Preprint ЕФИ-923(74)-86

ԵՐԵՎԱՆԻ ՖԻԶԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ
ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

R.G. BADALYAN

ON QUARK-GLUON SYSTEMS FRAGMENTATION FUNCTIONS

ЦНИИатоминформ

ЕРЕВАН-1986

Նախնատիպ ЕФИ-923(74)-86

ՔՎԱՐԿ-ԳԼՅՈՒՈՆԱՅԻՆ ՀԱՄԱԿԱՐԳԵՐԻ ՀԱՏՎԱԾԱՎՈՐՄԱՆ ՖՈՒՆԿՑԻԱՆԵՐԻ ՄԱՍԻՆ

Ռ.Գ.ԲԱԴԱԼՅԱՆ

Առաջարկված է վիճակագրական մոտեցում քվարկ-գլյուոնային S համակարգից h հադրոնի ,,փափուկ,, հատվածավորման վերաբերյալ: Ստացված են արտահայտություններ $\mathcal{D}_S^h(x)$ հատվածավորման $S \to h$ ֆունկցիաների համար: Մասնավորապես, որոշված են քվարկը հադրոնի հատվածավորման ֆունկցիաները: Բերված է առաջարկված վիճակագրական մոտեցման շրջանակում ստացված՝ խիստ ոչ առաձգական նեյտրինո/հականեյտրինո/-նուկլոն փոխազդեցություններում, π^+/π^- (π^-/π^+) մեզոնների ծնման կը-տրվածքների համեմատությունը փորձարարական տվյալների հետ:

Երևանի ֆիզիկայի ինստիտուտ

Երևան 1986

Preprint ЕФИ-923(74)-86

R.G. BADALYAN

ON QUARK-GLUON SYSTEMS FRAGMENTATION FUNCTIONS

A statistical approach to the question of "soft" fragmentation of quark-gluonic system S to hadron h is considered. Expressions for the fragmentation function $D_s^h(x)$ of the process $S \to h$ are obtained. The quark-to-hadron fragmentation functions are in particular defined. A comparison of the prediction for the considered statistical approach with experimental data is carried out with respect to $\pi^+/\pi^-(\pi^-/\pi^+)$ -mesons yields in deep-inelastic scattering of neutrino (antineutrino) on the nucleon.

Yerevan Physics Institute

Yerevan 1986

Препринт ЕФИ-923(74)-86

Р.Г.БАДАЛЯН

О ФУНКЦИЯХ ФРАГМЕНТАЦИИ КВАРК-ГЛЮОННЫХ СИСТЕМ

Рассматривается статистический подход к вопросу "мягкой" фрагментации кварк-глюонной системы S в адрон h. Получены выражения для функции фрагментации $D_S^h(x)$ процесса $S \to h$. В частности, определены функции фрагментации кварка в адрон. Приводится сравнение предсказания рассматриваемого статистического подхода с экспериментальными данными по отношению выходов π^+/π^- (π^-/π^+) - мезонов в глубоко-неупругом рассеянии нейтрино - (антинейтрино) на нуклоне.

Ереванский физический институт
Ереван 1986

1. Introduction

In order to define inclusive spectra of hadrons in deep-inelastic lepton-hadron collisions, spectra of hadrons with high transverse momenta P_T in hadron-hadron interactions as well as hadron distribution in e^+e^--annihilation, usually there are introduced fragmentation functions $D_q^h(x)$ of quark q to hadron h [1,2], where x is a ratio of longitudinal (relative to momentum direction of initial quark q) momentum of h hadron to momentum of q quark. Various phenomenological fragmentation models [3-5] also operate by quark (diquark)-to-hadron fragmentation functions in order to describe inclusive spectra of secondary hadrons with low P_T in hadron-hadron collisions.

In the mentioned processes with high momentum transfers at the process initial stage, there occurs a "knock-out" of the quark (antiquark) with large effective mass from the hadron and takes place radiation of hard gluons and quark-antiquark pairs. This stage can be described quantitatively within the theoretico-perturbative quantum chromodynamics (QCD). According to the modern representations, the knocked-out quark is assumed to lose its momentum (high virtuality) via the emission of gluons which in turn may transform into quark-antiquark pairs. The initial stage of such transformation of quark into quark-gluonic system can be described in the framework of evolutionary equations of Altarelli-Parisi [2, 6-9] , since at the initial stage the quark-gluon interaction constant is still small due to the quark high virtuality. With decreasing quark and gluon virtuality the quark-gluon interaction constant increases, hence the problem ceases to be theoretico-perturbative already. A second stage the present work is devoted to, namely the fragmentation of the produced compound quark-gluonic system to hadrons, is a "soft" process. The determination of the function of soft fragmentation of such a system to hadrons is essentially not a theoretico-perturbative problem and its solution within QCD is somewhat complicated due to quark and gluon confinement effects. However, due to strong coupling between quarks and gluons, one may expect that a statistical equilibrium takes place in quark-gluonic system prior to its fragmentation to hadrons. In the light of the above-said, the application of the statistical consideration to the question of soft fragmentation of quark-gluonic system to hadron seems reasonable, since the mentioned problem is essentially multi-particle.

Within the parton approach, the hadron (proton, π or K-meson) with high momentum is presented as a compound quark-gluonic system which is characterized by its valent composition and distribution functions of valent quarks and sea partons in Feynman variable X . Information about distribution functions of partons in hadrons is given by processes of deep-inelastic scattering of leptons on nucleons, and also by processes of Drell-Yan lepton pair production in hadron-hadron interactions [10] . Apparently, it seems impossible to derive (via studying hard processes with participation of leptons) distribution functions of valent (leading) quarks and sea partons in the quark-gluonic system S_q or $S_{q_1 q_2}$ to which quark q or diquark $q_1 q_2$ (the baryon remainder produced after knocking-out one valent quark from the latter) respectively transforms. However one may expect that the fragmentation function of quark q (diquark $q_1 q_2$) to hadron h is determined via distribution functions of multiparton subsystems with valent composition of hadron h in quark-gluonic system S_q ($S_{q_1 q_2}$) , just as it is the case for fragmentation of initial hadron H to the final hadron h with low P_T in hadron-hadron interactions [11-14] . In the latter case the distribution function (inclusive spectrum) of final hadron in the fragmentation region of initial $D^h_H(x) = 1/\sigma_{in}\, d\sigma/dx\,(H \to h)$ is an analog of fragmentation function of quark to hadron $D^h_q(x)$ (diquark to hadron $D^h_{q_1 q_2}(x)$). It seems possible that quark-gluonic systems (proton, π- and K -mesons as well as systems S_q and $S_{q_1 q_2}$), in which a statistical equilibrium is set up, fragmentize to final hadronic states similarly. The present work deals with a statistical approach to the question of soft hadronization of equilibrium quark-gluonic systems, since this problem, as mentioned, is essentially multi-particle.

