec{r}(D)$:

$$\vec{n} = \sin\theta\,\vec{e}_1 + \cos\theta\,\vec{e}_2 . \tag{7}$$

Substituting (7) into (4) one obtains equation for D:

$$\frac{dD}{d\theta} = \frac{\beta D[\sin\theta + D\varkappa\gamma^2\beta(\cos\theta-\beta)]}{1-2\beta\cos\theta+\beta^2+D\varkappa\gamma^2\beta^2\sin\theta(1-2\beta+\beta\cos\theta)+D^2\varkappa^2\beta^3\gamma^2(\cos\theta-\beta)}, \tag{8}$$

where $\varkappa = R^{-1}$.

The replacement

$$\cos\theta = \frac{\beta-\cos\Phi}{1-\beta\cos\Phi} \tag{9}$$

reduces (8) to the equation of the form

$$\frac{d\xi}{d\Phi} = \frac{\xi\beta(\sin\Phi-\xi\cos\Phi)}{1-\beta^2\cos^2\Phi+\xi\beta\sin\Phi(1-2\beta\cos\Phi)-\xi^2\beta\cos\Phi(1-\beta\cos\Phi)}, \tag{10}$$

where $\xi = D\varkappa\beta\gamma$.

The right hand side of (10) is periodical in Φ with a period of 2π, that is why it is sufficient to analyze the Eq.(10) for the values of Φ in the range of $[-\pi,\pi]$. Joining the end points of this interval, we shall obtain a cylindrical surface with the integral lines (10) to be drawn on.

Let us begin the analysis of the Eq.(10) by plotting of isoclines corresponding to the equally of $d\xi/d\Phi$ either to zero or to infinity (then isoclines "0" and "∞"). The "0" isoclines are determined by conditions

$$\xi = 0, \; \xi = \operatorname{tg}\Phi. \tag{11}$$

Let us find the "∞" isoclines. The discriminant in the denominator of the right-hand term in the Eq.(10) is nonnegative with respect to if

$$\cos\Phi \geqslant -\frac{\beta}{2(1-\beta^2)+\sqrt{4-3\beta^2}}. \tag{12}$$

Then the two branches of the " " isocline are determined by

$$\xi_{1,2} = \frac{\beta\sin\Phi(1-2\beta\cos\Phi) \pm \sqrt{\beta^2\sin^2\Phi + 4\beta(1-\beta^2)\cos\Phi(1-\beta\cos\Phi)}}{2\beta(1-\beta\cos\Phi)\cos\Phi} \tag{13}$$

and smoothly join in the point A where $\cos\Phi = -\beta\left[2(1-\beta^2)+\sqrt{4-3\beta^2}\right]^{-1}$ The value of $\Phi = -\pi/2$ is a singular point for ξ, where at $\Phi \to -\pi/2 - 0$, $\xi_2 \to +\infty$ and at $\Phi \to -\pi/2 + 0$ $\xi_2 \to -\infty$. $\Phi = \pi/2$ is an analogous point for ξ here $\xi_1 \to +\infty$ at $\Phi \to \pi/2 - 0$, $\xi_1 \to -\infty$ at $\Phi \to \pi/2 + 0$ (positive values of ξ only have physical meaning). $\Phi = 0$ also becomes a singular point at $\beta \to 1$ where $d\xi/d\Phi\big|_{\Phi=0} = (1-2\beta)/2(1-\beta) \to -\infty$.

In the interval $-\pi < \Phi < -\pi/2$ the "∞" isoclines are below the isocline "0"; at the point A $\xi_1=\xi_2=\mathrm{tg}\,\Phi[1-1/2(1-\beta\cos\Phi)]<\mathrm{tg}\,\Phi$.

Fig. 1 shows "0" and "∞" isoclines for $\beta=0.3,0.6,0.9$. "0" and "∞" isoclines devide the plane (ξ,Φ) into regions, where $d\xi/d\Phi$ has a definite sign. As is easy to check the right hand side of the Eq.(10) is never equal to unity. It means, that in ξ,Φ variables the equation of electric field lines has the form [5,6]:

$$\frac{d\xi}{d\Phi}=1, \tag{14}$$

and Poynting vector field lines, perpendicular to $\vec{E}$ in real coordinates, in ξ,Φ coordinates are, at least, not tangent. It means that $d\xi/d\Phi>1$ in the region restricted by the isocline "∞", and in regions beyond the isocline "0" $0<d\xi/d\Phi<1$ (shaded are in Fig.1). In points of intersection of integral curves with isoclines "0" and "∞" (expect for $\xi=0$) the curvature of integral lines differs from zero (the curvature sign is easily determined, see Fig.2). The integral line has a point of inflexion only in the point A.

