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ON QUARK-TO-HADRON FRAGMENTATION FUNCTIONS

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ՔՎԱՐԿԻ՝ ՀԱԴՐՈՆ ՀԱՏՎԱԾԱՎՈՐՄԱՆ ՖՈՒՆԿՑԻԱՆԵՐԻ ՄԱՍԻՆ

Առաջարկվում է քվարկ-գլյուոնային համակարգերի հատվածավորման հարցին՝ վիճակագրական մոտեցում: Որոշված են քվարկի քվանտային թվերով բազմամասնիկ քվարկ-գլյուոնային հադրոն հատվածավորման ֆունկցիաները: Բերված է առաջարկված մոդելում ստացված խորը ոչ-առաձգական նեյտրինո/հականեյտրինո/ և նուկլոն փոխազդեցություններում մեզոնների ծնման կտրվածքի համեմատությունը փորձարարական տվյալների հետ:

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ON QUARK-TO-HADRON FRAGMENTATION FUNCTIONS

A statistical approach to the question of fragmentation of quark-gluonic systems is considered. The functions of fragmentation of multiparton quark-gluonic system with quark's quantum numbers to hadrons are defined. A comparison of the model predictions with experimental data is carried out with respect to π^+/π^- -mesons yields in deep-inelastic scattering of neutrino (antineutrino) on the nucleon.

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Р.Г.БАДАЛЯН

О ФУНКЦИЯХ ФРАГМЕНТАЦИИ КВАРКА В АДРОН

Рассматривается статистический подход к вопросу фрагментации кварк-глюонных систем. Определены функции фрагментации многочастичной кварк-глюонной системы с квантовыми числами кварка в адроны. Приводится сравнение предсказаний модели с экспериментальными данными по отношению выходов π^+/π^- - мезонов в глубоко-неупругом рассеянии нейтрино (антинейтрино) на нуклоне.

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In order to define inclusive spectra of hadrons in deep-inelastic lepton-hadron collisions, spectra of hadrons with high transverse momenta P_T in hadron-hadron interactions as well as hadron distribution in e^+e^- -annihilation, usually there are introduced fragmentation functions $D_q^h(x)$ of quark q to hadron h [1,2], where x is a ratio of longitudinal (relative to momentum direction of initial quark q) momentum of h hadron to momentum of q quark. Various phenomenological fragmentation models [3-5] also operate by quark (diquark)-to-hadron fragmentation functions in order to describe inclusive spectra of secondary hadrons with low P_T in hadron-hadron collisions.

In the mentioned processes with high momentum transfers at the process initial stage, there occurs a "knock-out" of the quark (antiquark) with large effective mass from the hadron and takes place radiation of hard gluons and quark-antiquark pairs. This stage can be described quantitatively within the theoretico-perturbative quantum chromodynamics (QCD). According to the modern representations, the knocked-out quark is assumed to lose its momentum (high virtuality) via the emission of gluons which in turn may transform into quark-antiquark pairs. The initial stage of such transformation of quark into quark-gluonic system can be described in the framework of evolutionary equations of Altarelli-Parisi [2, 6-9] , since at the initial stage the quark-gluon interaction constant is still small due to the quark high virtuality. With decreasing quark and gluon virtuality the quark-gluon interaction constant increases, hence the problem ceases to be theoretico-perturbative already. A second stage the present work is devoted to, namely the fragmentation of the produced compound quark-gluonic system to hadrons, is a "soft" process. The determination of the function of soft fragmentation of such a system (possessing the q quark quantum numbers) to hadrons is essentially not a theoretico-perturbative problem and its solution within QCD is somewhat complicated due to quark and gluon confinement effects. However, due to strong coupling between quarks and gluons, one may expect that a statistical equilibrium takes place in quark-gluonic system. In the light of the above-said, the application of the statistical consideration to the question of soft fragmentation of quark (of quark-gluonic system the quark transforms to) to hadrons seems reasonable, since the mentioned problem is essentially multi-particle.

