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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

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GENERAL SOLUTION OF SUPERCONVERGENT SUM RULES FOR SCATTERING OF I=1 REGGEONS ON BARYONS

ЦНИИатоминформ

Նախնատիպ ЕФИ-934(85)-86

ԳԵՐԶՈՒԳԱՄԵՏ ԳՈՒՄԱՐՆԵՐԻ ԿԱՆՈՆՆԵՐԻ ԸՆԴՀԱՆՈՒՐ ԼՈՒԾՈՒՄԸ ԲԱՐԻՈՆՆԵՐԻ ՎՐԱ I = 1 ՌԵՋԵՈՆՆԵՐԻ ՑՐՄԱՆ ՀԱՄԱՐ

Ա.Ա.ԳՐԻԳՈՐՅԱՆ, Գ.Ն. ԽԱՉԱՏՐՅԱՆ

Մասնիկների վրա ռեջեոնների ցրման համար կիրառված են գերզուգամետ գումարների կանոնները ռեջեոնների α_i $(i = \pi, \rho, A_2)$ ցրման պրոցեսներում I = 1 իզոսպինով S = -1 տարօրինակությամբ բարիոնների վրա: Փորձարարական տվյալների հիման վրա որոշվում է այդ գումարների կանոնների հագեցման սխեման: Նախատեսված են բարիոնային ռեզոնանսների երկու սերիաներ I և J=I+1/2, J=I -1/2 կամայական սպիններով: Գտնված է այդ ռեզոնանսների α_i հետ փոխազդեցության գագաթների համար ընդհանուր լուծումը: $B_{\alpha_i} B'$ $(B, B' = \Lambda, \Sigma, \Sigma^*)$ կապի գագաթների համար կանխատեսումները գտնվում են փորձի հետ համաձայնության մեջ: Ցույց է տրված, որ գումարների կանոնների հագեցման պայմանները՝ ռեզոնանսների նվազագույն քանակով, հանգեցնում է փորձարարական տվյալներից հետևող հագեցման սխեմաներին: Վերլուծվում է գումարների կանոնների ընդհանուր լուծումը Ξ և Ω հիպերոնների վրա ռեջեոնների ցրման դեպքում:

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A.A. GRIGORYAN, G.N. KHACHATRYAN

GENERAL SOLUTION OF SUPERCONVERGENT SUM RULES FOR SCATTERING OF I=1 REGGEONS ON BARYONS

Superconvergent sum rules for reggeon-particle scattering are applied to scattering of reggeons $\alpha_i (i = \pi, \rho, A_2)$ with isospin I=1 on baryons with strangeness S=-1. The saturation scheme of these sum rules is determined on the basis of experimental data. Two series of baryon resonances with arbitrary isospins I and spins $J = I + 1/2$ and $J = I - 1/2$ are predicted. A general solution for vertices of interaction of these resonances with α_i is found. Predictions for coupling vertices $B\alpha_i B' (B, B' = \Lambda, \Sigma, \Sigma^*)$ agree well with the experiment. It is shown that the condition of sum rules saturation by minimal number of resonances brings to saturation schemes resulting from experimental data. A general solution of sum rules for scattering of α_i reggeons on Ξ and Ω hyperons is analyzed.

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ОБЩЕЕ РЕШЕНИЕ СВЕРХСХОДЯЩИХСЯ ПРАВИЛ СУММ ДЛЯ РАССЕЯНИЯ I = I РЕДЖЕОНОВ НА БАРИОНАХ

Сверхсходящиеся правила сумм для рассеяния реджеонов на частицах применяются к процессам рассеяния реджеонов α_i $(i=\pi,\rho,A_2)$ с изоспином I = I на барионах со странностью S = -I. На основе экспериментальных данных определяется схема насыщения этих правил сумм. Предсказываются две серии барионных резонансов с произвольными изоспинами I и спинами J = I + I/2, J = = I - I/2. Найдено общее решение для вершин взаимодействия этих резонансов с α_i. Предсказания для вершин связи $B\alpha_i B'$ $(B, B' = \Lambda, \Sigma, \Sigma^*)$ хорошо согласуются с экспериментом. Показано, что условие насыщения правил сумм минимальным количеством резонансов приводит к схемам насыщения, вытекающим из экспериментальных данных. Анализируется общее решение правил сумм для рассеяния реджеонов α_i на Ξ и Ω -гиперонах.

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1. Introduction.

Dispersion relations (DR) play a special role in study of properties of hadron physics. Being a result of rather general properties of S -matrix these relations represent, at first sight, only certain connections between averaged characteristics of processes. However, application of DR in different regions of strong interactions has shown that they can bring to successful development of model representations and theoretical approaches which explain the hadron physics dynamics.

Rather interesting is the application of DR to investigation of the hadron classification problem. In this connection, the bootstrap hypothesis should be mentioned. Following this hypothesis, there are certain hopes that all characteristics of elementary particles can be determined unambiguously, proceeding from first principles of S -matrix. Chew-Low statistical model was an attempt of practical realization of bootstrap program. A number of hopeful results were obtained within this model; however successive calculation of hadron properties encountered the problems of formulation of intrinsically non-contradictory bootstrap procedure and cumbersome mathematical apparatus.

The next step in application of DR to the study of hadron characteristics were finite energy sum rules (FESR) and superconvergent sum rules (SSR) for particle-particle scattering amplitudes (ab -amplitudes). Together with the duality concept, these sum rules served as a basis for construction of logically more closed bootstrap scheme. As an example, it is sufficient to mention Veneziano model and dual models.

Successes of the approach based on FESR and SSR indicate the necessity of systematical analysis of hadron properties in the framework of this approach and, in particular, the importance of spin generalization of the sum rules.

In Refs. [1,2] there were introduced sum rules for amplitudes of scattering of α_i and α_K reggeons on a and b particles with arbitrary spins. Reggeon-particle scattering amplitudes (αa -amplitudes) are a generalization of particle-particle scattering amplitudes (ab -amplitudes) for the case of non-integer spins of external particles. At the same time the - amplitudes have the same analytic properties as the ab -amplitudes.

Although the αa -amplitudes are not objects of direct measurement, nevertheless from the sum rules for these amplitudes there follow the relations between measurable quantities.