Let us qualitatively examine the behaviour of integral curves of Eq.(10). Isolate two seculiar integral curves which determine the class of curves intersecting the isocline "∞" once or thrice. The first one is tangent to isocline "∞" in the point A (the dashed line in Fig.3a). The second one crosses the isocline "∞" to the left from the axis $\Phi=-\pi/2$ (which has been proved by the numerical analysis) and without intersecting the axis $\Phi=-\pi/2$, tends to infinity. It is obvious that all the curves intersecting the isocline "∞" to the right from this curve tend to infinity. Let us consider one of them which intersects the isoline "∞" in the point H (in Fig.3b). The integral curve then turns to the right downwards and after intersecting the isocline "0" in the point B, it necessarily crosses the axis $\Phi=\pi$ at definite height; in the shaded area in Fig.1 $0<d\xi/d\Phi<1$. If the integral curve is continued from the axis $\Phi=-\pi$; then rising to the right, it intersects the isocline "0" in the point C at the final height (here too, $0<d\xi/d\Phi<1$). Then, it either intersects the isocline "∞", or running underneath it, crosses the isocline "0" in the point B' which lies lower than the point B. In the former case the integral curve intersects the isocline "∞" in the interval of $\Phi<-\pi/2$ twice - in the points D and E. Since at $\beta\to 0$ the ordinate of the point $A\sim 1/\beta$ and the ordinate of the point $F\sim 1/\sqrt{\beta}$, then at small β the point A is essentially higher than F, and F, in its turn, is higher than B_1. That is why at small β the integral curve, after crossing once the isocline "∞" above the point A, at its next turr-around (due to restriction $0<d\xi/d\Phi<1$ to the right from B_1) will pass underneath the isocline "∞". One should expect, that at any β the curve will not cross the isocline "∞" in the next turn-over (the latter case is realized) and then will asymptotically approach the axis $\xi=0$ (the number of turns-over $\to\infty$). Numerical analysis of Eq.(10) supports the mentioned qualitative conclusions. For the numerical solution of Eq.(10) the Runge-Kutta method cf the fourth order of accuracy has been used. In the range, where $|d\xi/d\Phi|>1$ the inverse equation has been solved. Integration steps were chosen to be

$\Delta\Phi = 0.05$, $\Delta\xi = 0.01, 0.02, 0.05, 0.005$. The behaviour of integral curves has been investigated from the initial points $(-\pi, a)$ for some values of $a \leqslant 30$, and from the points (b,5) for some values in the range $(-\pi, \pi)$. It turned out, that all the considered integral curves after crossing the axis $\Phi = -\pi$ pass underneath the isocline "∞" immediately or after one turn-over and then approach the axis $\xi = 0$ without closing (see Figs.4,5).

Let us supplement the qualitative and numerical analysis by the exact solution of Eq.(10) for $\beta=1$. Formally the Eq. (10) tolerates the limit $\gamma \to \infty$ $(\beta \to 1)$ and is essentially simplified

$$\frac{d\xi}{d\Phi} = -\frac{\xi}{\xi(1-\cos\Phi)+\sin\Phi}. \tag{15}$$

Passing to a new variable $\lambda = \mathrm{tg}(\Phi/2)$ one will find the general solution for the Eq.(15) (see Fig.6):

$$\lambda = \frac{2-\xi}{C-\xi^2} \tag{16}$$

where C is the integration constant. The transform (9) gives the dependence of Φ on θ in the form of a smoothed step which at $\beta \to 1$ becomes right-angled. Despite the fact that the replacement at $\beta=1$ is degenerate, it may admit the following physical meaning. The horizontal parts of the step $(\theta = 2\pi k,\ k = 0, \pm 1, \pm 2, \ldots)$ give the spatial sweep of signals emitted tangentially to the trajectory (in Ref. [3] called γ-spiral). The vertical lines correspond to circles with their centres on the trajectory. Such circles occupy the whole place and together with the γ-spiral form a system of Poynting vector field lines for $\beta = 1$. It is evident that at $\beta \to 1$ one should specify only the field around the γ-spiral. The motion of lines beyond these regions is conserved. It means, that at ultrarelativistic energies the field does not depend on γ at all beyond the circle of the γ - spiral.

Let us give the spatial representation of the Poyntings vector field lines of synchrotron radiation in spatial coordinates. Note, that the isocline "0" corresponds to the neutral line of magnetic field (here the poles of the Poynting vector field lines are situated). The isocline "∞" is a curve, surrounding the neutral line. The topology of the Poyntings vector field lines and of the electric field of synchrotron radiation is shown in Fig.7. Here the particle moves counterclockwise, it being at the present moment at the top point of the trajectory. The neutral line is not shown. (Only the first integral curve from the two plotted in Fig.3a is shown (the dashed line in Fig.3a). This line, indicated by a broad line in Fig.7, is unstable- the integral lines branch out of it in succession. At small distances all the integral curves coil around the charge in infinite number of turns. Asymptotical approach of lines to the charge is not shown.

