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Ed.Sh.EGORIAN, R.P.MANVELYAN

A NEW METHOD OF SUPERSYMMETRIC AND GAUGE-INVARIANT REGULARIZATION

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A NEW METHOD OF SUPERSYMMETRIC AND GAUGE-INVARIANT REGULARIZATION

A new method of gauge and N=2 supergauge invariant regularization based on stochastic quantization and stochastic regularization is proposed.

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1. Introduction.

The regularization method proposed in this work is based partly on the stochastic regularization method which was discussed in Refs. [1,2] .

The standard method of stochastic regularization regularizes five-dimensional stochastic Green functions that arise in the stochastic quantization scheme. The usual four-dimensional Green functions in the non-degenerate theories follow from the stochastic Green functions via the limiting transition $t \to \infty$ (t is the additional fifth time); so they are regularized as well. In Refs. [3,4,5] the standard method of stochastic regularization was used in the study of axial anomalies. Correct expressions for the anomalies in even-dimensional (without account of the additional t) spaces were obtained

In the case of non-degenerate theories (e.g. φ^4), it was proved that the corresponding five-dimensional stochastic theory is renormalizable owing to latent supersymmetry and at $t \to \infty$ reduces to the renormalized 4-dimensional theory [2] .

In the case of degenerate theories (gauge theories) there, however, arise problems. The main result of stochastic quantization in the case of gauge theories is the statement that the usual four-dimensional gauge-invariant averages are obtained in the limit $t \to \infty$ from stochastic averages, the ghost contribution being taken into account automatically (nonabelian case [7] , abelian case [8]). This statement is proved at a naive (nonregularized) level of Feynman diagrams and, as mentioned, holds only for gauge-invariant means; therefore the problems of nonrenormalizability and unitarity in the stochastic quantization scheme are complicated being not solved as yet. As to stochastic regularization in the case of gauge theories, it regularizes only the transverse part of the five-dimensional two-point Green function, while the longitudinal part (having no limit at $t \to \infty$) is not regularized. In addition, the naive Slavnov-Taylor identities in this scheme are not fulfilled (they can be fulfilled in the limit $t \to \infty$), and one should comprehend also the sense of regularization invariance.

To our mind, solution of the above-cited problems in the stochastic quantization scheme is highly urgent because in prospect stochastic quantization and stochastic regularization can be applied at least to supergauge theories with extended supersymmetry not only for solving the invariant regularization problem but also for getting rid of numerous ghosts in these theories.

In this paper we apply the technique of stochastic quantization and stochastic regularization for solution of the superinvariant regularization problem in the usual Faddeev-Popov quantization scheme.

We'll arrive at a formula for writing out the invariant regularizing action. Emphasize that two- and higher-loop diagrams containing no ghosts are regularized in our approach.

Our suggested regularization is generalized automatically also to the N=2 supergauge theory in formulation of Ogievetsky et al. and ensures supergauge-invariant regularization of two- and higher-loop diagrams containing no ghosts.

2. Yang-Mills Action As Five-Dimensional Functional Integral.

A formula to be discussed in this section is the basic tool of our proposed regularization. Suppose the Yang-Mills fields depend, like in stochastic quantization scheme, upon the additional time t . Then we have the following representation of classical Yang-Mills action (S_{YM}) through the five-dimensional functional integral:

$$\exp\left\{-S_{YM}(A_\mu^a(x))\right\} = \lim_{T\to\infty} \exp\left\{-S^T(A_\mu^a(x))\right\} \tag{2.1}$$

$$\exp\left\{-S^T(A_\mu^a(x))\right\} = \int \exp\left\{-\frac{1}{4}\int_0^T \left(\partial_t A_\mu^a(x,t) + \frac{\delta S_{YM}}{\delta A_\mu^a(x,t)}\right)^2 dt\, d^4x \right. \tag{2.2}$$

$$\times \det\left(\partial_t \delta^{ab} g_{\mu\nu}\, \delta(x-x')\,\delta(t-t') + \frac{\delta^2 S_{YM}}{\delta A_\mu^a(x,t)\,\delta A_\nu^b(x',t')}\right) \mathcal{D}A_\mu^a(x,t)$$

