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STOCHASTICITY OF TIME-DEPENDENT SPHERICALLY SYMMETRIC SOLUTIONS OF YANG-MILLS EQUATIONS

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ԺԱՄԱՆԱԿԱՅԻՆ ԿԱԽՈՒՄ ՈՒՆԵՑՈՂ ՅԱՆԳ-ՄԻԼՍԻ ՀԱՎԱՍԱՐՈՒՄՆԵՐԻ ԳՆԴԱՁԵՎ ՀԱՄԱՉԱՓ ԼՈՒԾՈՒՄՆԵՐԻ ՍՏՈԽԱՍՏԻԿՈՒԹՅՈՒՆԸ

Ցույց է տրված, որ ժամանակային կախում ունեցող Յանգ-Միլսի գրնդաձև համաչափ հավասարումները ոչ ինտեգրվող մի համակարգ է, մասնավորապես, փուլային տարածությունը Վու-Յանգի անշարժ լուծման մոտակայքում հանդիսանում է ստոխաստիկության տիրույթ:

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S.G. MATINYAN, E.B. PROKHORENKO, G.K. SAVVIDY

STOCHASTICITY OF TIME-DEPENDENT SPHERICALLY SYMMETRIC SOLUTIONS OF YANG-MILLS EQUATIONS

It is shown that time-dependent spherically symmetric Yang-Mills equations are a non-integrable system. In particular, the phase space in the vicinity of Wu-Yang static solution is the stochasticity region.

Yerevan Physics Institute

Yerevan 1986

Препринт ЕФИ-890(41)-86

С.Г.МАТИНЯН, Е.Б.ПРОХОРЕНКО, Г.К.САВВИДИ

СТОХАСТИЧНОСТЬ СФЕРИЧЕСКИ СИММЕТРИЧНЫХ РЕШЕНИЙ УРАВНЕНИЙ ЯНГА-МИЛЛСА, ЗАВИСЯЩИХ ОТ ВРЕМЕНИ

Показано, что сферически симметричные уравнения Янга-Миллса, зависящие от времени, представляют неинтегрируемую систему, в частности, фазовое пространство вблизи статического решения Ву-Янга является областью стохастичности.

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Ереван 1986

It is well known that Yang-Mills (Y.M.) equations dependent on time only ("Y.M. classical mechanics") are a non-integrable system [1-3] .

It should be said that time has come to investigate the general 3+1 - dimensional Y.M. field system from this point of view.

We present here the results of such investigation for spherical symmetry, i.e. when non-abelian vector potential A_μ^a depends on $r = |\vec{r}|$ and t . Meanwhile, one can obtain an answer to the question about stability of the well-known spherically symmetric static solutions of Wu-Yang type 4 .

The considered problem is reduced to studying a nonlinear string equation of the type:

$$u_{tt} - u_{xx} = F(x, u, u_x, u_t, \cdots) \qquad (1)$$

(u_t , u_{tt} , etc. are derivatives with respect to corresponding argument). The analytical investigation of the question on integrability of such a system in the general case is impossible.

The most appropriate method here is the approach suggested in the well-known work of Fermi-Pasta-Ulam 5 . The method consists in the replacement of continuous string (1) by its discrete analog - the chains of coupled anharmonic oscillators - and in the inspecting of the energy distribution in the string oscillations harmonics. In the general case, when the system is non-integrable, at small initial perturbations an energy transfer between several harmonics takes place, the system is "thermalized" anomalously slowly, which was just observed by the authors of Ref. 5. At larger perturbations, the system motion occurs in ergodic layer; this results in uniform energy distribution in harmonics, being observed in Fermi-Pasta-Ulam system 6. Exactly this approach we have chosen to investigate the system of spherically symmetric Y.M. equations whose general structure A_μ^a for the SU(2) group in 3+1 -space-time is given by the expression 7:

$$A_j^a = \frac{\varphi_1}{r^3}(\delta_{ja} r^2 - x_j x_a) + \frac{1+\varphi_2}{r^2}\varepsilon_{jak} x_k + A_1 \frac{x_j x_a}{r^2}$$
$$A_0^a = A_0 x_a / r , \tag{2}$$

where arbitrary functions $\varphi_{1,2}$ and $A_{0,1}$ depend on r and t. The gauge invariance allows to put $A_0 = 0$. In what follows we shall study in detail a special case when $\varphi_1 = A_1 = 0$, so we shall deal with nonlinear string of the form of (1) ($\varphi \equiv \varphi_2$):

$$(\partial_t^2 - \partial_r^2)\varphi = - r^{-2}\varphi(\varphi^2 - 1). \tag{3}$$

The static solution of (3) $\varphi = 0$ is the Wu-Yang monopole $A_j^a = r^{-2}\varepsilon_{jak} x_k$, $\varphi = -1$ is the vacuum solution of $A_j^a = 0$, $\varphi = 1$ gives the field which is gauge-equivalent to vacuum one.