Up to now we did not consider the direction of the Poynting vector field lines. It is easily seen, that every integral curve consists differently directed which splice on the neutral line. This line is the geometrical place of the poles of the spatial field of Poynting vectors. Consider the surface " "woven" by integral curves issued from the pole on the neutral line. This surface is closed for uniform and rectilinear motion - it is a sphere with its diameter on the trajectory. This is not so for synchrotron radiation. Special investigations are necessary for the determinating of shape of such surfaces, nevertheless, it is essential, that the surface is not closed. This follows from the fact, that in the plane of motion the Poynting vector field lines are not closed. The physical meaning is clear - for uniform circular motion there exists nonzero flux of energy of electromagnetic field, locally directed along these lines: consequently, there is no any surface consisting of Poynting vector field lines and surrounding the charge.

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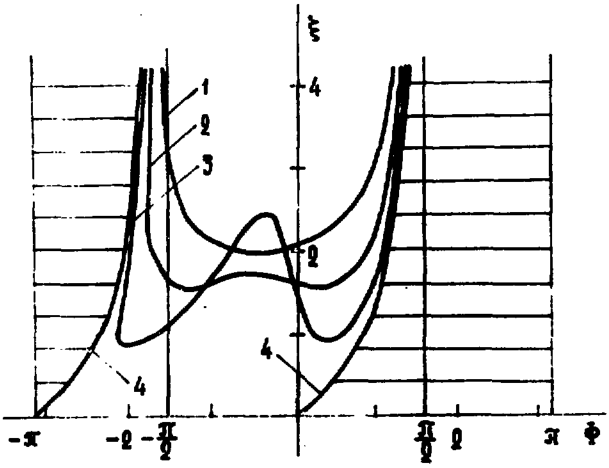

Fig.1. Isoclines "0" (4) and "∞" at β =0.3(1), β=0.6(2) and β= 0.9(3). In the shaded region $0<d\xi/d\Phi<1$.

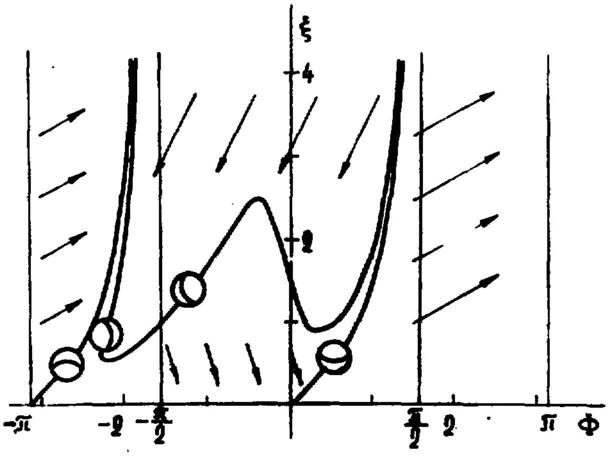

Fig.2. Curvature of integral curves at the points of intersection of isoclines "0" and "∞". The curvature is other than zero everywhere expect for the point A. Arrows show the direction of integral curves. For positive direction, regions being separated out, where the derivative $d\xi/d\Phi$ is more or less than unity.

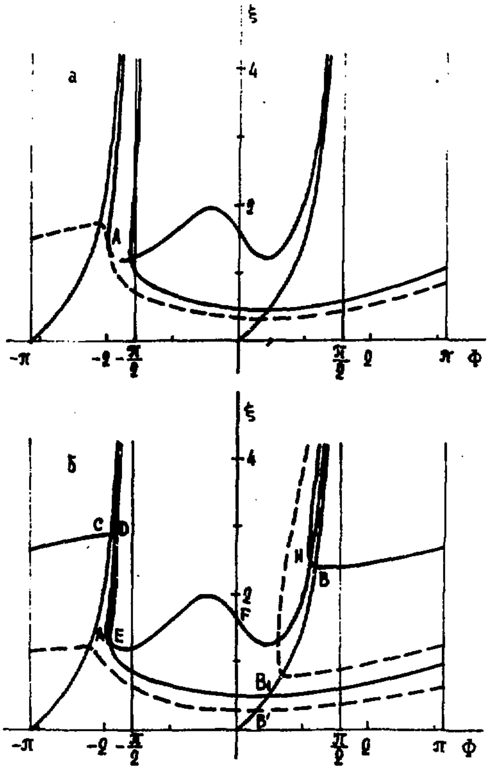

Fig.3a Singular integral curves separating out classes of integral curves which cross the isocline "∞" once or three times;

b-Integral curve, which crosses the isocline "∞" three times (broad line) and the one crossing the isocline only once (dashed line). The asymptotic tendency of curves to the axis $\xi=0$ is not shown.