In the functional integral (2.2) the following boundary conditions are imposed:

$$A_\mu^a(x,0) = 0$$
$$A_\mu^a(x,T) = A_\mu^a(x) \tag{2.3}$$

In (2.2) there is unexplicitly assumed a normalization to the similar functional integral with zero boundary conditions (i.e. $A_\mu^a(x) = 0$). The integrand in the r.h.s. of (2.2) is the measure of averaging of stochastic Green functions. But in calculation of stochastic averages one should integrate also over boundary fields $A_\mu^a(x)$. As we know, there are divergent parts

$T \to \infty$ in longitudinal parts of non-invariant Green functions. In (2.2) we fixed boundary fields by the condition (2.3) (i.e. we do not integrate over $A^a_\mu(x)$) and thus restricted fluctuations of field $A^a_\mu(x,t)$ at $0<t<T$; so no wonder that $\exp\{-S^T\}$ has finite limit at $T\to\infty$.

Make sure of correctness of formula (2.1) in the free (g = 0) case. For that we calculate $\exp\{-S^T\}$. By Feynman, in Gaussian integral the integral dependence on boundary conditions is determined by classical solution with the same boundary conditions:

$$\exp\{-S^T(A^a_\mu(x))\} = \exp\{-S(A^{cl}_\mu(x,t))\}$$

$$S = \int_0^T dt\, d^4x \frac{1}{4}\left(\partial_t A^a_\mu(x,t) + \frac{\delta S_{YM}(g=0)}{\delta A^a_\mu(x,t)}\right)^2 \tag{2.4}$$

where A^{cl} is a solution of classical equation of motion:

$$[\partial_t^2 g_{\mu\nu} - (p^2)^2 P_{\mu\nu}]A^a_\nu(x,t) = 0$$

$$A^a_\mu(p,0) = 0 \qquad A^a_\mu(p,T) = A^a_\mu(p) \tag{2.5}$$

where $P_{\mu\nu} = g_{\mu\nu} - p_\mu p_\nu/p^2$ and Fourier transformation over x is done. Solution of Eq.(2.5) has the form:

$$A^a_\mu(p,t) = \frac{sh(tp^2)}{sh(Tp^2)} P_{\mu\nu} A^a_\nu(p) + \frac{t}{T} L_{\mu\nu} A^a_\nu(p) \tag{2.6}$$

where $L_{\mu\nu} = p_\mu p_\nu / p^2$.

Substituting (2.6) into (2.4) we obtain:

$$\exp\{-S^T(A^a_\mu(x))\} = \exp\Big\{-\int\frac{d^4p}{(2\pi)^4}\Big[A^a_\mu(-p)\frac{1}{4}(1+ \\ + cth(Tp^2))p^2 P_{\mu\nu}A^a_\nu(p) + \frac{1}{T}A^a_\mu(-p)L_{\mu\nu}A^a_\nu(p)\Big]\Big\} \tag{2.7}$$

In the limit $T\to\infty$ the longitudinal part in (2.7) vanishes, and $cth(Tp^2)\to 1$, so we come to formula (2.1).

Turn now to the general proof of formula (2.1). For that we transform Eq.(2.2), by analogy with Refs. [2,8], to supersymmetric form:

$$\exp\{-S^T(A^a_\mu(x))\} = \int \exp\{-S(\Phi^a_\mu)\}\mathcal{D}\Phi^a_\mu \tag{2.8}$$

where $S(\Phi^a_\mu) = \int_0^T \mathcal{L}(\Phi^a_\mu)dt\, d\theta\, d\bar\theta\, d^4x$

$$\mathcal{L}(\Phi^a_\mu) = \mathcal{L}^{YM}(\Phi^a_\mu) - \Phi^a_\mu\left(\frac{\partial^2}{\partial\theta\partial\bar\theta} + \theta\frac{\partial}{\partial\theta}\partial_t - \partial_t\right)\Phi^a_\mu$$

$$\Phi^a_\mu = A^a_\mu(x,t) + \bar\theta\Psi^a_\mu(x,t) + \bar\Psi^a_\mu(x,t)\theta + \bar\theta\theta C^a_\mu(x,t) \tag{2.9}$$

In integral (2.8) the following boundary conditions are imposed:

$$A^a_\mu(x,T) = A^a_\mu(x)\,, \qquad A^a_\mu(x,0) = 0$$

$$\bar\Psi^a_\mu(x,0) = \bar\Psi^a_\mu(x,T) = \Psi^a_\mu(x,0) = \Psi^a_\mu(x,T) = 0 \tag{2.10}$$

Zero boundary conditions imposed on fields Ψ , $\bar\Psi$ in Eq.(2.10) ensure reality of the action in the r.h.s. of Eq.(2.9).