Qualitative analysis of Eq.(3) shows that spherically symmetric solutions of Y.M. equations already in the static limit ($\pi_\varphi = \partial\varphi/\partial t = 0$) are unstable: small perturbations of initial conditions ($\varphi(r)$ and $\partial\varphi/\partial r$) drastically change the behavior of solutions, resulting either in appearance of singularities or in change of their position. There are distinguished five static solutions of (3) which remain finite for all $r \geqslant 0$. These are already known $\varphi = \pm 1$ and $\varphi = 0$, and two separatrices of Eq.(3), $\varphi_{c_{1,2}}(r)$, which at $r \to 0$ coincide with the Wu-Yang solution, while at $r \to \infty$ with vacuum ones ($\varphi(r) = \pm 1$). $\varphi_{c_{1,2}}(r)$, as far as we know, were hitherto unknown, so these solutions are quite new ones.

The analysis of the system (3) phase trajectories near the static solutions exhibited the exponential time instability for solutions $\varphi(r) = 0$ and the new solutions $\varphi_{c_{1,2}}(r)$ as well as stability, relative to small perturbations, of vacuum solutions $\varphi(r) = \pm 1$. All singular (at $\pi_\varphi = 0$) solutions of (3) turn out exponentially unstable too.

Finite perturbations of Eq.(3) were investigated by us in the numerical experiments of Ref. 5 type. The continuous string of (3) was approximated by a set of nonlinear coupled oscillators $\varphi(i)$ ($i = 1,2,\ldots,N$) whose number N was taken equal to 64 and 128.

A discrete analog of Eq.(3)

$$\ddot{\varphi}(i,t) = \frac{\varphi(i+1,t) - 2\varphi(i,t) + \varphi(i-1,t)}{(\Delta r)^2} - \frac{\varphi(i,t)[\varphi^2(i,t) - 1]}{(i\Delta r)^2} \tag{4}$$

was numerically integrated (Δr is the string discretization step which we took equal to 0.1).

Perturbations near the static solutions $\varphi = 0$, $\varphi = \pm 1$, $\varphi_{c_{1,2}}(r)$ were studied by introduction of the corresponding initial and boundary conditions expressed via expansion in harmonics:

$$\varphi(i,t) = \sqrt{2/N} \sum_{i=1}^{N-1} \Psi(i,t) \sin(\pi i j/N).$$

Fig.1 gives examples of time dependence for energies of three modes $j = 8,9,10$ at small initial excitation of five modes (amplitude $A = 0.1$)

$j_o = 6 - 10$. Here $N = 64$. Total energy of "string" (4) $E_{tot} = 4.8$ ($\Delta E_{tot} / E_{tot} = 1\ \%$). Modes $j_o = 6, 7$ behave analogously, while all the rest ones are practically non-excited. One can see that the system has approximately quasi-periodical motion.

The picture changes essentially with increasing string energy. In Fig.2 ($N = 64$, $E_{tot} = 1100$, $\Delta E_{tot} / E_{tot} = 0.1\ \%$, $j_o = 30, 31, 32$) one can clearly see the process of equal distribution of energies for primarily excited (amplitude $A = 1$, $j_o = 30 - 32$) and the rest of modes (Fig.2 presents energies of some modes, $j = 29, 33, 34, 57$). The picture remains qualitatively the same at further (thrice-) increasing of energy for these modes.

In this case the system thermalization takes place. With increasing N (N was taken equal to 128) (i.e. as the investigation of Eq.(4) shows, at approaching to continuous limit) the string is "thermalized" at smaller perturbations.

The system stochasticity is exhibited in Fig.3 too, where we have given a distribution of energy $\overline{E}_i$ ($i = 1,2,\ldots,64$) averaged over large time (range of averaging 780 with a step 0.01) with respect to modes. One can see that on the average all modes became excited.

Thus, we have all grounds to claim that not only Y.M. classical mechanics [1,2] but also Y.M. classical field theory describing a system with infinite number of degrees of freedom is non-integrable, i.e. exhibits dynamical chaos.