In Ref. [1] it was mentioned that finite mass sum rules (FMSR) and SSR for αa -amplitudes can be written in the form of commutators for "reggeon operators" $\hat{\Omega}^{\alpha_j}$.

The FMSR have the form:

$$[\hat{\Omega}^{\alpha_i}, \hat{\Omega}^{\alpha_K +}] = \sum_{\substack{\alpha_\ell \\ (\sigma_\ell = -\sigma_i \sigma_K)}} f^{\alpha_i \alpha_K}_{\alpha_\ell^{\sigma_\ell}} (\bar{\nu}, q_i^2, q_K^2, q^2)\, \hat{\Omega}^{\alpha_\ell} \qquad (1.1)$$

where σ_j is α_j reggeon signature, $\bar{\nu}$ is upper limit of integration over low-energy region.

In case of SSR, $\hat{\Omega}^{\alpha_i}$ matrices commute

$$[\hat{\Omega}^{\alpha_i}, \hat{\Omega}^{\alpha_k +}] = 0 \tag{1.2}$$

Relations (1.1) and (1.2) represent the reggeon operators algebra and can form a dynamical basis for calculation of reggeon residues $\langle a|\hat{\Omega}|b\rangle$. Of particular interest are SSR (1.2) which do not depend on details of asymptotic behaviour.

To take into account a - and b -state spins, in Ref. [1] a formalism of S -channel helicity amplitudes in the infinite momentum frame (IMF) was proposed. A general method of SSR extraction for scattering of particles with arbitrary spins and isospins was formulated (see Section 2).

When working with the sum rules, the main problem consists in choice of saturation scheme. We shall saturate SSR with one-particle state contributions. The saturation procedure is similar to one suggested by R. Dashen and M. Gell-Mann [3] under consideration of sum rules resulting from current algebra.

Consider a commutator $[\Omega^{\alpha_i}, \Omega^{\alpha_k}]$ between a and b particle states and saturate it with a set of resonances $\{d\} = d_1, \ldots, d_n$. Further, the same commutator is taken between the particles a, d_k; b, d_m; d_m, d_k states, where d_k, d_m are the states from $\{d\}$ set, and is saturated with the sets $\{d\}'$, $\{d\}''$ and so on.

A question arising here is how to determine in a correct way a set of $\{d\}$ in each concrete case. In the absence of any sufficiently convincing theoretical argument, as a main criterion of correctness for a saturation scheme becomes agreement of its predictions with experiment.

Turn to the results of SSR applications to αa -amplitudes. Scattering of I=1 reggeons on N and Δ_{33} baryons was analyzed in Ref. [4]. The

$\alpha_i N \to \alpha_K N$ and $\alpha_i N \to \alpha_K \Delta_{33}$ processes ($i, K, = \rho, \pi, A_2$) were considered, and $S(u)$-channel states with I=1/2 and 3/2 were saturated with N and Δ_{33} contributions, respectively, i.e. only the lowest states were taken into account from the whole possible set of $\{d\}$. Results of this saturation scheme appear to be in good agreement with experimental data: the Δ_{33} width, helicity structure of $N\alpha_i B$ vertices as well as relations between $N\alpha_i B$ residues ($B = N, \Delta_{33}$; $i = \rho, \pi, A_2$) are correctly predicted. In addition, this solution satisfies also the SSR for I=1/2 and 3/2 states in $\alpha_i \Delta_{33} \longrightarrow \alpha_K \Delta_{33}$ processes.

In Ref. [5] there was analyzed the scattering of I=0 reggeons ($\mathbb{P}$, f , ω -reggeons) on N and Δ_{33} baryons. $\alpha_i N \to \alpha_K N$ and $\alpha_i \Delta_{33} \to \alpha_K \Delta_{33}$ ($i, K = \mathbb{P}, f, \omega$) processes were considered, and SSR for these processes were also saturated by contributions of only N and Δ_{33}. Predictions of this scheme also agree well with experiment.

It is seen that in both applications considered, there is realized a self-consistent closed scheme when under consideration of N and Δ_{33} - particle scattering the intermediate I=1/2 and I=3/2 states are saturated with the same particles, without involvement of new ones. This fact is rather nontrivial, since the sets of residue equations resulting from the sum rules saturation turn out to be overconstrained.

Successful application of the αa -amplitude SSR method to processes of boson reggeon scattering on N and Δ_{33} indicates importance of consideration of other processes in the framework of this method. The present paper is devoted to sum rules as applied to scattering processes of I=1 reggeons on strange baryons *. Section 2 presents the results of Refs. [1,2], necessary

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* Some of the results presented here were published earlier in Ref. [6].

for further analysis. In Section 3 the sum rules saturation scheme for scattering of $S = -1$ hyperons is grounded. The contribution of Σ-resonances to the imaginary part of amplitude $B^{(+)}_{\pi\Lambda}$ of pion scattering on a lowest state among $S = -1$ baryons - Λ hyperon is considered. It turns out that sum rules for $\mathrm{Im}\, B^{(+)}_{\pi\Lambda}$ are saturated mainly by the contributions of two lowest states - $\Sigma(1193)$ and $\Sigma^*(1385)$ hyperons. The contribution of high-lying states can be neglected (a similar situation takes place also in sum rules for $B^{(+)}_{\pi N}$ amplitude of $\pi N \to \pi N$ scattering where the contributions of lowest N and $\Delta(1232)$ states dominate).

Although $B^{(+)}_{\pi\Lambda}$ amplitude is not cuperconvergent, one can naturally expect that a similar saturation scheme is valid also for SSR for I=1 reggeon scattering, since just in the SSR, the contribution of high-lying states must be suppressed. So, we will saturate the SSR for $\alpha_i \Lambda \to \alpha_k \Lambda$ processes by contributions of $\Sigma(1193)$ and $\Sigma^*(1385)$ resonances. Consider, next, SSR for $\alpha_i \Lambda \to \alpha_k \Sigma_\ell$, $\alpha_i \Sigma_\ell \to \alpha_k \Sigma_m$ (Σ_ℓ, $\Sigma_m = \Sigma$ and Σ^*) processes and saturate I=0 and 1 $S(u)$-channel states by Λ, Σ and Σ^* contributions. General analysis of this scheme is done in Section 4. It turns out that self-consistency of sum rules requires introduction of $I \geqslant 2$ states (exotic baryons with $S = -1$ strangeness), in each I two baryons with $J = I \pm 1/2$ and positive parity being predicted.