The additional fields enter (2.9) quadratically, i.e. in Eq.(2.8) the integral over fields C_μ^a is gaussian. This implies that C_μ^a -integration is equivalent to substitution $C_\mu^a = \frac{1}{2}(\partial_t A_\mu^a + \delta S_{YM}/\delta A_\mu^a)$, this being a solution of classical equation of C_μ^a fields. Therefore the boundary contions imposed on C_μ^a field at $t = 0$ and $t = T$ are determined by the value of expression $1/2(\partial_t A_\mu^a + \delta S_{YM}/\delta A_\mu^a)$ on classical solution of the equation of motion for fields A_μ^a. At $g = 0$, with the help of solution (2.6), one can readily be convinced in validity of equation

$$\lim_{T\to\infty} (C_\mu^a(t) - \partial_t A_\mu^a(t))_{t=0,T} = 0 \tag{2.11}$$

From the general analysis of the equation of motion one can easily understand that condition (2.11) is preserved also in the case $g \neq 0$ (one may become sure of that when having solved the equation of motion by the perturbation theory). Under such analysis or solution by perturbation theory an important role is played by the boundary condition $A_\mu^a(x,T) = A_\mu^a(x)$, this restricting the values of classical solutions of the equations of motion at $0 < t < T$. The condition (2.11) means that there is no independent integration over boundary values of $C_\mu^a(0)$ and $C_\mu^a(T)$.

By analogy with Refs. [10,11] we introduce a one-parametrical family of actions $S^{T,\lambda}$:

$$\exp\{-S^{T,\lambda}(A_\mu^a(x))\} = \int \exp\{-\int_0^T dt\, d\theta\, d\bar\theta\, d^4x\, \mathcal{L}^\lambda(\Phi_\mu^a)\}\mathcal{D}\Phi_\mu^a \tag{2.12}$$

$$\mathcal{L}^\lambda(\Phi_\mu^a) = \{\lambda + (1-\lambda)\bar\theta\theta\delta(t-T)\}\mathcal{L}(\Phi_\mu^a)$$

where integral (2.12) has the former boundary conditions (2.10), (2.11). We want to prove the independence of the actions family $S^{T,\lambda}$ upon λ in the

limit $T \to \infty$. For that, it is enough to prove that the action derivative over λ is zero. Differentiating (2.12) over λ we obtain:

$$\frac{\partial}{\partial\lambda}\exp\{-S^{T,\lambda}\} = -\int_0^T dt\, d\theta\, d\bar\theta (1-\bar\theta\theta\delta(t-T)) < S^\lambda(\Phi_\mu^a) > \tag{2.13}$$

where $< S^\lambda(\Phi_\mu^a) >$ is the density average integrated over d^4x. The exponent in the r.h.s. of Eq.(2.12) as well as boundary conditions (2.10), (2.11) except $\Psi_\mu^a(0) = \Psi_\mu^a(T) = 0$ are invariant under the following supertransformations:

$$t \to t - \bar\theta\varepsilon$$
$$\theta \to \theta + \varepsilon \tag{2.14}$$

As to variation of $\Psi_\mu^a(t)$ under transformations (2.14), it is proportional to expression $C_\mu^a - \partial_t A_\mu^a$, and hence in the limit $T \to \infty$ the boundary condition $\Psi_\mu^a(0) = \Psi_\mu^a(T) = 0$ is also invariant owing to (2.11). Due to this supersymmetry, $< S^\lambda(\Phi_\mu^a) >$ in Eq.(2.13) in the limit $T \to \infty$ depends on t, θ, $\bar\theta$ parameters via superinvariant $\tau = t + \theta\bar\theta$. This latter and Eq.(2.11) taken into account, Eq.(2.13) takes the form:

$$\frac{\partial}{\partial\lambda}\exp\{-S^{T,\lambda}\} \sim -< S^\lambda(\Phi_\mu^a) > \Big|_{t=\theta=\bar\theta=0} = A_\mu^a(C_\mu^a - \partial_t A_\mu^a)\Big|_{t=0} \xrightarrow{T\to\infty} 0$$

Thus, $S^{T,\lambda}$ in the limit $T \to \infty$ is λ -independent. Comparing the values of $S^{T,\lambda}$ at $\lambda = 0$ and $\lambda = 1$ with respect to Eq.(2.11) we come to Eq.(2.1) required.