In the infinite momentum frame the quark-gluonic system S_q to which the q quark transforms owing to QCD evolution, is characterized by its "valent" composition (by the type of q quark) and distribution functions of "valent" or "leading" quark and sea partons (quark-antiquark pairs and gluons). The number of sea partons in the S_q system is not fixed and may vary from zero to infinity *). The produced quark-gluonic system S_q has many features common to the compound one (valon) in hadron [9,10] - the presence of leading (valent) quark and statistically distributed sea partons.

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*) Energy restrictions to the number of sea partons are not considered, since the mean transverse mass of parton $\mu \ll p$, where p is a momentum of the initial quark

Therefore, to describe such a system, one presumably can apply the analogy to the Kuti-Weisskopf model [11] . The distribution density of (1 + $\bar N_S$)-parton configuration of the S_q system ($\bar N_S$ is the number of sea partons in the considered configuration) is determined by the expression [10,11] :

$$dS_{\bar N_S} = V_q(x_q)\,dx_q \prod_a \frac{1}{(\bar N_a)!} S_a(x_{aj})\,dx_{aj}\,\delta\Big(1-x_q-\sum_a \sum_{j=1}^{\bar N_a} x_{aj}\Big) \qquad (1)$$

The quark-gluonic system S_q with the q quark quantum numbers is defined as a sum of all possible (1 + $\bar N_S$)-parton configurations:

$$dS_q = \frac{1}{Z} \sum_{\bar N_S=0}^{\infty} dS_{\bar N_S} \qquad (2)$$

In expressions (1) and (2), x is the Feynman variable (portion of parton momentum relative to momentum of initial quark q), $a=u,d,s,\dots,\bar u,\bar d,\bar s,\dots,G$ denotes a sort of sea parton, $\bar N_a$ is the number of partons of sort of a , $\bar N_S = \sum_a \bar N_a$, Z is statistical weight (normalization factor). $V_q(x)$ and $S_a(x)$ are so-called input or noncorrelated distribution functions respectively for leading (valent) quark and sea partons (the distribution functions without account of the longitudinal momentum conservation law). In the general case, these functions have the form as follows [10-12] :

$$V_q(x) = x^{\beta_q} Q_q(x)/\sqrt{x^2+x_T^2}, \quad Q_q(0)=1, \quad \beta_q>0 \qquad (3a)$$

$$S_a(x) = g_a P_a(x)/\sqrt{x^2+x_T^2}, \quad P_a(0)=1, \quad g_a>0 \qquad (3b)$$

Here $x_T=\mu/p$, where μ is effective (mean) transverse mass of parton, p is momentum of initial quark q ($p \gg \mu$). Parameters β_q , g_a and the form of functions $Q_q(x)$, $P_a(x)$ can be defined from the comparison [12] of results obtained for the distribution of leading quark and sea partons, or quark-to-hadron fragmentation function with experimental data available. To simplify the formulae and to obtain qualitative estimates, we shall assume $Q_q(x) = 1$ and $P_a(x) = 1$ by analogy with Ref.[11] .

With respect to expressions (1)-(3), for the distribution of leading quark q and sea partons of quark-hadronic system S_q we find [10-12] :

$$q(x) = V_q(x)(1-x)^{-1+\gamma}/B(\beta_q, \gamma) \tag{4a}$$

$$a_s(x) = S_a(x)(1-x)^{-1+\gamma+\beta_q} \tag{4b}$$

where $B(x,y)$ is the Euler beta function, $\gamma = \sum_a g_a$. Parameters β_q and g_a indicate in what proportion the longitudinal momentum of initial quark q is shared between the leading quark q (the mean momentum of leading quark q is $\bar{x}_q = \beta_q/(\gamma + \beta_q)$) and sea partons (the mean momentum of sea parton a is $\bar{x}_a = g_a/(\gamma + \beta_q)$).