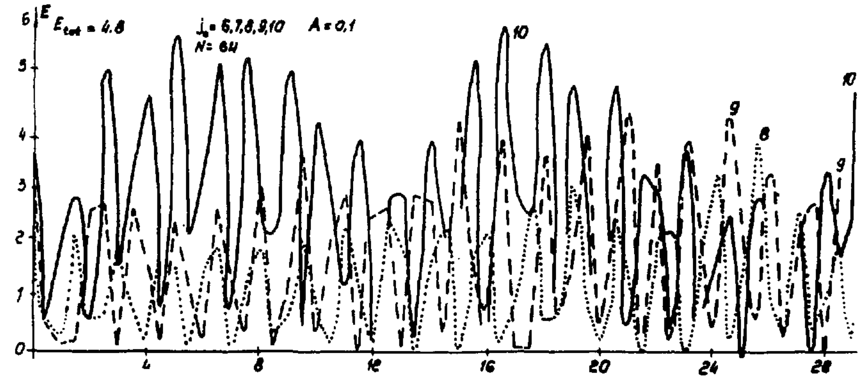

Fig.1. Quasi-periodical motion at small amplitude of initial perturbation (A = 0.1; j_o = 6,7,8,9,10; N = 64).

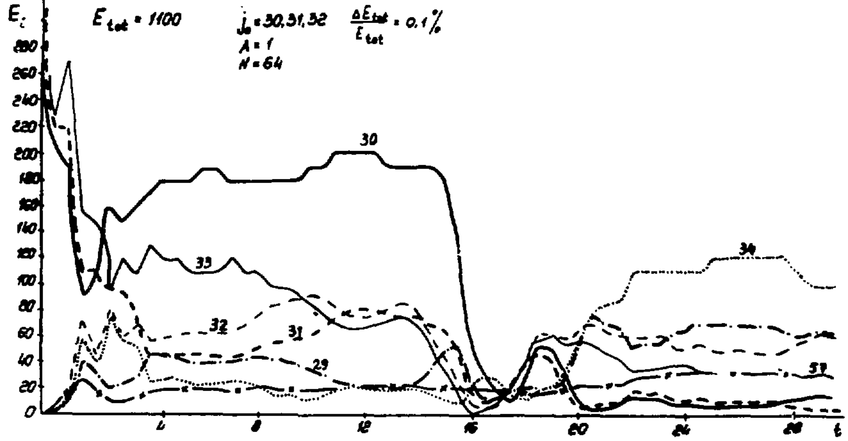

Fig.2. Energy of modes j = 29,33,34,57 as a function of time at large amplitude of initial perturbation (A = 1; j_o = 30,31,32; N = 64).

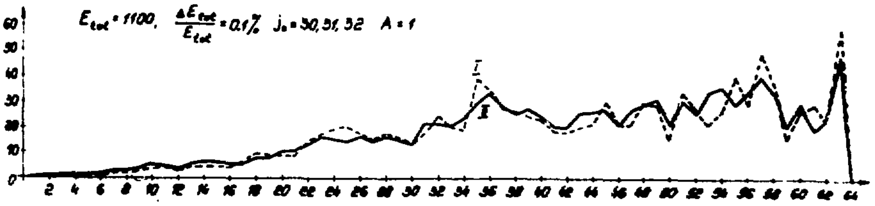

Fig.3. Distribution of averaged over large time interval (total interval of integration 780, step 0.01) energies $\overline{E}_j$ (j = 1,2,..., 64) with respect to modes.

Curve I - averaging interval t_i = 40, t_f = 410.

Curve II - averaging interval t_i = 40, t_f = 780.

REFERENCES

1. Matinyan S.G., Savvidy G.K., Ter-Arutunyan-Savvidy N.G. Yang-Mills Classical Mechanics. Color Non-Linear Oscillations. - ZhETF, 1981, v.80, p.830.
2. Matinyan S.G., Savvidy G.K., Ter-Arutunyan-Savvidy N.G. Stochasticity of Yang-Mills Classical Mechanics and Its Elimination by Higgs Mechanism. - Pis'ma v ZhETF, 1981, v.34, p.613.
3. Чириков Б.В., Шепелянский Д.Л. Стохастические колебания классических полей Янга-Миллса. Письма в ЖЭТФ,1981, т.34,с.171.
4. Wu T.T., Yang C.N. In: Properties of Matter under Unusual Conditions / ed. H.Mark, S.Fernbach, Interscience, N.Y.: 1969, p.349.
5. Fermi E., Pasta J.R., Ulam S. Studies of Non-Linear Problems. - Los Alamos Scientific Lab. Rep. No. LA-1940, May 1955.
6. Израилев Ф.М., Хисанутдинов А.И., Чириков Б.В. Численные эксперименты с нелинейной цепочкой. Препринт №252, ИЯФ СО АН СССР, 1968.
7. Witten E. Some Exact Multipseudoparticle Solutions of Classical Yang-Mills Theory. - Phys.Rev.Lett., 1977, v.38, p.121.

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УРАВНЕНИЙ ЯНГА-МИЛЛСА, ЗАВИСЯЩИХ ОТ ВРЕМЕНИ
(на английском языке, перевод Н.З.Асланян)

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