Valid is also a more general statement: Eq.(2.1) remains true if in functional integral (2.2) the first of the two boundary conditions (2.3) is replaced by pure gauge, the second one being unchanged:

$$\mathcal{A}_\mu^a(x,0)=\frac{1}{g}\Omega^{-1}(x)\partial_\mu\Omega(x)$$

$$\mathcal{A}_\mu^a(x,T)=\mathcal{A}_\mu^a(x) \tag{2.15}$$

This can be proved by analogy with the case $\mathcal{A}_\mu^a(x,0)=0$, but more easily can be derived from the statement (2.1), (2.2), (2.3) already proved. Indeed, if boundary conditions (2.15) are imposed on integral (2.2), then after the relevant gauge transformation in the r.h.s. of Eq.(2.2) we can proceed to boundary conditions:

$$\mathcal{A}_\mu^a(x,0)=0$$

$$\mathcal{A}_\mu^a(x,T)=[\mathcal{A}^\Omega]_\mu^a$$

where superscript Ω denotes transformed field, the integrand in (2.2) being unchanged for it is invariant under t-independent gauge transformations. Then, according to Eq.(2.1), in the limit $T\to\infty$ we'll obtain an expression $\exp\{-S_{YM}(\mathcal{A}^\Omega)\}$ coinciding with $\exp\{-S_{YM}(\mathcal{A})\}$. The statement has been proved. The last statement (i.e. Eqs.(2.1), (2.2), (2.15)) implies that $\exp\{-S^T\}$ at $T\to\infty$ forgets the initial boundary condition (the first condition of Eq.(2.15)). Such effect takes place if there is no degeneracy in the given direction, and this in our case takes place owing to cutoff of integral (2.2) over t at boundary point T. This cutoff actually denotes infrared regularization along t ; therefore there is no degeneracy in this direction.

Explain also why boundary condition imposed at T brings to infrared regularization. In calculating integral (2.2) we must expand field $\mathcal{A}_\mu^a(x,t)$ near the classical solution with boundary conditions (2.3) or (2.15). Then

in the term quadratic over quantum fluctuations there will arise a mass-type contribution proportional to the second derivative of action on classical solution. The mean value of this contribution (roughly, mass) is determined by the boundary field $\mathcal{A}_\mu^a(x)$, and therefore, if there is no integration over $\mathcal{A}_\mu^a(x)$, the infrared divergences along t are absent.

Now we'll use (2.1) for regularization as follows. We regularize S^T by inserting the quadratic over fields term with "highest derivatives" over t into the exponent of the r.h.s. of Eq.(2.8). Clear that this term will improve the propagators behaviour, and on the other hand it is invariant under t-independent gauge transformations. With account of these additional terms in the limit $T\to\infty$ we shall obtain the following expression instead of the l.h.s. of Eq.(2.1):

$$\exp\{-S_{YM}(\mathcal{A}_\mu^a(x))-S^{reg}(\mathcal{A}_\mu^a(x))\} \tag{2.16}$$

where S^{reg} is invariant under 4-dimensional gauge transformations. Next we must add to the weight of (2.16) the Faddeev-Popov determinant and gauge-fixing condition and then work in a standard way.

Turn now to discussion of regularization at greater length.

3. Invariant Regularization of Yang-Mills Theory.

Regularization is achieved, first, by replacement of S and $\mathcal{L}$ in (2.8), (2.9) by regularized S^Λ and $\mathcal{L}^\Lambda$, this bringing to regularized $S^{T,\Lambda}$, and second, by replacement of S^T by $S^{T,\Lambda}$ in (2.1). Finally we obtain:

$$\exp\{-S_{YM}^\Lambda(\mathcal{A}_\mu^a(x))\}=\lim_{T\to\infty}\exp\{-S^{T,\Lambda}(\mathcal{A}_\mu^a(x))\} \tag{3.1}$$

where

$$\exp\left\{-S^{T,\Lambda}(A_\mu^a(x))\right\} = \int \exp\left\{-S^\Lambda(\Phi_\mu^a)\right\}\mathcal{D}\Phi_\mu^a \tag{3.2}$$