The considered scheme brings to definite predictions for $S = -1$ hyperons interaction characteristics: helicity coupling vertices with I=1 reggeons, decay widths. Ratios of $\Sigma^* \to \Sigma\pi$ and $\Sigma^* \to \Lambda\pi$ decays widths as well as of $G^{\Sigma^*\pi\Lambda}$ and $G^{\Sigma\pi\Lambda}$ constants, which can be compared with experiment, are in good agreement with experimental data. The other SSR predictions can be checked in hyperon-beam experiments.

Hence, just as in case of scattering on N and Δ_{33}, the SSR saturation scheme for processes of I=1 reggeon scattering on $S = -1$ hyperons turns out to be logically closed in all sum rules for $\alpha_i a \to \alpha_K b$ ($ab = \Lambda$, Σ and Σ^*) processes; one can saturate the states with I=0 and 1 by contributions of the same Λ , Σ and Σ^*.

It was mentioned in [4] that substitution of SSR solutions for scattering of I=1 reggeons on nucleons and Δ_{33}-isobar in FMSR for these processes results in cancellation of the last sum rules. This points out the consistency of the SSR-based approach to that of Ref. [7] where it was assumed that in all FMSR there works the above-described picture - summary contribution of nucleon and Δ_{33}-isobar is small being locally dual to Regge contribution. Similar consistency between SSR and FMSR takes place also for I=1 reggeon - $S = -1$ hyperon scattering. Moreover, it turns out (see Section 5) that starting with scattering of I=1 reggeons on $a(1/2,1/2, +)$ state with $I = \mathcal{J} = 1/2$ and positive parity (nucleon) as well as on $a(0, 1/2, +)$ state with I=0, $J = 1/2$ and positive parity (Λ -hyperon) and accepting the condition that the SSR and FMSR are saturated with the minimal number of resonances, we are necessitated to introduce $a(3/2, 3/2, +)$ as well as $a(1, 3/2, +)$ and $a(1, 1/2, +)$ states. The quantum numbers as well as properties of these states predicted from sum rules allow one to identify them with Δ_{33} , Σ^* and Σ resonances, respectively.

Section 6 is devoted to αa -amplitudes sum rules as applied to scattering of I=1 reggeons on hyperons with $S = -2$ and -3. Saturation schemes and corresponding solutions of sum rules are discussed.

Series of states predicted in [8] and in this paper coincide with representations of non-compact $SU_I(2) \otimes SU_{\mathcal{J}}(2) \times T_g$ group where T_g is a group of translations introduced in Ref. [9] under analysing of strong-coupling limit in static Chew-Low model (see Conclusion).

2. Superconvergent Sum Rules for αa -Amplitudes.

The superconvergent dispersion relations for the αa -scattering amplitude have a usual form:

$$\int_0^\infty \Im m\, T_{a\beta}^{\alpha_i \alpha_\kappa (-)} (q_i^2, q_\kappa^2, q^2, \nu) = 0 \qquad (2.1)$$

where $\nu = \frac{s-u}{4M_N}$, $T_{a\beta}^{\alpha_i\alpha_\kappa(-)}$ is crossing-antisymmetric amplitude

$$T_{a\beta}^{\alpha_i\alpha_\kappa (-)} = T_{a\beta}^{\alpha_i\alpha_\kappa (s)} - T_{a\beta}^{\alpha_i\alpha_\kappa (u)}$$

To take into account the scattered particle spins, authors of Ref. [1] considered a formalism of S -channel helicity amplitudes in the infinite momentum frame (IMF). The vertices of couplings of reggeons with particles and resonances contribution to the sum rules have a simple form in this formalism.

Helicity particle-reggeon-particle vertex is as follows:

$$\Gamma_{\lambda_a \lambda_\beta}^{a\alpha\beta}(q) = |q^\perp|^{|\lambda_a - \lambda_\beta|} e^{i\varphi(\lambda_a - \lambda_\beta)} G_{\lambda_a\lambda_\beta}^{a\alpha\beta}(q^{\perp 2}) \qquad (2.2)$$

where $q = p_\beta - p_a$, φ is the angle between $q^\perp$ and x axis.

Residues $G_{\lambda_a\lambda_\beta}^{a\alpha\beta}(q^{\perp 2})$ have no kinematic singularities. In what follows, the small $(q_{ik}^\perp)^2$ are considered and quantities $G_{\lambda_a\lambda_\beta}^{a\alpha\beta}(0)$ are discussed.

The reggeon residues $G_{\lambda_a\lambda_\beta}^{a\alpha\beta}$ have the following symmetry properties:

$$G_{\lambda_a\lambda_\beta}^{a\alpha\beta} = \sigma P (-1)^{s_a - s_\beta - \lambda_a + \lambda_\beta} \eta_a \eta_\beta G_{-\lambda_a -\lambda_\beta}^{a\alpha\beta} \qquad (2.3)$$

$$G_{\lambda_a\lambda_\beta}^{a\alpha\beta} = G(-1)^I P(-1)^{\lambda_a - \lambda_\beta} \tilde{G}_{\lambda_\beta\lambda_a}^{\beta\alpha a} \qquad (2.4)$$

where σ , P , G and I are signature, P - and G -parities and isospin of α -reggeon (nonstrange boson trajectories are considered);

η_i is intrinsic P -parity of i -particle. Vertices $\tilde{G}^{\beta\alpha a}_{\lambda_\beta \lambda_a}$ describe an inverse transition $\beta \to a$. For amplitudes of α_i , α_K reggeons scattering on a and β particles with spin, there exist the following SSR [1,2] .

