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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

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ON THE INSTABILITY OF MOTION IN THE GALAXY

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ՇԱՐԺՄԱՆ ԱՆԿԱՅՈՒՆՈՒԹՅՈՒՆԸ ԳԱԼԱԿՏԻԿԱՅՈՒՄ

Հիդրոդինամիկական մոտավորությամբ հետազոտված է շարժման բը-ը Գալակտիկայի սկավառակում: Օգտվելով Ռիմանի չափադիր/մետրի-/ տարածությունների դիֆֆեոմորֆիզմների խմբի որոշակի հատկու-ներից՝ ցույց է տրված շարժման խիստ անկայուն բնույթը: Որոշ ցնող ենթադրությունների դեպքում գնահատված է անկայուն բնու-ժամանակը մեկ պտույտի տևողության $\frac{2}{\pi}$ մասը կամ մոտավորապես [illegible]ն տարի:

Երևանի ֆիզիկայի ինստիտուտ
Երևան 1986

[illegible]ный научно-исследовательский институт информации [illegible] экономических исследований по атомной науке [illegible] (ЦНИИатоминформ) 1985г.

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V.G. GURZADYAN, A.A. KOCHARYAN

ON THE INSTABILITY OF MOTION IN THE GALAXY

In the hydrodynamical approximation the character of motion in the Galaxy is discussed. Using the properties of the group of diffeomorphisms of Riemann metrical spaces, the existence of exponential instability is shown. In the case of several simplifying assumptions the characteristic time scale of instability is estimated being of the order of $\frac{2}{\pi}$ part of rotational period, i.e. 130 million yrs.

Yerevan Physics Institute
Yerevan 1986

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В.Г.ГУРЗАДЯН, А.А.КОЧАРЯН

К НЕУСТОЙЧИВОСТИ ДВИЖЕНИЯ В ГАЛАКТИКЕ

В гидродинамическом приближении исследуется характер движения в диске Галактики. Показано наличие экспоненциальной неустойчивости движения и при некоторых упрощающих предположениях оценено характерное время неустойчивости, составляющее $\frac{2}{\pi}$ долю от времени одного оборота, т.е. порядка 130 млн.лет.

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The character of stellar motion in the Galactic disk at least at Solar neighbourhood is close to stationarity. Despite the necessity of obtaining more specific data on the regularity of stellar peculiar velocity distribution, their spatial concentration, etc., this fact generally is not disputed and points out inevitable existence of mixing mechanism. It is well known that, say, the effect of binary stellar encounters cannot lead to the observed pattern, so far as predicts time scales much exceeding the age of Galaxy.

This unresolved up to now problem is discussed in this paper using the methods of theory of dynamical systems; the principal possibility of "rapid" mixing in the Galaxy at present epoch is shown and even at several simplifying assumptions the corresponding characteristic time scale is estimated. We have applied the methods used here earlier while investigating distinct aspects of structure at evolution of stellar systems [1-6] . It has been shown [1,2] that spherical systems of N gravitating bodies possess strong instability peculiar to K-systems, while disk ones are more regular. The study of disk systems being dynamical systems with non-negative curvature is performed in [3] .

Below, for investigating the stability of motion we shall proceed from Arnold's theorems [7,8] concerning the group of diffeomorphisms S Diff D of Riemannian manifold D with one-side invariant metric. The main simplification is the adopted hydrodynamical approximation, namely the identification of the motion in Galactic disk with a flow of fluid on a two-dimensional torus

$$T^2 = \left\{ (x, y) \bmod 2\pi \right\} .$$

Fridman, Morozov and co-authors (see [9] and references therein) describe by hydrodynamical theory many features of spiral structure of galaxies.