$$S^\Lambda(\Phi_\mu^a) = \int_0^T\int_0^T \mathcal{L}^\Lambda(\Phi_\mu^a)\,dt\,dt'\,d\theta\,d\bar\theta\,d^4x \tag{3.3}$$

$$\mathcal{L}^\Lambda(\Phi_\mu^a) = \mathcal{L}^{YM}(\Phi_\mu^a)\delta(t-t') - \Phi_\mu^a(t,x,\theta)\Big[\delta_\Lambda(t-t')\frac{\partial^2}{\partial\theta\partial\bar\theta} + $$

$$+\delta(t-t')\Big(\frac{\partial}{\partial\theta}\partial_{t'} - \partial_{t'}\Big)\Big]\Phi_\mu^a(t',x,\theta,\bar\theta) \tag{3.4}$$

In (3.2) the boundary conditions (2.10) are imposed, and in (3.4) δ^Λ is a regularized delta function:

$$\lim_{\Lambda\to\infty}\delta^\Lambda(t-t') = \delta(t-t')$$

Let us now prove that the replacement of $\exp\{-S\}$ by $\exp\{-S^\Lambda\}$ actually realizes invariant regularization of Yang-Mills theory.

Start with invariance. Substitute in (3.1), (3.2), (3.3) $[A^\Omega(x)]_\mu^a$ instead of $A_\mu^a(x)$. Make gauge replacement of integration variables $A_\mu^a(x,t) \to [A^\Omega(x)]_\mu^a$. Then we come back to formulae (3.1), (3.2), (3.3) with the boundary value of $A_\mu^a(x)$ instead of $[A^\Omega(x)]_\mu^a$, but the boundary condition (2.3) will transfer to (2.15). At $T\to\infty$ the initial boundary condition is forgotten and we arrive at equality:

$\exp\{-S^\Lambda(A^\Omega)\} = \exp\{-S^\Lambda(A)\}$, i.e. the action S^Λ is gauge-invariant.

Now turn to regularization. For that, calculate S^Λ at a free ($g = 0$) level. Formula (3.2) after integration over C_μ^a fields can be presented as follows:

$$\exp\left\{-S^{T,\Lambda}\right\} = \int\exp\left\{-\frac{1}{4}\int(\partial_t A_\mu^a(t) + p^2P_{\mu\nu}A_\nu^a(t)) \times \right.$$

$$\left.\times\,\delta_\Lambda^{-1}(t-t')(\partial_{t'}A_\mu^a(t') + p^2P_{\mu\lambda}A_\lambda^a(t'))\,dt\,dt'\right. \tag{3.5}$$

where Fourier transformation over x is done and four-dimensional integration over p momentum is omitted. As δ_Λ^{-1} we choose, e.g. the following expression

$$\delta_\Lambda^{-1}(t-t') = \left(1 + \left(-\frac{\partial_t^2}{\Lambda^2}\right)^n\right)\delta(t-t') \tag{3.6}$$

where Λ is dimension 2 parameter.

Substituting the solution (2.6) (which is also a solution of classical equation in the (3.5) case) into Eq.(3.5) we find the action $S^{T,\Lambda}$ at $g = 0$:

$$S^{T,\Lambda}(A_\mu^a(p)) = \frac{1}{2}A_\mu^a(-p)p^2\left(1 + \left(-\frac{p^4}{\Lambda^2}\right)^n\right)\times$$

$$\times\left(\frac{1+\mathrm{cth}\,Tp^2}{2}\right)P_{\mu\nu}A_\nu^a(p) + \frac{1}{T}A_\mu^a(-p)L_{\mu\nu}A_\nu^a(p) \tag{3.7}$$

Hence we come to the following expression for S_{YM}^Λ:

$$S_{YM}^\Lambda(A_\mu^a(p)) = \frac{1}{2}A_\mu^a(-p)p^2\left(1+\left(-\frac{p^4}{\Lambda^2}\right)^n\right)P_{\mu\nu}A_\nu^a(p) \tag{3.8}$$

i.e. at a free level our regularization coincides with one with highest derivatives over four-dimensional x. However at $g \neq 0$ it may not reduce to regularization with highest covariant derivatives of Slavnov [12] (the exact answer for $\hat{S}_{YM}$ at $g \neq 0$ was not obtained by us because of complicacy in construction of perturbation for unordinary functional integral (3.2)).