a) SSR which take place if $\alpha_\ell - \alpha_i - \alpha_K < -1$, where α_ℓ is the rightmost in J -plane singularity with the given t -channel quantum numbers:

$$\sum_{I_s} X_{ts} \sum_{d_s} G^{a\alpha_i d_s}_{\lambda_a \lambda_a + n} G^{d_s \alpha_K \beta}_{\lambda_a + n\, \lambda_\beta} - \sum_{I_u} X_{tu} \sum_{d_u} G^{a\alpha_K d_u}_{\lambda_a \lambda_\beta - n} G^{d_u \alpha_K \beta}_{\lambda_\beta - n\, \lambda_\beta} = 0 \quad (2.5)$$

and

$$\sum_{I_s} X_{ts} \sum_{d_s} \left[G^{a\alpha_i d_s}_{\lambda_a \lambda_a - 1} G^{d_s \alpha_K \beta}_{\lambda_a - 1\, \lambda_a} - G^{a\alpha_i d_s}_{\lambda_a \lambda_a + 1} G^{d_s \alpha_K \beta}_{\lambda_a + 1\, \lambda_a} \right] +$$
$$+ \sum_{I_u} X_{tu} \sum_{d_u} \left[G^{a\alpha_K d_u}_{\lambda_a \lambda_a - 1} G^{d_u \alpha_i \beta}_{\lambda_a - 1\, \lambda_a} - G^{a\alpha_K d_u}_{\lambda_a \lambda_a + 1} G^{d_u \alpha_i \beta}_{\lambda_a + 1\, \lambda_a} \right] = 0 \quad (2.6)$$

In (2.5) is integer

$$\min\left(|\lambda_a|, |\lambda_\beta|\right) \leqslant |\lambda_a + n| \left(|\lambda_\beta - n|\right) \leqslant \max\left(|\lambda_a|, |\lambda_\beta|\right) \quad (2.7)$$

SSR of (2.5) and (2.6) are in definite isospin state of t -channel; $X_{ts} \equiv X(I_t, I_s)$ and $X_{tu} \equiv X(I_t, I_u)$ are isotopic crossing matrices; d_s and d_u are S - and U -channel resonances with the given I_s and I_u .

Quantities $G^{c\alpha_j f}_{\lambda_c \lambda_f}$ are reduced residues. They are connected with the physical residues as follows:

$$\left(G^{c\alpha_j f}_{\lambda_c \lambda_f}\right)^{\varphi u a} = \varepsilon_c \varepsilon_j \varepsilon_f (-1)^{I_c - I_j + m_f} \begin{pmatrix} I_c & I_j & I_f \\ m_c & m_j & -m_f \end{pmatrix} G^{c\alpha_j f}_{\lambda_c \lambda_f} \qquad (2.8)$$

where $I_i\,(m_i)$ is isospin (its third projection) of the particle; ε_i is a phase that connects the particle state with the basic isospin state; $\begin{pmatrix} I_1 & I_2 & I_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ - $3jm$ is symbol. SSR (2.5) and (2.6) take place for amplitudes with definite value of naturality $\sigma_\ell P_\ell$: for (2.5) $\tau \equiv (\sigma_\ell P_\ell)(\sigma_i P_i)(\sigma_K P_K) = +1$, while for (2.6) $\tau = -1$.

Apart from (2.5) and (2.6), there exist SSR which hold at weaker restrictions on position of α_ℓ in J -plane, namely $\alpha_\ell - \alpha_i - \alpha_K < 0$. These sum rules have the form:

$$\left\{\sum_{I_s} X_{ts} \sum_{d_s} G^{a\alpha_i d_s}_{\lambda_a \lambda_a + n} G^{d_s \alpha_K \ell}_{\lambda_a + n \lambda_\ell} - \sum_{I_u} X_{tu} \sum_{d_u} G^{a\alpha_K d_u}_{\lambda_a \lambda_\ell - n} G^{d_u \alpha_i \ell}_{\lambda_\ell - n \lambda_\ell}\right\} \Big/ \binom{|n|}{|\lambda_a - \lambda_\ell|} =$$

$$= \left\{\sum_{I_s} X_{ts} \sum_{d_s} G^{a\alpha_i d_s}_{\lambda_a \lambda_a + n'} G^{d_s \alpha_K \ell}_{\lambda_a + n' \lambda_\ell} - \sum_{I_u} X_{tu} \sum_{d_u} G^{a\alpha_K d_u}_{\lambda_a \lambda_\ell - n'} G^{d_u \alpha_i \ell}_{\lambda_\ell - n' \lambda_\ell}\right\} \Big/ \binom{|n'|}{|\lambda_a - \lambda_\ell|}$$

(2.9)

In (2.9) n and n' are integer numbers satisfying conditions $n \neq n'$ and (2.7); $\binom{|m|}{|\lambda_a - \lambda_\ell|}$ is binomial coefficient. SSR of (2.9) do not depend on sign of τ .

In order to determine the asymptotic behaviour of the given sum rule, it is necessary to find position of α_ℓ in J -plane. In case of scattering of nonstrange boson reggeons α_i , α_K with $I_{i,K} = 1$ (ρ , π , A_2), a simple analysis(which takes into account the possible contribution of moving cuts) shows [2] that in case of SSR (2.5) and (2.6)

$$\begin{aligned} \alpha_\rho(0) &< 0 && \text{at} \quad I_t = 0 \\ \alpha_\rho(0) &\approx 0 && \text{at} \quad I_t = 1 \\ \alpha_\rho(0) &\approx -0{,}5 && \text{at} \quad I_t = 2 \end{aligned} \tag{2.10}$$

In case of sum rules (2.9)

$$\begin{aligned} \alpha_\rho(0) &\approx 0.5 && \text{if} \quad I_t = 0.1 \\ \alpha_\rho(0) &\approx 0 && \text{at} \quad I_t = 2 \end{aligned} \tag{2.11}$$

Finally we give the values of crossing isospin matrices X_{ts} and X_{tu} for scattering of $I=1$ reggeons on a and b particles with arbitrary isospins. The s-channel crossing matrix has the form [2]:

$$X_{ts} = (-1)^{I_t - I_s - I_b} \begin{Bmatrix} 1 & 1 & I_t \\ I_b & I_a & I_s \end{Bmatrix}$$

where

$$\begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix} \quad \text{- } 6J \text{ - symbol}$$

Correspondingly X_{tu} is

$$X_{tu} = (-1)^{I_b - I_u} \begin{Bmatrix} 1 & 1 & I_t \\ I_a & I_b & I_u \end{Bmatrix}$$

The values of $6J$ -symbols are listed in Table 1.