In Sect.2, a general scheme of construction of multiparton subsystems distribution in quark-gluonic systems is described. Multiparton distributions in π -meson are constructed. Sect.3 is devoted to definition of fragmentation function of quark-gluonic system to hadron. Quark-to-hadron fragmentation functions are obtained.

2. Distribution of Partons and Multiparton Subsystems in Quark-Gluonic Systems.

In the infinite momentum frame the quark-gluonic system S is characterized by its valent composition $q_1, \dots, q_{\bar{N}_v}$ (q is valent quark, $\bar{N}_v$ is a total number of valent quarks in system S) and distribution functions of valent quarks and sea partons in Feynman variable x . The number of sea partons in system S is not fixed and may vary from zero to infinity *). In case of (anti)baryon $\bar{N}_v = 3$, for meson and diquark $\bar{N}_v = 2$, while for quark $\bar{N}_v = 1$. To describe such systems, one may apply the Kuti-Weisskopf model [15]. The distribution density of ($\bar{N}_v + \bar{N}_s$)-parton configuration of system S ($\bar{N}_s$ is number of sea partons in the considered configuration) is determined by the expression **) [15]:

$$dS_{\bar{N}_s} = T_{\bar{N}_v \bar{N}_s}(x_1, \ldots, x_{\bar{N}_v}; x_{a1}, \ldots, x_{a\bar{N}_a}) \delta\Big(1 - \sum_{j=1}^{\bar{N}_v} x_j - \sum_a \sum_{j=1}^{\bar{N}_a} x_{aj}\Big) \times$$

$$\times \prod_{j=1}^{\bar{N}_v} \frac{dx_j}{\sqrt{x_j^2 + x_T^2}} \prod_a \frac{1}{(\bar{N}_a)!} \prod_{j=1}^{\bar{N}_a} \frac{dx_{aj}}{\sqrt{x_{aj}^2 + x_T^2}} \tag{1}$$

The quark-gluonic system S is defined as a sum of all possible ($\bar{N}_v + \bar{N}_s$)-parton configurations:

$$dS = \frac{1}{Z} \sum_{\bar{N}_s = 0}^{\infty} dS_{\bar{N}_s} \tag{2}$$

In expressions (1) and (2) X is the Feynman variable, $a = u, d, s, \ldots, \bar{u}, \bar{d}, \bar{s}, \ldots, G$ stands for the kind of sea parton, $\bar{N}_a$ is number of partons of the a sort, $\bar{N}_s = \sum_a \bar{N}_a$. $x_T = \mu / P$, where μ is the effective transverse mass of parton, P is momentum of quark-gluonic system $S\,(P \gg \mu)$. Z is statistical weight (normalization

---

*) Energy restrictions to the number of sea partons are not considered, since the mean transverse mass of parton $\mu \ll P$, where P is the system momentum.

**) If there are $\bar{N}_{qv}$ identical particles between valent quarks, then the corresponding factor in (1) should be multiplied by $1/(\bar{N}_{qv})!$

factor). Square module of matrix element of transition of system S to $(\bar{N}_V+\bar{N}_S)$-parton configuration $T_{\bar{N}_V\bar{N}_S}(x_1,\dots,x_{\bar{N}_V};\ x_{a1},\dots,x_{a\bar{N}_a})$ is assumed factorized over longitudinal momenta of valent quarks and sea partons [15-17] :

$$T_{\bar{N}_V\bar{N}_S}(x_1,\dots,x_{\bar{N}_V};\ x_{a1},\dots,x_{a\bar{N}_a})=\prod_{j=1}^{\bar{N}_V} V_{q_j}(x_j)\prod_{a}\prod_{j=1}^{\bar{N}_a} S_a(x_{aj}) \qquad (3)$$

where functions $V_q(x)$ and $S_a(x)$ are so-called noncorrelated (input or primitive) distribution functions (those without account of the longitudinal momentum conservation law) respectively for valent quarks and sea partons. They determine asymptotics of correlated distribution functions (or, simply, distribution functions) respectively for valent quarks $q(x)$ and sea partons $a_S(x)$ at $x\to 0$. General expressions for $V_q(x)$ and $S_a(x)$ are as follows [11-17] :

$$V_q(x)=x^{\beta_q}Q_q(x)\ ,\quad Q_q(0)=1,\quad \beta_q>0 \qquad (4a)$$

$$S_a(x)=g_a P_a(x)\ ,\quad P_a(0)=1\ ,\quad g_a\geqslant 0 \qquad (4b)$$

The form of $Q_q(x)$ and $P_a(x)$ functions as well as values of parameters β_q and g_a can be determined from the comparison of results obtained for distribution functions of $q(x)$ and $a_S(x)$ with experimental data.