Now we derive the diagram divergence index and prove convergence of regularized diagrams above one loop. We calculate regularized diagrams by means of the following effective action:

$$S_{eff} = \hat{S}_{YM}(A_\mu^a) + S_{GF} + S_{FP}$$

where S_{FP}, S_{GF} are respectively Faddeev-Popov's and gauge-fixing actions. One can easily understand that owing to gauge-fixing terms the Green functions can be calculated, first, by means of the action $S_{eff}^T = S^{\Lambda,T} + S_{GF} + S_{FP}$, and only then one can pass to the limit $T \to \infty$. This is due to the fact that in the presence of gauge-fixing finite longitudinal components in the action S_{eff}^T the longitudinal terms of $\frac{1}{T} A_\mu^a L_{\mu\nu} A_\nu^a$ type (Eq.(3.7)) do not yield singular at $T \to \infty$ contributions to the Green functions. Therefore we shall calculate divergence index of diagrams determined by action S_{eff}^T, i.e. averages of the following type:

$$\int A(x_1,T) \dots A(x_n,T) \exp\Big\{-S_{GF}(A(x,T)) - S_{FP}(A(x,T)) - \int_0^T\int_0^T dt\,dt' \Big(\partial_t A + \frac{\delta S_{YM}}{\delta A}\Big)\Big(1 + \Big(-\frac{\partial_t^2}{\Lambda^2}\Big)^n\Big)\Big(\partial_t A + \frac{\delta S_{YM}}{\delta A}\Big) \tag{3.9}$$

where isotopical, vector indices of fields A_μ^a as well as four-dimensional integration over x in the action are omitted. In (3.9) we omitted also $\det(\partial_t + \delta^2 S_{YM}/\delta A^2)$. One can readily understand that in calculating the averages only over fields A_μ^a this determinant is inessential, though it plays an important part in supersymmetrization of five-dimensional theory (in the supersymmetric version there arise new fields Ψ_μ^a and C_μ^a which have nontrivial averages). The statement that this determinant is inessential in (3.9) can be grounded as follows. One can readily calculate the mentioned determinant:

$$\det\Big(\partial_t + \frac{\delta^2 S_{YM}}{\delta A^2}\Big) = \det\Big(1 + \theta(t-t')\frac{\delta^2 S_{YM}}{\delta A^2}\Big) = \exp\Big\{\frac{1}{2} S_P \frac{\delta^2 S_{YM}}{\delta A^2}\Big\}$$

The last expression in the chain of equalities contains $\delta(0)$ (delta function of four-dimensional null). Now if we consider that from the very beginning prior to introduction of stochastic regulator the four-dimensional integrals were regularized so that $\delta(0) = 0$ (e.g. dimensional regularization), then the determinant will turn to unity and may be omitted in formulae (2.1), (2.2). Further we can already introduce the stochastic regulator (i.e. highest derivatives over t) and remove the initial one. Then we'll arrive at expression (3.9).

Now calculate divergence index of theory described by Eq.(3.9). In calculating the divergence index, it is convenient to make Fourier transformation over x_μ, and as to the additional dimension, to work here in coordinate representation. One can readily make sure that then propagator $\langle A(p,t)\, A(p',t')\rangle$ behaves as $\Big[p^2\Big(1 + \Big(-\frac{p^4}{\Lambda^2}\Big)^n\Big)\Big]^{-1}$. The interaction contained in the integrand exponent can schematically be presented in the form:

$$(p^4)^n (g p^3 A^3 + g^2 p^2 A^4 + g^3 p A^5 + g^4 A^6)$$

where the momenta powers denote derivatives over x and t (t has dimensions of 2).

The conservation laws take place only over four-dimensional momenta, and the diagrams vertices contain integration over t . Now one can readily calculate the divergence index of the loop diagrams. It turns out equal to $4-\ell_{ext}-4n(M-1)$, where M is the number of loops, i.e. diagrams with $M>1$ (two- and higher-loop ones) are regularized.

Thus, the action S^{Λ} (Eqs.(3.1)-(3.4)) realizes gauge-invariant regularization of two- and higher-loop Yang-Mills (containing no ghosts) diagrams.