3. Scattering of I=1 Reggeons on S =-1 Hyperons. Saturation Scheme.

Consider scattering of I=1 reggeons on Λ -hyperon which is the lowest state among baryons with S=-1 strangeness:

$$\alpha_i \Lambda \longrightarrow \alpha_K \Lambda \tag{3.1}$$

Processes (3.1) are determined by contributions of s-channel states with I=1 isospin. At determination of SSR saturation scheme for these procedures, an important information can be obtained when considering real π -meson scattering. Analysis of sum rules for $\pi N \to \pi N$ scattering case was carried out in Ref. [7]. It was shown that the nucleon and Δ_{33}-isobar have large and mutually cancelling contributions to amplitude $\Im m\, B^{(+)}_{\pi N}$ (sum rules (2.6) correspond to this amplitude). Contribution of high-lying states in $\Im m\, B^{(+)}_{\pi N}$ is small; one can neglect it taking into account only N and Δ_{33}. Such mechanism provides small contribution to $B^{(+)}_{\pi N}$ of t-channel singularities with I = 0.

Consider now $\pi\Lambda \to \pi\Lambda$ scattering. Fig.1 presents Σ -resonances contribution to imaginary part of amplitude $B^{(+)}_{\pi\Lambda}$ of this process, calculated with the assumption of Breit-Wigner distribution shape. The pole contribution of Σ(1193)-hyperon is marked by a rectangle. Constant $G^{\Sigma\pi\Lambda}$ is derived from data on $\Sigma^{\pm}$-hyperon β-decay $\Sigma^{\pm} \to \Lambda e^{\pm} \nu$ using the Goldberger-Treiman relation.

First of all one can see that Σ(1193) and Σ^{*}(1385) make large and sign-opposite contributions to $\Im m\, B^{(+)}_{\pi\Lambda}$. Numerical values of Σ(1193) and Σ^{*}(1385) contributions to sum rules for $\Im m\, B^{(+)}_{\pi\Lambda}$ are as follows: the contribution of $\Sigma(1193) - \frac{1}{4\pi}\left(G^{\Sigma\pi\Lambda}\right)^2 = 11\pm1$, the contribution of $\Sigma^{*}(1385) = -11\pm0.8$, i.e. these contributions cancel each other with high accuracy. Turn to contribution of highest states to $\Im m\, B^{(+)}_{\pi\Lambda}$. For its calculation we used data on total widths and modes of Σ -resonance decay into $\Lambda\pi$ -channel, given in [10]. Unfortunately, the accuracy of experimental measurements is not high, and data on decay to $\Lambda\pi$ -channel are absent for some resonances. In these cases we assumed that the width of decay into $\Lambda\pi$ was 15% of total width, this being equal to average value of a portion of measured partial widths of highest excitations [10]. Under this assumption, the summary contribution of highest states into the sum rule for $\Im m\, B^{(+)}_{\pi\Lambda}$ is ~ 0.1.

Thus, in crossing-antisymmetric sum rules for $\Im m\, B^{(+)}$ $\pi\Lambda \to \pi\Lambda$ -scattering there works a mechanism of resonance saturation, similar to πN -scattering case: large contributions of lowest states $\Sigma(1193)$ and $\Sigma^*(1385)$ cancel each other, while high-lying resonance contribution can be neglected.

For evident reasons the contribution of Regge singularities to $\Im m\, B^{(+)}_{\pi\Lambda}$ amplitude is unknown. It is most likely, however, that its behaviour is the same as in πN -scattering case.

Together with the cancellation mechanism which takes place for $\Im m\, B^{(+)}$ amplitude, in other cases a mechanism is possible when smallness of Regge contribution is ensured by smallness of each low-energy state contribution separately. Such mechanism takes place at saturation of sum rules for $\Im m\, A^{(-)}$ amplitude of $\pi N \to \pi N$ scattering by contributions of the nucleon and Δ_{33}-isobar [7].

The above-cited examples demonstrate validity of so-called "semi-local duality" according to which the Regge contribution coincides with averaged contribution of resonances at all ν.

It is natural to assume that similar mechanisms work also in case of saturation of SSR for I=1 trajectory scattering on nucleons and Λ -hyperon. The neglect of high-lying states contribution is natural just in case of SSR because of convergence of these sum rules. Hence, when considering SSR for scattering of I=1 reggeons on S=-1 hyperons we shall saturate the $s(u)$ - channel states with I=1 by contributions of $\Sigma(1193)$ and $\Sigma^*(1385)$ hyperons and the states with $I_{s(u)}$ = 0 by contribution of Λ -hyperon.

4. Scattering of I=1 Reggeons on S = -1 Hyperons. General Solution of SSR.

In accordance with the above-described scheme consider the processes

$$\alpha_i \Lambda \to \alpha_K \Lambda \qquad (i, K = \rho, \pi, A_2) \tag{4.1}$$

and take into account Σ and $\Sigma^*(1385)$ in SSR for these processes. Consider, further, SSR for the reactions

$$\alpha_i \Lambda \to \alpha_K \Sigma \tag{4.2}$$

$$\alpha_i \Lambda \to \alpha_K \Sigma^* \tag{4.3}$$

$$\alpha_i \Sigma \to \alpha_K \Sigma \tag{4.4}$$

$$\alpha_i \Sigma \to \alpha_K \Sigma^* \tag{4.5}$$

$$\alpha_i \Sigma^* \to \alpha_K \Sigma^* \tag{4.6}$$

and saturate $I_{s(u)}$= 0.1 by contributions of Λ , Σ , Σ^*, respectively. In reactions (4.4)-(4.6) s and u -channel isospins $I_{s(u)}$ can be equal to 2. If we assume that I=2 resonances (exotic resonances) do not exist, then it turns out that the system of equations for Regge residues $G^{a\alpha_j b}_{\lambda_a \lambda_b}$ ($a, b = \Lambda, \Sigma, \Sigma^*$; $j = \rho, \pi, A_2$) which follows from SSR, has a trivial (nonphysical) solution either for all residues of ρ and A_2 or for all residues of π -meson (coupling constants of hyperons and pions vanish). Now take into account the contribution of states with $I = 2 - \{d^{I=2}\}$.