Using multiparton distribution (1)-(4) one can determine one-parton and arbitrary (N_V+N_S)-parton distributions in quark-gluonic system S (N_V is number of valent quarks $0\leqslant N_V\leqslant\bar{N}_V$). Such distribution is obtained after $(\bar{N}_V+\bar{N}_S-N_V-N_S)$-multiple integration of $d\sigma_{\bar{N}_S}$ ar summation over $\bar{N}_S$ from N_S to infinity *) ($N_S \leq \bar{N}_S < \infty$). For example, for the distribution of the K-th valent quark ($N_v = 1$, $N_S = 0$) and sea partons ($N_v = 0$, $N_S = 1$) we have **) (denote $q_{K} = q$) [11,12] :

$$xq(x) = V_q(x)(1-x)^{-1+\gamma+\sum'^{\bar{N}_v}_{j=1}\beta_{q_j}} B_{(\bar{N}_v-1)}(1;1-x) \tag{5a}$$

$$x a_s(x) = S_a(x)(1-x)^{-1+\gamma+\sum^{\bar{N}_v}_{j=1}\beta_{q_j}} B_{\bar{N}_v}(1;1-x) \tag{5b}$$

Here $\gamma = \sum_a g_a$, $B_N(1;x) = C_N(1;x)/C_{\bar{N}_v}(1;1)$. For function $C_N(\omega;x)$, where $0 \leq N \leq \bar{N}_v$, $0 \leq \omega \leq 1$ (the physical meaning of parameter ω will be explained below), one can obtain various representations depending on the region the integration over x_j and x_{aj} is carried out in expression (1). While integrating in (1) over variables x_j and x_{aj} from zero to x , for function $C_N(\omega;x)$, we have the following expression [11,12] :

$$C_N(\omega;x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\xi e^{i\xi} \prod_{j=1}^{N} \int_0^1 z^{\beta_{q_j}} \tilde{q}_{q_j}(xz) e^{-i\xi z} \frac{dz}{z} \times$$

$$\times \exp\left(\omega \sum_a g_a \int_0^1 [P_a(xz)e^{-i\xi z} - 1] \frac{dz}{z}\right) \tag{6}$$

---

*) The account of correlation between the numbers of sea quarks and antiquarks of a certain type does not change the final expressions for one-parton and ($N_v + N_S$)-parton distribution functions [17,18] .

**) A prime at the sum (product) symbol denotes that the term with $j = K$ is not taken into account in the sum (product).

If in expression (1) integration over x_j and x_{aj} is carried out from zero to infinity, then for function $B_N(1;x)$ we find $B_N(1;x) = \mathcal{T}_N(1;x)/\mathcal{T}_{\bar{N}_v}(1;1)$, where $\mathcal{T}_N(\omega;x)$ is determined by the expression:

$$\mathcal{T}_N(\omega;x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\xi\, e^{i\xi}\, \xi^{-\omega\gamma}\prod_{j=1}^{N}\int_0^\infty z^{\beta_{q_j}}\, Q_{q_j}(x\cdot z)\, e^{-i\xi z}\frac{dz}{z}\times$$

$$\times\exp\left(\omega\sum_a g_a\int_0^\infty [P_a(xz)-1]\, e^{-i\xi z}\frac{dz}{z}\right) \tag{7}$$

Expressions analogous to (5) one can obtain from the arbitrary (N_v+N_S)-parton distribution function.

Up to here we considered only distributions of multiparton subsystems, in which number of particles is fixed (one-parton, (N_v+N_S)-parton). However the multiparton distribution (1)-(4) allows one to determine also x -distributions of arbitrary multiparton subsystem with a variable number of particles [11-14] . A particular case of such system is valon containing one valent quark and on the average the $W=1/\bar{N}_v$ fraction of sea partons of the initial quark-gluonic system S (a probability for some sea parton of system S to belong to the valon is $W=1/\bar{N}_v$).

To determine the distribution function of valon containing the K -th valent quark of system S , one should make replacement in expression (1) for $dS_{\bar{N}_S}$ [11-14] :

$$\delta\Big(1-\sum_{j=1}^{\bar{N}_v}x_j-\sum_a\sum_{j=1}^{\bar{N}_a}x_{aj}\Big)\to\prod_a C_{\bar{N}_a}^{N_a}\,W^{N_a}(1-W)^{\bar{N}_a-N_a}\times$$

$$\times\,\delta\Big(x-x_K-\sum_a\sum_{j=1}^{N_a}x_{aj}\Big)\times\delta\Big(1-x-\sum_{j=1}^{\bar{N}_v}x_j-\sum_a\sum_{j=1}^{\bar{N}_a-N_a}x_{aj}\Big) \tag{8}$$

where N_α is number of sea partons of type α belonging to the valon, ξ is the valon momentum. $C_{\bar{N}_\alpha}^{N_\alpha}$ are binomial coefficients. The expression $\sum_{j=1}^{\bar{N}_\alpha - N_\alpha} x_{\alpha j}$ is formal being a definition of addition to the sum $\sum_{j=1}^{N_\alpha} x_{\alpha j}$ up to a total sum $\sum_{j=1}^{\bar{N}_\alpha} x_{\alpha j}$. The distribution density of the K-th valon results from (1)-(4) with account of (8) after integration over longitudinal momenta of all partons and summation over $\bar{N}_S$ from N_S to infinity ($N_S \leq \bar{N}_S < \infty$) and over $N_S = \sum_\alpha N_\alpha$ from zero to infinity [11-14] :

$$Q(x) = x^{-1+W\gamma+\beta_q}(1-x)^{-1+(1-W)\gamma+\sum_{j=1}^{\bar{N}_v}{}'\beta_{qj}} B_1(W, 1-W; x, 1-x) \tag{9}$$

where are introduced notations $q_K = q$ and

$$B_{N_v}(\omega_1, \omega_2; x_1, x_2) = \left[C_{N_v}(\omega_1; x_1)\, C_{(\bar{N}_v - N_v)}(\omega_2; x_2) / C_{\bar{N}_v}(1;1)\right] \times$$

$$\times\, \delta(1-\omega_1-\omega_2)\, \delta(1-x_1-x_2) \tag{10}$$

or

$$B_{N_v}(\omega_1, \omega_2; x_1, x_2) = \left[\tilde{C}_{N_v}(\omega_1; x_1)\, \tilde{C}_{(\bar{N}_v - N_v)}(\omega_2; x_2) / \tilde{C}_{\bar{N}_v}(1;1)\right] \times$$

$$\times\, \delta(1-\omega_1-\omega_2)\, \delta(1-x_1-x_2) \tag{11}$$

Functions $C_N(\omega; x)$ and $\tilde{C}_N(\omega; x)$ are determined by expressions (6) and (7). The first argument of function $C_N(\omega; x)$ ($\tilde{C}_N(\omega; x)$) is determined by quantity W , i.e. by a probability for the sea parton of system S to belong to the extracted multiparton substate with variable number of particles, in the given case to valon. In other words, W characterizes the system S sea average fraction which belongs to valon. Note that at $W \to 0$ expression (9) for the K-th valon distribution transforms to expression for the K-th valent quark distribution (5a) [11].