Consider the group of diffeomorphism of torus $T^2 - S\mathrm{Diff}\,T^2$ preserving the volume element (for notations and details see Arnold's papers cited above). Lie algebra corresponding to a group $S\mathrm{Diff}\,T^2$ contains all vector fields with zero divergence onto T^2. Scalar product of two elements of Lie algebra is defined as follows

$$< u, v > = \int_{T^2} (v^1 u^1 + v^2 u^2)\, dx\, dy . \qquad (1)$$

The motion of homogeneous ideal fluid on T^2 is described by the curve $t \to g_t$ on an infinite-dimensional group $S\mathrm{Diff}\,T^2$, where the diffeomorphism g_t is a map transforming every particle of the fluid from an initial point to the point corresponding to the time t.

Kinetic energy of the moving fluid induces right-invariant Riemannian metric on $S\mathrm{Diff}\,T^2$, i.e. when the derivative of left shift does not change the vector's value. From the principle of least action it follows that the motion of ideal fluid occurs by the geodesics of metric (1) on the group $S\mathrm{Diff}\,T^2$. This principle is analogous here to that of Maupertuis, which also transfers the motion in phase space onto geodesical flow in a suitable Riemann space.

Consider now Lie algebra $S_0\mathrm{Diff}\,T^2$ with current function of a velocity field ξ defined in the following way:

$$v = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathrm{grad}\, \xi . \qquad (2)$$

If the flow is stationary, the geodesics form one-parametric subgroup of the group defined above. Then the deviation of geodesics is determined by the sign of two-dimensional curvature in Lie algebra and hence by velocity field.

Approximating the "flow" in the Galaxy with a velocity field (cf. e.g. [10])

$$v = \left(0, \sin \frac{x}{2} \right) \qquad (3)$$

and using Arnold's expression for the curvature of the group $S_0\mathrm{Diff}\,T^2$ in a section defined by current functions $\xi = \cos kx$, $\eta = \cos lx$

$$K = - \frac{k^2 + l^2}{8S} \sin^2 \alpha \sin^2 \beta ; \quad \alpha = (\widehat{k, l}), \beta = (\widehat{k+l, k-l})$$

one can find the "mean curvature"

$$K_0 = - \frac{1}{8S} < 0 , \qquad (4)$$

where $S = 4\pi^2$ is the area of torus.

Evidently, in general, there exist two-dimensional directions at which the curvature is positive; however it is important that in this case the mean curvature K_0 is strongly negative.

Then the characteristic path of mixing l, i.e. the mean path, at which the errors in initial data grow e times will equal

$$l = (- K_0)^{-1/2} = 2\sqrt{2S} . \qquad (5)$$

The motion velocity in the group is

$$V = \langle v, v \rangle^{1/2} = \left[\int_{T^2} v \cdot v \, dx \, dy \right]^{1/2} = \left(\frac{\xi}{2} \right)^{1/2}, \quad (6)$$

From here one can estimate the time during which the most rapid particles cover the characteristic path (5):

$$t = 4 \quad (7)$$

i.e. $\frac{2}{\pi}$ part of the rotational period.

Thus the considered motion is exponentially unstable in the sense of the loss of information on initial data of the particles within the flow, and is stable with respect to the velocity field. This fact particularly reflects the result of the paper [1] concerning the sign of the two-dimensional curvature of disk systems consisting of N gravitating particles.

At rotational period of Galaxy $2 \cdot 10^8$ years the characteristic time scale of mixing yields $\tau \sim$ 130 million years. For example, during the time of existence of Solar system ($5 \cdot 10^9$ years) the initial deviation ε_0 must grow up to

$$\varepsilon_t \sim e^{\frac{25}{2}\pi} \varepsilon_0 \sim 10^{17} \varepsilon_0 .$$

Paraphrasing Arnold one can say that "the Galactic weather is unpredictable for times exceeding τ ".

It is worth mentioning that the estimation of instability time scale is rough, so far as rather simplified model was considered allowing analytical estimation of the curvature. The necessity of a more detailed analysis is evident. At the same time, the conclusion on relatively rapid mixing can be strict enough to throw light on the problems of structure and evolution of real galaxies.

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