When saturating in (4.4)-(4.6) processes $I_{S(u)} = 2$ by $d_n^{I=2}$ resonances contributions, we must simultaneously consider the processes:

$$\alpha_i a \to \alpha_K d_n^{I=2} \quad (a = \Lambda, \Sigma, \Sigma^*) \tag{4.7}$$

It turns out that the SSR saturation scheme for (4.1)-(4.7) processes is self-consistent (brings to nontrivial solution for residues) if we take into account in $I_{S(u)} = 2$ a contribution at the minimum of two states from $\{d^{I=2}\}$ - call them S_E and S_E^*.

Our purpose is to determine spins and parities of these states as well as to solve equations for helicity residues $G^{a\alpha_i b}_{\lambda_a \lambda_b}$ ($a, b = \Lambda, \Sigma, \Sigma^*, S_E, S_E^*$).

First of all from SSR (2.6) for the process (4.6) at $\lambda_{S_E^*} = 3/2$ ($I_t = 0$) which has the form

$$\begin{aligned} &\left(G^{\Sigma^*\rho\Lambda}_{3/2\ 1/2}\right)^2 + \left(G^{\Sigma^*\rho\Sigma}_{3/2\ 1/2}\right)^2 + \left(G^{\Sigma^*\rho\Sigma^*}_{3/2\ 1/2}\right)^2 + \\ &+ \sum_{d_n^{I=2} = S_E, S_E^*} \left\{ \left(G^{\Sigma^*\rho d_n^{I=2}}_{3/2\ 1/2}\right)^2 - \left(G^{\Sigma^*\rho d_n^{I=2}}_{3/2\ 5/2}\right)^2 \right\} = 0 \end{aligned} \tag{4.8}$$

it follows that at least one resonance (call it S_E^*) must have spin:

$$J_{S_E^*} \geqslant 5/2 \tag{4.9}$$

We shall use the restriction (4.9) involving the values of S_E^*-resonance helicities up to $\pm 5/2$ ($|\lambda_{S_E^*}| \leqslant 5/2$). Not writing out the whole set of equations for helicity residues, which is rather cumbersome, we note that this set is simplified very much, assuming that

$$G^{\Lambda\pi\Sigma}_{1/2\ 1/2} = 0 \qquad (4.10)$$

The condition (4.10) represents a condition of equality of and masses

$$\frac{M_\Sigma - M_\Lambda}{M_\Sigma} = 0 \qquad (4.11)$$

since $G^{\Sigma\pi\Lambda}_{1/2\ 1/2} \sim G^{\Sigma\pi\Lambda}(M_\Sigma - M_\Lambda)/\sqrt{\mu_\Sigma M_\Lambda}$, where $G^{\Sigma\pi\Lambda}$ is constant of $\Sigma\pi\Lambda$ -interaction. It is evident that, due to the closeness of Σ and Λ masses, relations (4.11) and correspondingly (4.10) are true with a high degree of accuracy. Accounting (4.10), the solution of the set of equations is as follows:

a) residues with helicity non-flip are zero:

$$G^{\alpha\alpha_j\beta}_{\lambda\lambda} = 0 \quad \left(\alpha,\beta = \Lambda, \Sigma, \Sigma^*, S_E, S_E^*;\ j = \rho, \pi, A_2\right) \qquad (4.12)$$

b) residues with single-flip helicity satisfy the following relations:

$$G'^{\alpha\alpha_i\beta}_{\lambda\ \lambda\pm1} = (-1)^{\frac{\sigma_i P_i - \sigma_k P_k}{2}} G'^{\alpha\alpha_k\beta}_{\lambda\ \lambda\pm1} \qquad (4.13)$$

where

$$G'^{\alpha\alpha_j\beta}_{\lambda\ \lambda\pm1} \equiv G^{\alpha\alpha_j\beta}_{\lambda\ \lambda\pm1} \Big/ G^{\Sigma\alpha_j\Lambda}_{1/2\ -1/2}$$

The relation (4.13) reduces equations for single-flip residues to the case of scattering of identical reggeons ($i = K$). As this case we consider the π -trajectory scattering.

In our further analysis we shall use results of Ref. [11] where helicity vertices $G^{a\alpha_\pi b}_{\lambda_a \lambda_b}$ were studied using invariant expression for the vertex $a\pi b$.

It was shown that if

$$G^{a\alpha_\pi b}_{\lambda\,\lambda} = 0 \qquad \text{and} \qquad G^{a\alpha_\pi b}_{\lambda\,\lambda\pm1} \neq 0 \tag{4.14}$$

then necessarily

$$P_b = P_a \quad , \qquad J_a \leq J_b \leq J_a + 1 \tag{4.15}$$

$$M_b = M_a \tag{4.16}$$

and the residues $G^{a\alpha_\pi b}_{\lambda\,\lambda\pm1}$ are expressed only through one invariant function $G^{a\pi b}$ and

a) if $J_b = J_a$, then

$$G^{a\alpha_\pi b}_{\lambda\,\lambda\pm1} = (-1)^{J_a - 1/2}\, G^{a\pi b}\, \frac{1}{2J_a}\sqrt{(J_a \mp \lambda)(J_a \pm \lambda + 1)} \tag{4.17}$$

b) if $J_b = J_a + 1$, then

$$G^{a\alpha_\pi b}_{\lambda\,\lambda\pm1} = \mp(-1)^{J_a - 1/2}\, M_a\, G^{a\pi b}\sqrt{\frac{(J_a \pm \lambda + 2)(J_a \pm \lambda + 1)}{(2J_a + 2)(2J_a + 1)}} \tag{4.18}$$

c) if $G^{a\alpha_\pi b}_{\lambda\,\lambda} = 0$ and $J_b \geqslant J_a + 2$, then

$$G^{a\alpha_\pi b}_{\lambda\,\lambda\pm1} = 0 . \tag{4.18a}$$

Turning back to the system of equations for residues $G^{a\alpha_\pi b}_{\lambda\,\lambda\pm1}$ ($ab = \Lambda, \Sigma, \Sigma^*, S_E, S_E^*$), we note that from this solution it follows that

$$G_{\lambda\ \lambda\pm1}^{\Sigma^*\alpha_\pi S_E^*} \neq 0 \tag{4.19}$$

$$G_{\lambda\ \lambda\pm1}^{\Sigma\alpha_\pi S_E} \neq 0 \quad , \qquad G_{\lambda\ \lambda\pm1}^{\Sigma^*\alpha_\pi S_E} \neq 0 \tag{4.20}$$