Turn now to the consideration of hadroproduction in the fragmentation region of initial quark q . To be concrete, let us consider the process of fragmentation of initial u -quark to π^+-meson. Isolate in the quark-gluonic system S_u into which transforms the initial u -quark a substate which contains the leading u -quark, the sea $\bar{d}$ -quark and arbitrary number of sea partons "belonging" to the considered substate with a probability W . The hadron, in this case the π^+-meson, is produced via recombination of all the partons belonging to the considered multiparton substate. By analogy with Ref.[13] , we assume that the recombination stage is preceded by a formation of constituent objects - valons (in this case, U - and $\bar{D}$ -valons which just recombine to the final π^+-meson) via statistical grouping [10] of partons inside the multiparton substate considered. Here the probabilities for the sea parton of the considered multiparton substate to belong to U - or $\bar{D}$ -valon are W_1 and W_2 , respectively ($W = W_1 + W_2$). It is assumed, just as in Ref.[12] , that at formation of the valon not containing the leading (valent) quark the value of W_2 is zero. At formation of U -valon, which contains the leading (valent) u -quark, the value of $W_1 = W$ may differ from zero. Then the density of probability that U - and $\bar{D}$ -valons carry respectively x_1/x and x_2/x portions of total momentum $x\ (x = x_1 + x_2)$ of the isolated multiparton substate is determined by the expression [10,12]:

$$F^{V}_{u\bar{D}}(x_1, x_2) = g_{\bar{d}}(1-x)^{-1+(1-W)\gamma}\, x_1^{-1+W\gamma+\beta_u}\, x_2^{-1}\, B^{-1}(W\gamma+\beta_u, (1-W)\gamma) \qquad (5)$$

The function of fragmentation of initial u -quark to π^+-meson, for the case when in formation of the latter participates the leading u -quark, can be determined as follows:

$$x D^{\pi^+}_{u}(x) = \int F^{V}_{u\bar{D}}(x_1, x_2)\, R^{\pi}\left(\frac{x_1}{x}, \frac{x_2}{x}\right)\delta\left(1 - \frac{x_1}{x} - \frac{x_2}{x}\right) dx_1\, dx_2 \qquad (6)$$

The recombination function $R^{\pi}(y_1, y_2)$ [14] defines the probability of recombination of U - and $\bar{D}$ -valons with momenta x_1 and x_2 , respectively, to the final π^+-meson with a longitudinal momentum $x = x_1 + x_2\ (y_{1,2} = x_{1,2}/x)$. The function $R^{\pi}(y_1, y_2)$ is defined by analogy with Ref.[14] , via the expression [13] :

$$R^{\pi}(y_1, y_2) = y_1 y_2 \qquad (7)$$

If we take into account that the π^+-mesons in the fragmentation region of the initial u -quark can be produced also on the sea (nonleading) u -quarks, then for the total $u \rightarrow \pi^+$ fragmentation function we find:

$$x D_u^{\pi^+}(x) \doteq \int \left[F_{u\bar{D}}^{V}(x_1,x_2) + F_{u\bar{D}}^{S}(x_1,x_2) \right] R^{\pi}\left(\frac{x_1}{x}, \frac{x_2}{x}\right) \delta\left(1 - \frac{x_1}{x} - \frac{x_2}{x}\right) dx_1 \, dx_2 \quad (8)$$

where

$$F_{u\bar{D}}^{S}(x_1,x_2) = g_u \, g_{\bar{d}} \, (1-x)^{-1+\gamma+\beta_u} \, (x_1 x_2)^{-1} \quad (9)$$

The second term in expression (8) defines also the fragmentation function of initial u-quark to π^--meson (not containing the leading u-quark):

$$x D_u^{\pi^-}(x) = \int F_{D\bar{U}}^{S}(x_1,x_2) \, R^{\pi}\left(\frac{x_1}{x}, \frac{x_2}{x}\right) \delta\left(1 - \frac{x_1}{x} - \frac{x_2}{x}\right) dx_1 \, dx_2 \quad (10)$$

Using expressions (8) and (10), for the fragmentation function of initial u-quark to $\pi^{\pm}$-mesons we find:

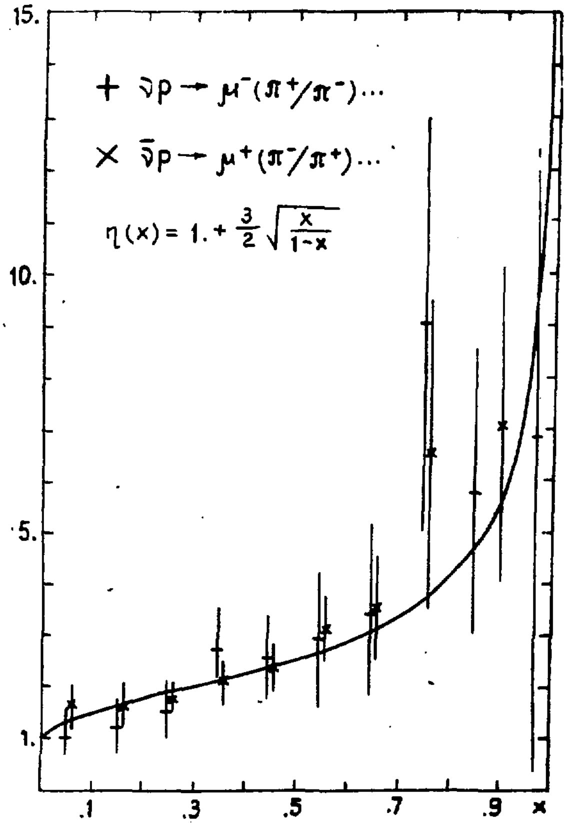
$$x D_u^{\pi^+}(x) = g_{\bar{d}} \, x^{W\gamma+\beta_u} (1-x)^{-1+(1-W)\gamma} \, \frac{B(W\gamma+\beta_u+1,1)}{B(W\gamma+\beta_u,(1-W)\gamma)} + g_u \, g_{\bar{d}} \, (1-x)^{-1+\gamma+\beta_u} \quad (11a)$$

$$x D_u^{\pi^-}(x) = g_d \, g_{\bar{u}} \, (1-x)^{-1+\gamma+\beta_u} \quad (11b)$$

As was mentioned, the quark-gluonic system S_u, to which the initial u-quark transforms due to evolution, has many features common to U-valon in the proton [9,10,12]. If we make use of this analogy, we can determine the values of parameters: $\beta_u = 0.5$, $W = 0$ and $g_u = g_{\bar{u}} = g_d = g_{\bar{d}} \simeq \simeq \gamma/2(N_f+1)$, where N_f is the number of types of quarks. Then, for the ratio $\eta(x) = D_u^{\pi^+}(x)/D_u^{\pi^-}(x)$ we find:

$$\eta(x) = 1 + A(\gamma; N_f)\sqrt{\frac{x}{1-x}} \quad (12)$$

where $\mathcal{A}(\gamma; N_f) = 4(N_f+1)\Gamma(\gamma+0.5)/3\sqrt{\pi}\,\Gamma(\gamma+1)$, $\Gamma(x)$ is the Euler gamma function. The values of the factor $\mathcal{A}(\gamma; N_f)$ for $N_f = 2$ (u -, d -quarks), $N_f = 3$ (u -, d -, s -quarks) as well as various values of γ $(1 \leq \gamma < 3.5)$ are listed in the Table. The Figure presents a comparison of $\eta(x)$ function for $\mathcal{A}(\gamma; N_f)=1.5$ with experimental data on deep-inelastic neutrino (antineutrino) production of $\pi^{\pm}$ -mesons.



The ratio of inclusive production, $\pi^+/\pi^-(\pi^-/\pi^+)$, in deep-inelastic interaction of neutrino (antineutrino) with the proton. Experimental points are taken from Ref.[1] . The curve corresponds to $\mathcal{A}(\gamma; N_f) = 1.5$.

Table

γ	$A(\gamma ; N_f)$	
	$N_f = 2$	$N_f = 3$
1.0	2.000	2.667
1.5	1.698	2.264
2.0	1.500	2.000
2.5	1.358	1.811
3.0	1.250	1.667
3.5	1.164	1.552

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О ФУНКЦИЯХ ФРАГМЕНТАЦИИ КВАРКА В АДРОН

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