Thus, following from distribution (1) for the quark-gluonic system (microcanonical distribution) one can determine one-parton and arbitrary $(N_v + N_s)$-parton distributions of quark-gluonic subsystems with a fixed number of particles (canonical distribution) as well as the distribution of compound quark-gluonic subsystems with variable number of particles in them (big canonical distribution), in particular valons.

The way of dividing the quark-gluonic system S into two parts (valon + "remainder") determined by transformation (8) directly refers to the case of dividing the system S into an arbitrary number of $m \geqslant 2$ parts:

$$\delta\Big(1-\sum_{j=1}^{\bar{N}_v} x_j - \sum_a \sum_{j=1}^{\bar{N}_a} x_{aj}\Big) \to \prod_a \frac{\bar{N}_a!}{N_{1a}! \ldots N_{ma}!} W_1^{N_{1a}} \ldots W_m^{N_{ma}} \times$$

$$\times\, \delta\Big(z_1 - \sum_{j=1}^{N_{1v}} x_j - \sum_a \sum_{j=1}^{N_{1a}} x_{aj}\Big) \ldots \delta\Big(z_m - \sum_{j=1}^{N_{mv}} x_j - \sum_a \sum_{j=1}^{N_{ma}} x_{aj}\Big) \qquad (12)$$

Expressions $\sum_{j=1}^{N_{Kv}} x_j$ and $\sum_{j=1}^{N_{Ka}} x_{aj}$ denote sums over longitudinal momenta of all valent quarks and sea partons, respectively, entering the substates under number $K (1 \leqslant K \leqslant m)$. Using distribution (1)-(4) with account of the division rule (12) for the distribution function of "complete division" of quark-gluonic system S into m multiparton subsystems

$G_{(N_v, N_s; m)}(z_1, \ldots z_m; W_1, \ldots W_m)$, we find:

$$G_{(N_v,N_s;m)}(z_1,\dots,z_m;W_1,\dots,W_m)=\prod_{K=1}^{m} z_K^{-1+W_K\gamma+\sum_{j=1}^{N_{KV}}\beta_{q_j}} \times$$

$$\times\ B_{(N_v,N_s;m)}\ (W_1,\dots,W_m;\ z_1,\dots,z_m) \qquad (13)$$

where the notation ($N_v, N_s; m$) is introduced for the set ($N_{1V}, N_{1S}; \dots; N_{mV}, N_{mS}$). Each multiparton substate S_K contains N_{KV} valent quarks of system S, N_{KS} sea quarks (antiquarks) of a definite type and arbitrary number of sea partons of the quark-gluonic system S which belong to substate S_K with a probability W_K. A total longitudinal momentum of multiparton substate S_K equals z_K. Functions $B_{(N_v,N_s;m)}(\omega_1,\dots,\omega_m;x_1,\dots,x_m)$ are determined via $C_N(\omega;x)$ or $\tilde{C}_N(\omega;x)$ by analogy with expressions (10) or (11), respectively:

$$B_{(N_v,N_s;m)}(\omega_1,\dots,\omega_m;x_1,\dots,x_m)=\left[\prod_{K=1}^{m} C_{N_{KV}}(\omega_K;x_K)/C_{\bar{N}_v}(1;1)\right]\times$$

$$\times\,\delta\left(1-\sum_{K=1}^{m}\omega_K\right)\delta\left(1-\sum_{K=1}^{m}x_K\right)\delta\left(\bar{N}_v-\sum_{K=1}^{m}N_{KV}\right) \qquad (14)$$

or analogous expression with the replacement $C_N(\omega;x)\to\tilde{C}_N(\omega;x)$
The valon distribution function $Q(x)$ is connected with the complete division functions by the following relations:

$$Q(x)=G_{(N_v,N_s;2)}(x,1-x;W,1-W) \qquad (15)$$

$$Q(x)=\int G_{(N_v,N_s;m)}(x,x_2,\dots,x_m;W,W_2,\dots,W_m)\prod_{K=2}^{m}dx_K \tag{15b}$$

where $(N_v,N_s;m)=(1,0;N_{2v},N_{2s};\dots;N_{mv},N_{ms})$. In expression (15b) $m>3$.

Expressions (15a) and (15b) determine the valon distribution function via functions of complete division of the quark-gluonic system S into $m>2$ multiparton subsystems. The fact that both these expressions give one and the same result follows from the identity ($W=W_1$, $N_v=N_{1v}$, $N_s=N_{1s}$, $x=z_1$):

$$\sum_{\substack{N_{1a}\dots N_{ma}\\ \sum_{K=1}^{m}N_{Ka}=\bar{N}_a}}(\bar{N}_a)!\left[\prod_{K=1}^{m}\frac{W_K^{N_{Ka}}}{(N_{Ka})!}\,\delta\Big(z_K-\sum_{j=1}^{N_{Kv}}x_j-\sum_a\sum_{j=1}^{N_{Ka}}x_{aj}\Big)\times\right.$$

$$\times\prod_{K=2}^{m}dz_K\,\delta\Big(1-\sum_{K=1}^{m}z_K\Big)=\sum_{N_a\,\sum_{K=2}^{m}N_{Ka}}\frac{(\bar{N}_a)!\;W^{N_a}}{(N_a)!\,\Big(\sum_{K=2}^{m}N_{Ka}\Big)!}\times$$

$$\times\sum_{\substack{N_{2a}\dots N_{ma}\\ \sum_{K=2}^{m}N_{Ka}=(\bar{N}_a-N_a)}}\frac{\Big(\sum_{K=2}^{m}N_{Ka}\Big)!}{(N_{2a})!\dots(N_{ma})!}W_2^{N_{2a}}\dots W_m^{N_{ma}}\times\delta\Big(x-\sum_{j=1}^{N_v}x_j-\sum_a\sum_{j=1}^{N_a}x_{aj}\Big)\times$$