4. Supersymmetric and Gauge-Invariant Regularization of the N=2 Supersymmetric Yang-Mills Theory.

The formulae of the previous section can be applied to regularization of extended N=2 supersymmetric Yang-Mills theory defined in Ogievetsky's harmonic superspace, introduced in Refs. [13,14]. What is essential to us in this formulation is that the action of N=2 theory depends on one independent gauge superfield $V^{++}(\zeta,u)$, being invariant under linear gauge transformation:

$$V^{++}\to \exp\{i\lambda\}(V^{++}-iD^{++})\exp\{-i\lambda\} \qquad (4.1)$$

where D^{++} is harmonic derivative.

Besides, the action in linearized approximation (i.e. at g = 0) has a very simple form [13]:

$$S^{N=2}_{\ell in}=\frac{1}{2}\int d\zeta_1^{-4}du_1 d\zeta_2^{-4}du_2 V^{++}_{(1)}(-\Box_1)\Pi^{(2,2)}_{(1/2)}V^{++}_{(2)} \qquad (4.2)$$

where $\Pi^{(2,2)}_{(1/2)}$ is projector.

And these two properties are enough to apply our regularization to the N=2 supersymmetric Yang-Mills theory.

For that recall that in deriving the general formula (3.1) and in proving the regularization gauge invariance we nowhere were interested in concrete form of the Lagrangian. We used gauge invariance of Lagrangian (3.4) only, and essential was a demand for the gauge transformation to be linear, since the second term in (3.4) is invariant only under linear transformations of field ϕ^a_μ. Hence it is clear that regularization can be applied only within Ogievetsky's formulation, since in the latter the independent field V^{++} transforms by linear law (4.9). In addition, $S^{N=2}_{\ell in}$ has the same form as for the N=0 case to the accuracy of replacement of $\Pi^{(2,2)}_{(1/2)}$ by $P_{\mu\nu}$, i.e. all our calculations for the free case are valid.

Thus, all formulae in the second and third sections can be applied to the N=2 case if fields ϕ^a_μ in them are replaced by

$$\phi^{++}(\zeta,u,t,\theta)=V^{++}(\zeta,u,t)+\bar\theta\psi^{++}(\zeta,u,t)+\bar\psi^{++}(\zeta,u,t)+\bar\theta\theta C^{++}(\zeta,u,t) \qquad (4,3)$$

and projector $P_{\mu\nu}$ by $\Pi^{(2,2)}_{(1/2)}$, projector $L_{\mu\nu}$ by $\delta_A^{(2,2)}(1/2)$ - - $\Pi^{(2,2)}_{(1/2)}$, where function $\delta_A^{(2,2)}(1/2)$ is introduced in Ref.[14].

Further, the action S_{YM} should be replaced by the action $S^{N=2}_{YM}$ = = $\int d^4x d^4\bar\theta\, \bar W^2$, where $\bar W$ depends on V^{++} in the following way [13,14]:

$$\bar W=-\frac{i}{4}\exp\{-i\vartheta\}\left[(D^+)^2(\exp\{i\vartheta\}D^{--}\exp\{-i\vartheta\})\right]\exp\{i\vartheta\} \qquad (4.4)$$

Here field ϑ is related to the independent V^{++} field via the following equation:

$$\exp\{i\vartheta\}(D^{++}\exp\{-i\vartheta\})=iV^{++} \qquad (4.5)$$

The action $S^{N=2}_{YM}$ will have linearized limit (4.2) which in the regularized form (after transition to the limit $T \to \infty$) can be written as:

$$S^{\Lambda}_{lin} = \frac{1}{2} \int d\zeta_1^{-4} du_1 d\zeta_2^{-4} du_2 V^{++}(1) p^2 \left(1 + \left(-\frac{p^4}{\Lambda^2}\right)^n\right) \Pi^{(2,2)}_{(1/2)} V^{++}(2) \quad (4.6)$$

Regularization of complete action is achieved by modification of integral formula (3.2).

5. Conclusion.

Note in conclusion that the problem of ghost regularization remains open. While the problem of invariant regularization of matter fields can be solved via evident generalization of formulae given above: one must enlarge the matter fields in five dimensions leaving the interacting gauge field in four-dimensional space, then impose analogous to (2.3) boundary conditions on the matter fields - and formulae (3.1)-(3.4), after replacement of by appropriate supersymmetric Lagrangian of matter fields, will work (i.e. regularized action will be gauge-invariant).

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