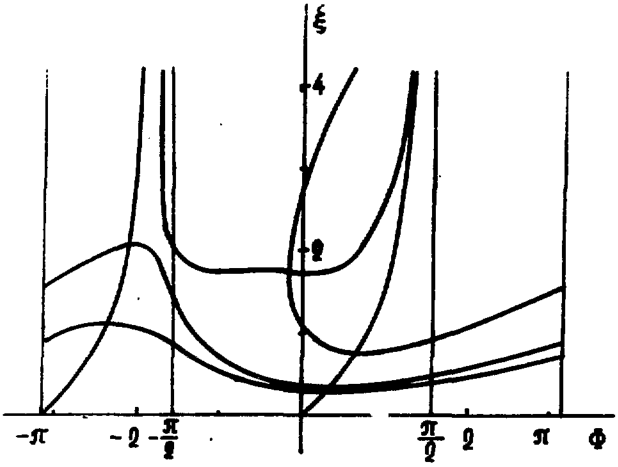

Fig.4. Integral curve obtained by the numerical solution of the Eq.(10) for initial values of $\Phi_0 = 1.5$, $\xi_0=5$.

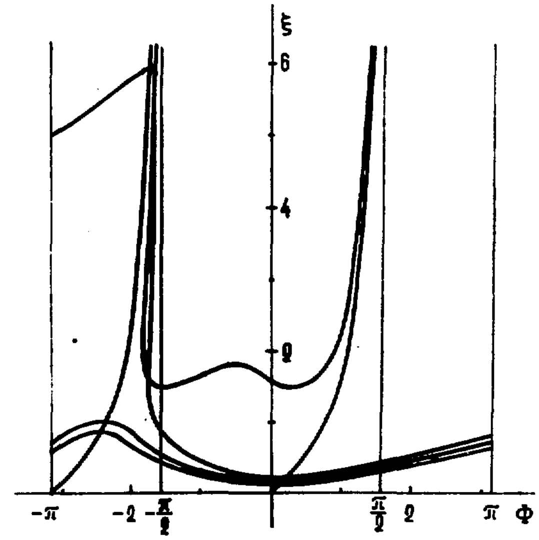

Fig.5. Integral curve, obtained as a result of numerical solution of the Eq.(10) for initial values of $\Phi_0=-\pi$, $\xi_0=5$.

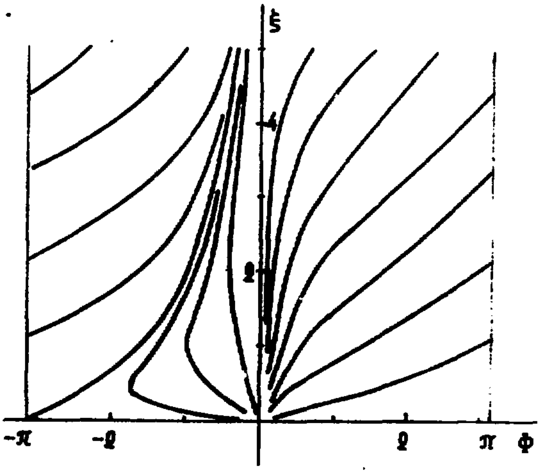

Fig.6.Integral curves of the Eq.(10) at $\beta=1$.

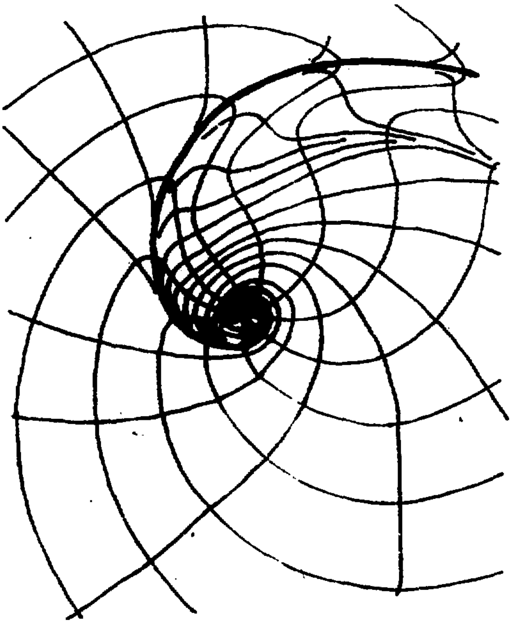

Fig.7.Field lines of Poynting vectors and of the electric field of synchrotron radiation.

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ЛИНИИ ПОЛЯ ВЕКТОРОВ ПОЙНТИНГА СИНХРОТРОННОГО ИЗЛУЧЕНИЯ
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