$$\times\,\delta\Big(1-x-\sum_{j=1}^{\bar{N}_v-N_v}x_j-\sum_a\sum_{j=1}^{\bar{N}_a-N_a}x_{aj}\Big)=\sum_{N_a}\frac{(\bar{N}_a)!}{(N_a)!\,(\bar{N}_a-N_a)!}W^{N_a}\times$$

$$\times\Big(\sum_{K=2}^{m}W_K\Big)^{(\bar{N}_a-N_a)}\delta\Big(x-\sum_{j=1}^{N_v}x_j-\sum_a\sum_{j=1}^{N_a}x_{aj}\Big)\delta\Big(1-x-\sum_{j=1}^{\bar{N}_v-N_v}x_j-\sum_a\sum_{j=1}^{\bar{N}_a-N_a}x_{aj}\Big). \tag{16}$$

The relation (16) points out also that one-parton ($q(x)$, $a_s(x)$), arbitrary ($N_v + N_s$)-parton and multiparton distributions with variable number of particles are normalized in a proper way. For example, $\int_0^1 q(x)dx = \bar{N}_{qv}$, where $\bar{N}_{qv}$ is the number of valent quarks q in the quark-gluonic system S .

To be concrete, consider a distribution of multiparton subsystems in the π -meson. For the valent quarks of the pion, the parameters $\beta^{\pi}_{u(d)}$ characterizing structural functions behaviour at $x \to 0$ equal [19] $\beta^{\pi}_{u(d)} = 0.5$. The value of γ_{π} is determined [14] from data on Drell-Yan lepton pair production in interactions πp [10] $\gamma_{\pi} = 1.6$ (Fig.1).

Thus, in π -meson the distribution of multiparton subsystem, which contains one of valent quarks of meson and on the average a fraction W of sea partons, is determined by the expression [14] :

$$q(x;W) = x^{-1+W\gamma_{\pi}+\beta^{\pi}_{u(d)}} (1-x)^{-1+(1-W)\gamma_{\pi}+\beta^{\pi}_{d(u)}} \times$$

$$\times B^{-1}\left(W\gamma_{\pi} + \beta^{\pi}_{u(d)} , \quad (1-W)\gamma_{\pi} + \beta^{\pi}_{d(u)}\right) \tag{17}$$

Functions $q(x;W)$ and $xq(x;W)$ at various values of parameter W are presented in Fig.2. At $W = 0$ expression (17) determines distribution function of valent quark in π -meson (Fig.1 and also curve 1 in Fig.2). In case when besides valent quark of π -meson also the fraction $W = 1/2$ (on the average) of sea partons of pion enters multiparton substate, function $q(x; 0.5)$ determines distribution of constituent quark or valon in π -meson (curve 6 (3) in Fig.2a (b)) which coincides with the valon distribution obtained from the analysis of π -meson formfactor behaviour [20, 21] .

3. Quark-Gluonic System Fragmentation Functions.

Using the multiparton distribution (1)-(4) and transformation (8) one can construct the distribution density $F_{(N_v,N_s;1)}(X;W)$ of multiparton substate with a longitudinal momentum X, containing N_v valent quarks of quark-gluonic system S, a fixed number N_S of sea quarks of definite type and arbitrary number of sea partons belonging to this substate with a probability W (longitudinal momentum X of this substate is summed from longitudinal momenta of all valent quarks and sea partons that constitute this substate). Particular cases of this function, namely $F_{(1,0;1)}(X;0)$ and $F_{(0,1;1)}(X;0)$ represent distribution functions of valent quark $q(X)$ and sea partons $a_S(X)$, respectively. In another particular case, at $N_v=1$, $N_S=0$ and $W=1/\bar{N}_v$, the function $F_{(1,0;1)}(X;1/\bar{N}_v)$, as shown in Sect.2 (see also [11,12]), describes distribution of constituent quark or valon in the quark-gluonic system S.

Let us define the distribution function of m multiparton subsystems (with variable number of particles in each) with momenta $X_1,\dots,X_m$ - $F_{(N_v,N_s;m)}(X_1,\cdots,X_m;W_1,\cdots,W_m)$, where the subscript $(N_V,N_S;m)$ denotes the set $(N_{1V},N_{1S};\cdots;N_{mv},N_{ms})$. Each multiparton substate S_K contains N_{KV} valent quarks of system S, N_{KS} sea quarks of definite type and arbitrary number of sea partons entering the substate S_K with a probability W_K. A summary longitudinal momentum of substate S_K equals X_K. In the general case $\sum_{K=1}^{m} N_{KV} \leq \bar{N}_v$, $\sum_{K=1}^{m} W_K \leq 1$ and $\sum_{K=1}^{m} X_K \leq 1$, where $\bar{N}_V$ is total number of valent quarks in the quark-gluonic system S. In case $\sum_{K=1}^{m} N_{KV} = \bar{N}_v$, $\sum_{K=1}^{m} W_K = 1$ and $\sum_{K=1}^{m} X_K = 1$ (complete division of quark-gluonic system), function $F_{(N_v,N_s;m)}(X_1,\cdots,X_m;W_1,\cdots,W_m)$ coincides with the complete division function $G_{(N_v,N_s;m)}(x_1,\dots,x_m;W_1,\dots,W_m)$ of system S . In the general case, the "incomplete division" function F and complete division function G of system S are connected by relations analogous to (15):

$$F_{(N_v,N_s;m)}(x_1,\dots,x_m;W_1,\dots,W_m)=$$

$$=G_{(N_v,N_s;m+1)}(x_1,\dots,x_m,1-\sum_{K=1}^{m}x_K;W_1,\dots,W_m,1-\sum_{K=1}^{m}W_K) \quad (18a)$$

$$F_{(N_v,N_s;m)}(x_1,\dots,x_m;W_1,\dots,W_m)=$$

$$=\int G_{(N_v,N_s;M)}(x_1,\dots,x_M;W_1,\dots,W_M)\prod_{K=m+1}^{M}dx_K \quad (18b)$$

It follows from (16) that the definitions of function F given by expressions (18a) and (18b) are self-consistent.

