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SINAI BILLIARDS AS A PSEUDORANDOM-NUMBER GENERATOR

ЦНИИатоминформ

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БИЛЬЯРД СИНАЯ КАК ДАТЧИК ПСЕВДОСЛУЧАЙНЫХ ЧИСЕЛ

Приведено численное моделирование на ЭВМ динамической системы, представляющей собой двух-и трехмерный бильярд Синая. Исследованы статистические свойства таких систем.

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1. Introduction

In computer simulation of physical systems with a Monte-Carlo technique, various pseudorandom-number generators are used [1]. The dynamical systems with strong statistical properties were suggested to use [2,3] for generation of pseudorandom numbers. To such systems there refer the Kolmogorov-Anosov-Sinai K-and U-systems [4]. In particular, automorphism of compact commutative groups was used in [2,3]. Statistical analysis has shown that thus obtained pseudorandom sequences possess the best statistical properties and weak correlations [3].

In this work we suggest to study two- and three-dimensional systems of the Sinai-billiard type [4].

2. Sinai Billiard

Dynamical systems with elastic reflections from the boundary, or the so-called billiards [5] occupy a special place among dynamical systems with strong statistical properties. We shall be interested in a system whose boundary is a unit square or unit cube with a circle or sphere of radius R in the centre [5]. Systems in a space of higher dimensions can, in principle, be considered as well. In Sinai's works a two-dimensional case was considered [4,5] . An analytical study of three- and higher-dimensional billiards offers great difficulties, which, as known, are not overcome yet. Therefore, a computer simulation of such systems is of certain interest.

Consider, first of all, some general properties of motion of scattering billiards. If point trajectory does not s-atter on the circle or sphere, but scatters only on the square or cube walls, then such motion, as is well known, is quasi-periodic, being of no interest to us. Hence it is natural to consider only those reflections from the square or cube boundary which occurred after scattering on the sphere or circle. In the two-dimensional case the point's position after reflection is characterized by a single number - its coordinate being in the interval (0.1), while in the three-dimensional case the point is reflected on one of the six plaquetts and has two coordinates. If we consider billiards in spaces of higher dimension, d, then after being reflected on a sphere of (d-1) dimension, the point is elastically reflected from one of the plaquetts of a d-dimensional cube and is characterized by d-1 numbers lying in the interval (0.1).

These numbers can be arranged in the infinite sequence which is the subject of our statistical analysis. Note that the above-described procedure of obtaining a pseudorandom sequence is none other than the Poincare cross section which, as is well known, reflects properties of the dynamical system[4].

3. A Statistical Analysis of the Poincare Cross Sections

Thus, the billiard trajectory is described by a sequence

$$P_o , P_1 , P_2 , \dots , P_N , \dots \tag{1}$$

where P_i are coordinates of the reflection points on the d-dimensional cube. P_i are d-1 numbers lying in the interval (0.1).

$$P_i = (P_i^{(1)}, \ \dots \ P_i^{(d)}) . \tag{2}$$

The simulation is based on the simple scheme. Initially the trajectory is given by the point P_o on one of the plaquetts and by a unit vector n_o. The scattering sphere is given by the equation

$$(x_1 - \frac{1}{2})^2 + \dots + (x_d - \frac{1}{2})^2 = R^2 . \tag{3}$$

If the trajectory with initial data P_o , n_o does not scatter on the sphere, one can readily find a new reflection point $\tilde{P}_o$, $\tilde{n}_o$ on the cube plaquett which, as mentioned before, is not included in our sequence (1). We proceed to calculate the trajectory until there occurs scattering on the sphere. Now after scattering the coordinates of point P_1 with the vector (n_1) are inserted into sequence (1) and so on. The statistical investigations of these sequences are the same as in [3] .

The given construction is based on the fact that the stochastic instability occurs namely owing to scattering on the sphere [6,4,5] . In the two-dimensional case the essential characteristic of the motion is the correlation decoupling time [6,4,2] :

$$\tau \sim \frac{1}{\ln \rho / R} , \tag{4}$$

where ρ is the free path length. The nontrivial moment is that ρ , naturally, depends on R and, generally speaking, increases with decreasing R . Or, in other words, the scattering cross section falls off with decreasing R , and hence the free path length increases. With increasing the ratio $\rho/R > 1$ the system becomes more unstable.

It is not evident how the system will behave with increasing its dimensions. Simple physical reasons prompt that at the same radii of spheres the systems in spaces with large number of dimensions are more unstable.

The statistical analysis has shown that the Sinai billiards give less uniform pseudorandom sequences than in the case of automorphisms of compact commutative groups [2,3] (see Tables 1 and 2). This analysis has shown also that indeed, the three-dimensional systems are more unstable than the two-dimensional ones (Tables 1 and 2)*, for the radii from 0.3 and higher. While for the radii less than 0.3 the situation is vice versa. Apparently, this reflects the nontrivial dependence of ρ on R, which is contained in formula (4) if we believe that the latter is true also for the three-dimensional case.

Note to add. Recently, Yu. Sinai informed one of us (G.K.S.) that now he has some new results for higher dimensions.

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* The procedure of statistical studies as well as the Tables' structure entirely correspond to [3].

Table 1

	0.1	0.2	0.3	0.4	0.45
χ^2_9	10.9	298	20	22.1	46.8
χ^2_{99}	328	374	509	1028	1980
χ^2_{999}	4444	5892	9985	30036	68482
χ^2_{9999}	59783	79638	180579	704874	2067657

Table 2

	0.1	0.2	0.3	0.4	0.45
χ^2_9	19	30	21	28.5	26.3
χ^2_{99}	435	439	487	408	370
χ^2_{999}	6043	6343	6735	8810	12945
χ^2_{9999}	80391	82344	95840	167487	373932

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