Taking (4.12), (4.14), (4.15), (4.19) and (4.20) into account we immediately obtain

$$J_{S_E^*}^{P} = 5/2^{+} \quad , \qquad J_{S_E}^{P} = 3/2^{+} \tag{4.21}$$

For constants $G^{\prime\alpha\pi\beta} = G^{\alpha\pi\beta} / G^{\Sigma\pi\Lambda}$ the following values are predicted:

$$G^{\prime\Lambda\pi\Sigma^*} = -\mu\frac{\sqrt{3}}{2} \,,\quad G^{\prime\Sigma\pi\Sigma} = \mu\nu\sqrt{2} \,,\quad G^{\prime\Sigma\pi\Sigma^*} = \mu\frac{1}{2}\sqrt{\frac{3}{2}} \,,\quad G^{\prime\Sigma^*\pi\Sigma^*} = -\mu\nu\frac{3}{\sqrt{2}} \tag{4.22a}$$

$$G^{\prime\Sigma\pi S_E} = \nu\frac{1}{2}\sqrt{\frac{15}{2}} \,,\quad G^{\prime\Sigma^*\pi S_E} = \frac{3}{\sqrt{10}} \,,\quad G^{\prime\Sigma^*\pi S_E^*} = -\mu\frac{3}{2} \;; \tag{4.22b}$$

$$G^{\prime S_E\pi S_E} = \mu\nu\frac{9}{\sqrt{10}} \,,\quad G^{\prime S_E\pi S_E^*} = \mu\frac{1}{2} \,,\quad G^{\prime S_E^*\pi S_E^*} = -\mu\nu\sqrt{10} \tag{4.22c}$$

where $\mu, \nu = \pm 1$.

Helicity residues $G_{\lambda\ \lambda\pm1}^{\prime\alpha\pi\beta}$ satisfy relations (4.17) and (4.18).

Just as in case of scattering on nonstrange baryons, the SSR solution is self-consistent in the limit of equal masses ($M_{S_e^*} = M_{S_c} = M_{\Sigma^*} = M_{\Sigma} = M_{\Lambda}$).

Let us discuss the SSR solution at greater length.

a) From formulae (4.22a) the relation between constants $G^{\prime\Sigma^*\pi\Lambda}$ and $G^{\prime\Sigma^*\pi\Sigma}$ can be checked experimentally. The following ratio of $\Sigma^* \to \pi\Lambda$ and $\Sigma^* \to \pi\Sigma$ decay widths is predicted:

$$\frac{\Gamma_{\Sigma^* \to \pi\Sigma}}{\Gamma_{\Sigma^* \to \pi\Lambda}} = \frac{|\vec{P}_{\Sigma}|^3 (E_{\Sigma} - M_{\Sigma})}{2|\vec{P}_{\Lambda}|^3 (E_{\Lambda} + M_{\Lambda})} \sim 0.12 \tag{4.23}$$

where $\vec{P}_{\Lambda}$ and $\vec{P}_{\Sigma}$ are the corresponding decay momenta.

The prediction (4.23) agrees well with experimental data [10] .

It should be noted that our prediction for the ratio $X = (G^{\Sigma^*\pi\Lambda}/G^{\Sigma^*\pi\Sigma})^2$ differs from the value which is predicted from SU(3)-symmetry ($X_{SU(3)} = \frac{3}{4} X_{cnc}$) and which also agrees with experiment.

To explain the deviation of X_{cnc} from $X_{SU(3)}$, one may assume, e.g., that the state Σ^* is a mixture of 10 and 27 representations that enter to $8 \otimes 8$ decomposition. A detailed analysis of experimental data on $\Sigma^* \to \pi\Lambda$ and $\Sigma^* \to \pi\Sigma$ decays shows that their accuracy is rather low. At the same time, even at precise measurements it seems impossible to distinguish which of the predictions is more suitable since both our and SU(3) calculations cannot pretend on accuracy higher than 20%.

b) The SSR solution brings to mutual cancellation of contributions of Σ and Σ^* -hyperons in the sum rule for amplitude $B^{(+)}_{\pi\Lambda}$, i.e. to relation $(G^{\Sigma\pi\Lambda}/G^{\Sigma^*\pi\Lambda})^2$ that agrees with experiment (see Sect. 3).

c) In a number of works there were analyzed dispersion relations for

$A^{(-)}$ and $B^{(+)}$ amplitudes of $\pi\Sigma \to \pi\Sigma$ scattering (see, e.g. Ref.[12] where the references to other works are cited) through saturating these relations by contributions of known resonances with I=0 and I=1. The main results of the analysis are as follows: i) for the ratio of $g^{\Sigma\pi\Lambda}$ and $g^{\Sigma\pi\Sigma}$ constants ($g^{\Sigma\pi\Lambda}$ and $g^{\Sigma\pi\Sigma}$ are physical constants (see (2.8))) there is predicted a value close to one $(g^{\Sigma\pi\Lambda}/g^{\Sigma\pi\Sigma})^2 \approx 1$; ii) self-consistency of dispersion relations requires the account in $I_{s(u)} = 2$ of the contribution of resonance with $J^P = 3/2^+$ - the P_3 state in $\Sigma\pi$ system. In Ref. [10] the value of 250 MeV at a mass 1600 MeV is given for the width of this resonance.