In the framework of statistical approach to the question of quark-gluonic system fragmentation considered in this work, it is assumed that the fragmentation function of quark-gluonic system S to hadron h is proportional to a probability to find in system S a substate having valent composition of hadron h and possessing a longitudinal momentum x equal to longitudinal momentum of hadron h , i.e. proportional to $F_{(N_v,N_s;1)}(x;W)$. This probability depends on the number N_v of valent quarks common for system S and hadron h , on the number N_s of sea quarks of system S which enter hadron h as valent ones and on the value of probability W for sea parton of system S to enter the hadron h sea composition ($N_v+N_s=N_h$ is number of valent quarks in hadron h). Further, in the considered approach there is used a hypothesis [21,22] reading that the transition of extracted substate to hadron h is preceded by formation of constituent objects - valons $V_1, \dots, V_{N_h}$ which respectively carry fractions $x_1/x, \dots, x_{N_h}/x$ of longitudinal momentum of this substate ($x = \sum_{k=1}^{N_h} x_k$). Such hypothesis allows one to introduce the recombination function [23] of valons $V_1, \dots, V_{N_h}$ into hadron h - $R_h(x_1/x, \dots, x_{N_h}/x)$, which, following Refs. [21,22] , can be related to the hadron h complete division function determined, as was shown above, by its structure:

$$R_h(z_1, \dots, z_{N_h}) = A_h\Big(\prod_{k=1}^{N_h} z_k\Big)\, G_{(1,0;N_h)}\Big(z_1, \dots, z_{N_h}; \frac{1}{N_h}, \dots, \frac{1}{N_h}\Big) \quad (19)$$

Here A_h is a coefficient independent of kinematical variables, which in the general case may be different for hadrons belonging to different multiplets [13,14] .

Then the fragmentation function $D_S^h(x)$ of quark-gluonic system to hadron h can be defined by the following formula *) :

$$x D_S^h(x) = \sum_{N_v=0}^{N_v^{max}} \int F_{(N_v,N_s;N_h)}(x_1, \dots, x_{N_h}; W_1, \dots, W_{N_h}) \times$$

$$\times R_h\Big(\frac{x_1}{x}, \dots, \frac{x_{N_h}}{x}\Big)\, \delta\Big(1 - \sum_{k=1}^{N_h} \frac{x_k}{x}\Big) \prod_{k=1}^{N_h} dx_k \quad (20)$$

---

*) Summation in (20) over N_v implies that to the $S \to h$ fragmentation contribute the processes in which from S to h there transfer from $N_v = 0$ to $N_v = N_v^{max}$ valent quarks, where N_v^{max} is a maximally possible number of valent quarks common for quark-gluonic system S and hadron h .

Note that functions F and R that enter the expression (20) for the fragmentation function $D_S^h(x)$ are determined via the complete division functions of the quark-gluonic system S and hadron h, respectively, which, in turn, are determined by multiparton distributions in systems S and h.

In case when the quark-gluonic system S is hadron H, the expression (20) determines the hadron h inclusive spectrum with low P_T in the fragmentation region of initial hadron H in hadron-hadron interactions in the framework of multiparton recombination model (MRM) [11-14]. Inclusive spectra of meson and baryon resonances in fragmentation regions of proton, pion and kaon within the MRM are determined in Refs. [13,14]. Experimental data on inclusive spectra of hadron resonances in the fragmentation regions of proton, π and K-mesons in $pp-$, $\pi^{\pm}p-$, $K^{\pm}p-$ interactions are described satisfactorily by expression (20) [13,14].

Turn now to the case when the quark-gluonic system S is formed due to quark evolution. With the help of expression (20), in Ref. [24] are constructed fragmentation functions of u-quark (quark-gluonic system S_u the u-quark transforms to) into $\pi^{\pm}$-mesons:

$$xD_u^{\pi^+}(x) = g_{\bar{d}}\, x^{W\gamma+\beta_u}(1-x)^{-1+(1-W)\gamma}\,\frac{B(W\gamma+\beta_u+\alpha_\pi+1,\alpha_\pi+1)}{B(W\gamma+\beta_u,(1-W)\gamma)\,B(\alpha_\pi+1,\alpha_\pi+1)} +$$

$$+ g_u\, g_{\bar{d}}\,(1-x)^{-1+\gamma+\beta_u} \qquad (21a)$$

$$xD_u^{\pi^-}(x) = g_{\bar{d}}\, g_{\bar{u}}\,(1-x)^{-1+\gamma+\beta_u} \qquad (21b)$$

where $\alpha_\pi = -1 + \gamma_\pi/2 + \beta^{\pi}_{u(d)}$, $\gamma_\pi = 1{,}6$, $\beta^{\pi}_{u(d)} = 0{,}5$ [14].

$\beta_u = 0{,}5$, $W=0$ and $g_u = g_{\bar{u}} = g_d = g_{\bar{d}} \simeq \gamma/2(N_f+1)$,

where N_f is number of kinds of quarks [24]. For relation $\eta(x) = D^{\pi^+}_u(x)/D^{\pi^-}_u(x)$ we find:

$$\eta(x) = 1 + A(\gamma; N_f)\sqrt{\frac{x}{1-x}} \qquad (22)$$

where $A(\gamma; N_f) = 0{,}38(N_f+1)\Gamma(\gamma+0{,}5)/\Gamma(\gamma+1)$, $\Gamma(x)$ is the Euler gamma function. The values of coefficient $A(\gamma; N_f)$ at $N_f = 2$ (u, d-quarks), $N_f = 3$ (u, d, s-quarks) and at different values of γ ($0.2 \leq \gamma \leq 2.0$) are listed in the Table. Fig.3 presents a comparison of function $\eta(x)$ for the value $A(\gamma; N_f) = 1.5$ with experimental data on deep-inelastic neutrino (antineutrino) production of $\pi^{\pm}$-mesons.

Table

γ	$A(\gamma; N_f)$	
	$N_f = 2$	$N_f = 3$
0.2	1.62	2.15
0.4	1.38	1.84
0.6	1.22	1.62
0.8	1.10	1.47
1.0	1.01	1.35
1.5	0.86	1.15
2.0	0.76	1.01

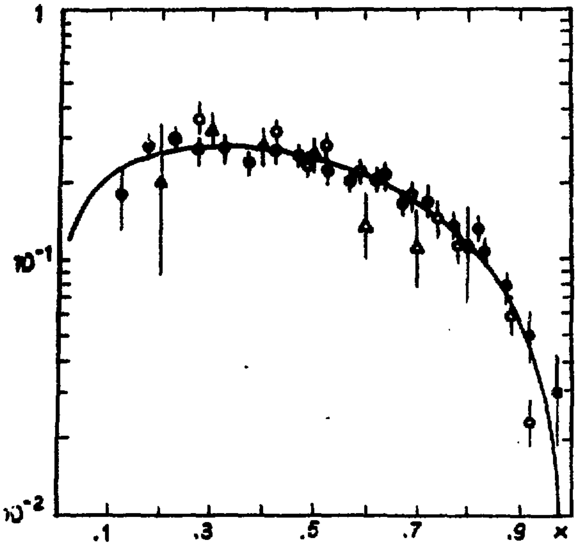

Fig.1. Valent quark distribution function in π -meson determined by $\mu^+\mu^-$ -pair production in πp -interactions: ● - NA3, 200 GeV/c; ○ - CIP, 225 GeV/c; △ - GOLIATH, 175 GeV/c [10] . The curve is calculated by formula (17) at $W = 0$.

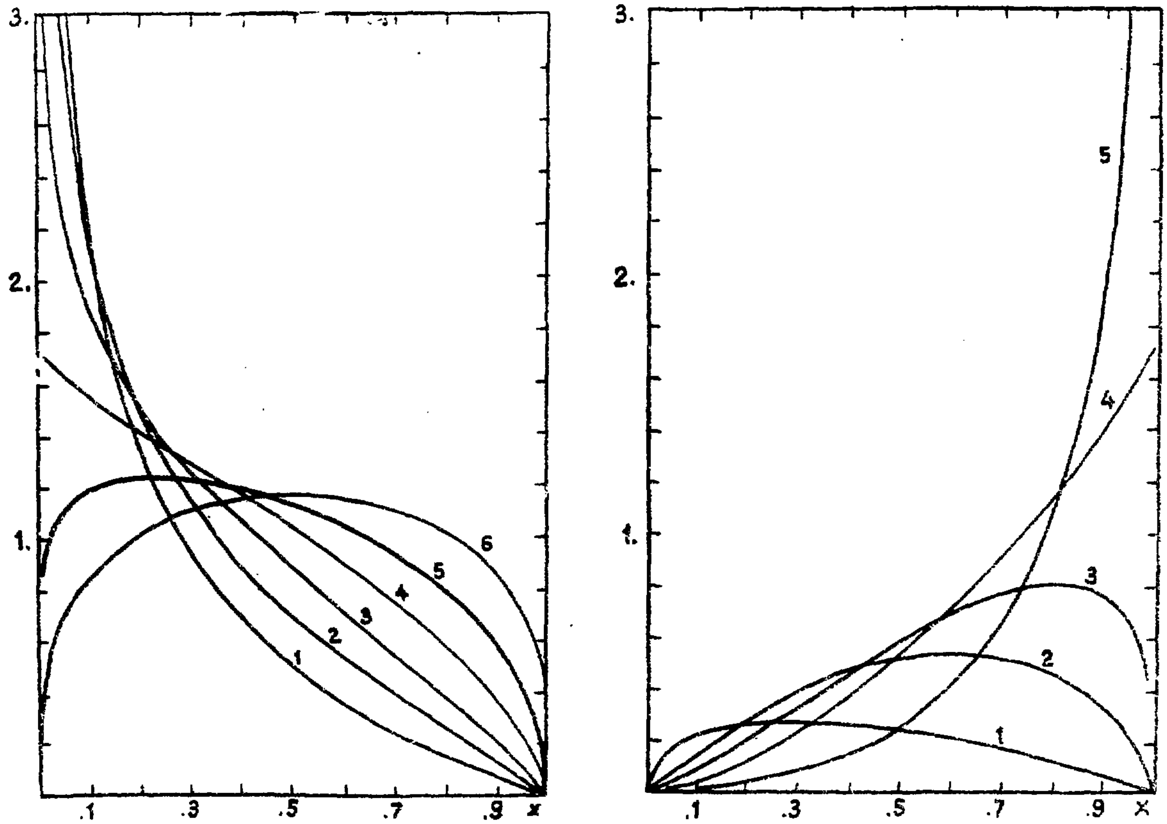

Fig.2. Multiparton subsystems distributions in π -meson:

a) $q_v(x;W)$ distribution. Curve 1 - W = 0,
2 - W = 0.1, 3 - W = 0.2, 4 - W = 0.3,
5 - W = 0.4, 6 - W = 0.5.

b) $xq_v(x;W)$ distribution. Curve 1 - W = 0,
2 - W = 0.3, 3 - W = 0.5, 4 - W = 0.7,
5 - W = 1.

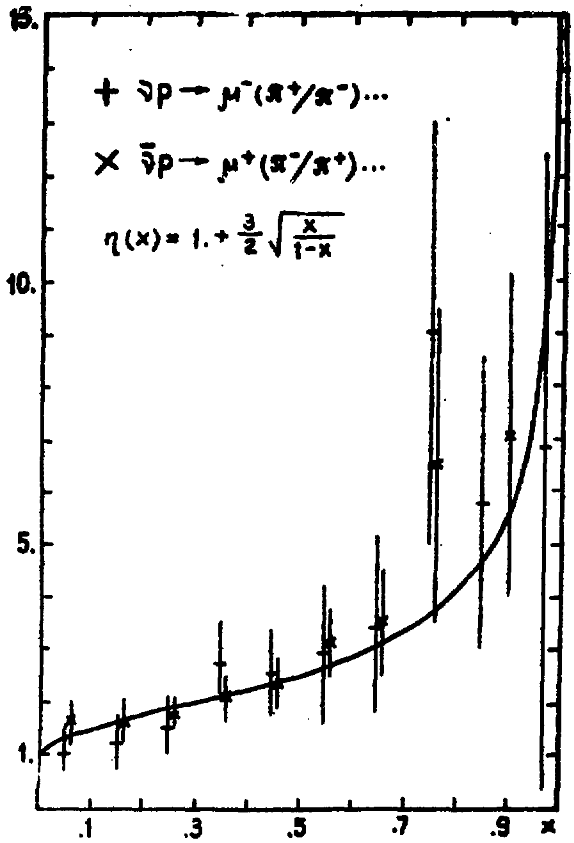

Fig.3. Ratio of inclusive production π^+/π^- (π^-/π^+) in deep-inelastic interaction of neutrino (antineutrino) with proton. Experimental points are taken from Ref. [1]. The curve refers to the value $A(\gamma; N_f) = 1.5$.

REFERENCES

1. Field R.D., Feynman R.P. Quark elastic scattering as a source of high-transverse-momentum mesons. - Phys.Rev.D, 1977, v.15, N.9, p.2590-2616.
2. Reya E. Perturbative quantum chromodynamics. - Phys.Rep., 1981, v.69, N.3, p.195-333.
3. Capella A., Sukhatme U., Tran Thanh Van J. Soft multihadron production from partonic structure and fragmentation functions. - Z. Phys.C, 1980, v.3, N.4, p.329-337.
4. Cohen-Tannoudji G., Hassouni A.El., Kalinowski J. et al. Partons at low - P_T . - Phys.Rev.D., 1980, v.21, N.9, p.2699-2705.
5. Andersson B., Gustafson G., Ingelman G., Sjostrand T. Parton fragmentation and string dynamics. - Phys.Rep., 1983, v.97, N. 2/3, p.31-145.
6. Altarelli G., Parisi G. Asymptotic freedom in parton language. - Nucl. Phys.B, 1977, v.126, N.2, p.298-318.
7. Buras A.T., Gaemers K.J.F. Simple parametrizations of parton distributions with Q^2 dependence given by asymptotic freedom. - Nucl.Phys.B, 1978, v.132, N. 3/4, p.249-267.
8. Owens J.F., Reya E. Hadronic Υ production, parton distributions, and quantum chromodynamics. - Phys.Rev.D, 1978, v.17, N.11, p.3003-3009.
9. Hwa R.C., Zahir M.S. Parton and valon distribution in the nucleon. - Phys.Rev.D, 1981, v.23, N.11, p.2539-2553.
10. Stroynowski R. Lepton pair production in hadron collisions. - Phys.Rep., 1981, v.71, N.1, p.1-50.
11. Badalyan R.G., Gulkanyan H.R. Multiparton recombination model. - Preprint EPI-753(68)-84, Yerevan 1984.
12. Бадалян Р.Г., Гулканян Г.Р. Многопартонная рекомбинационная модель образования адронов с малыми P_T.- ЯФ, 1985, т.41, вып.6, с.1611-1621.

13. Badalyan R.G., Gulkanyan H.R., Korchagin S.A. Inclusive spectra of hadron resonances in multiparton recombination model: I. Proton fragmentation. - Preprint EPI-854(5)-86, Yerevan 1986.

14. Badalyan N.N., Badalyan R.G., Gulkanyan H.R. Inclusive spectra of hadron resonances in multiparton recombination model: II. Pion and kaon fragmentation. - Preprint EPI-913(64)-86, Yerevan 1986.

15. Kuti T., Weisskopf V.F. Inelastic lepton-nucleon scattering and lepton pair production in the relativistic quark-parton model. - Phys.Rev.D, 1971, v.4, N.1, p.3418-3439.

16. Takasugi E., Tata X., Chiu C.B., Kaul R. Fast-meson production and the recombination model. - Phys.Rev.D, 1979, v.20, N.1, p.211-220.

17. Gerhold H.R. On the generalized Kuti-Weisskopf model for parton distributions. - Nuovo Cim.A, 1980, v.59, N.4, p.373-386.

18. Бедняков В.А., Исаев П.С., Коваленко С.Г.Мультипартонные распределения в адронах. ЯФ, 1984,т.40, вып.5(II), с.I3I2-I320.

19. Chliapnikov P.V., Kartvelishvili V.G., Kniazev V.V., Likhoded A.K. Symmetry violation and quark distributions in mesons. - Nucl.Phys.B, 1979, v.148, N. 3/4, p.400-412.

20. Hwa R.C., Lam C.S. Hadron wave functions and pion decay constant. - Phys.Rev.D, 1982, v.26, N.9, p.2338-2346.

21. Hwa R.C. Hadron structure and soft processes. - Preprint OITS-165, 1981.

22. Hwa R.C. Clustering and hadronization of quarks: A treatment of low-P_T problem. - Phys.Rev.D, 1980, v.22, N.7, p.1593-1608.

23. Das K.P., Hwa R.C. Quark-antiquark recombination on the fragmentation region. - Phys.Lett.B, 1977, v.68, N.5, p.459-462.

24. Badalyan R.G. On quark-to-hadron fragmentation functions. - Preprint EPI-900(51)-86, Yerevan 1986.

The manuscript was received 18 July 1986

Р.Г.БАДАЛЯН
О ФУНКЦИЯХ ФРАГМЕНТАЦИИ КВАРК-ГЛЮОННЫХ СИСТЕМ
(на английском языке, перевод З.Н.Асланян)
Редактор Л.П.Мукаян
Технический редактор А .Абрамян

Подписано в печать 4/XI-86г. ВФ-05716 Формат 60x84
Офсетная печать.Уч.изд.л.I,0 Тираж 290 экз.Ц.I5 к.
Зак.тип.№ 572 Индекс 3624

Отпечатано в Ереванском физическом институте
Ереван 36, Маркаряна 2

индекс 3624

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