All these results are in agreement with our predictions, namely: from (4.22a) and (2.8) it follows that: i)

$$(g^{\Sigma\pi\Lambda} / g^{\Sigma\pi\Sigma})^2 = 1 \tag{4.24}$$

ii) although from our SSR there follows existence of two states with I=2, nevertheless only S_E -resonance with $J^P = 3/2^+$ contributes to sum rules for $\alpha_i\Sigma \to \alpha_K\Sigma$ processes. For its decay width a value is predicted close to that given in Ref. [12] (see the formula for Γ_{S_E} at the end of this section).

d) SSR solution for helicity residues of I=1 reggeons $G^{a\alpha_i b}_{\lambda_a \lambda_b}$ ($a, b = \Lambda, \Sigma, \Sigma^*$) has the form analogous to solution in the case of $a, b = N, \Delta_{33}$ (which agrees well with experiment [4]), namely residues with helicity non-flip are zero, and relations between residues with $\Delta\lambda = \pm 1$ are given by formulae (4.17) and (4.18). These relations are to be checked in hyperon-beam experiments.

e) SSR predicts existence of two exotic baryons with strangeness S=-1 and isospin I=2. In the framework of our approach S_E and S_E^* are analogs of Λ , Σ and Σ^* resonances in the world of hyperons with S=-1 and I=2, i.e. lowest states among these hyperons. S_E^* -resonance is the S=-1 partner of exotic E_{55} -baryon - of $\alpha(5/2, 5/2, +)$ state predicted in [11] . $\Sigma^*\pi$ -channel is dominant in S_E^* -resonance decay, whereas S_E can decay both into $\Sigma^*\pi$ and $\Sigma\pi$ -channels. The properties and possibilities of experimental search for S_E^* and S_E states are discussed at greater length in Ref. [13] .

The next step is to consider the processes:

$$\alpha_i a \rightarrow \alpha_K b \qquad (a, b = S_E , S_E^*) \tag{4.25}$$

These processes proceed via $S(u)$ -channel isospin states with $I_{S(u)}$=1,2,3. Taking into account the Σ , Σ^* and S_E , S_E^* contributions in $I_{S(u)}$ =1,2 and substituting the earlier determined corresponding vertices into the sum rules, one can make sure that the account of resonance contribution with $I_{S(u)}$ = 3 is necessary to satisfy the sum rules for the processes (4.25). A minimal necessary number of resonances is two - denote them $\alpha(3, J_x, P_x)$ and $\alpha(3, J_x^*, P_x^*)$, where J_x , J_x^* , P_x and P_x^* are unknown spins and parities. SSR unambiguously fixes spins and parities of these resonances $J_x = 5/2$, $J_x^* = 7/2$, $P_x = P_x^* = +1$ as well as the structure of helicity vertices of their coupling with I=1 reggeons.

Successive application of this procedure shows that the sum rule solution requires to account two resonances with spins J = I±1/2 and positive parity in each isospin state I. The procedure to find out the general solution is analogous to procedure described in [8] for the case of I=1 reggeon scattering on nonstrange baryons. Omitting intermediate calculations the results are:

a) Residues with helicity non-flip are zero:

$$G_{\lambda\ \lambda}^{\alpha(I,J,+)\alpha_i\alpha(I',J',+)} = 0$$

b) Residues $G_{\lambda\ \lambda\pm1}^{a(I,J,+)\alpha_1 a(I',J',+)}$ satisfy relations (4.13), (4.17)--(4.18a).

c) With account of (4.13), (4.17)-(4.18a) the SSR set for single-flip residues reduces to the set of equations for six independent invariant residues $\begin{pmatrix} I & I \\ I\pm 1/2 & I+1/2 \end{pmatrix}$, $\begin{pmatrix} I & I+1 \\ I\pm 1/2 & I+1/2 \end{pmatrix}$, $\begin{pmatrix} I & I+1 \\ I+1/2 & I+3/2 \end{pmatrix}$ and

$\begin{pmatrix} I & I \\ I-1/2 & I-1/2 \end{pmatrix}$ (Here a notation $G^{a(I,J,+)\pi a(I',J',+)} \equiv \begin{pmatrix} I & I' \\ J & J' \end{pmatrix}$ is introduced). This set is given in Table 2 where $I' = I+1$, I , $I+1$; $J = I \pm 1/2$; while $X_{I_t I_s}$ are isospin crossing-matrices (see Sect. 2 and Table 1).

The solution is as follows:

$$\begin{pmatrix} I & I \\ I-1/2 & I-1/2 \end{pmatrix}' = \mu\nu\sqrt{3}(2I-1)\sqrt{\frac{I+1}{I(2I+1)}} \ ; \quad \begin{pmatrix} I & I \\ I+1/2 & I+1/2 \end{pmatrix}' = -\mu\nu\sqrt{3}\sqrt{\frac{I(2I+1)}{I+1}} \ ; \tag{4.26}$$

$$\begin{pmatrix} I & I+1 \\ I-1/2 & I+1/2 \end{pmatrix}' = \nu\frac{\sqrt{3}}{2}\sqrt{\frac{I(2I+3)}{I+1}} \ ; \quad \begin{pmatrix} I & I+1 \\ I+1/2 & I+3/2 \end{pmatrix}' = -\mu\frac{\sqrt{3}}{2}\sqrt{2I+1}$$

$$\begin{pmatrix} I & I \\ I-1/2 & I+1/2 \end{pmatrix}' = \mu\frac{\sqrt{3}}{2}\frac{1}{\sqrt{I+1}} \ ; \quad \begin{pmatrix} I & I+1 \\ I+1/2 & I+1/2 \end{pmatrix}' = \sqrt{3}\sqrt{\frac{2I+1}{(2I+3)(I+1)}} \ , \tag{4.27}$$

where $\begin{pmatrix} I & I' \\ J & J' \end{pmatrix}' = \begin{pmatrix} I & I' \\ J & J' \end{pmatrix} \Big/ G^{\Sigma\pi\Lambda}$; $\mu, \nu = \pm 1$

Note that sets of equations both for helicity and invariant residues are strongly overconstrained, therefore the fact that they have nonzero solution is rather nontrivial.

So, just like in the case with nonstrange baryons, a general solution of the problem of SSR as applied to scattering of I=1 boson reggeons on S=-1 hyperons brings to the prediction of $I \geqslant 2$ exotic hyperons existence. The difference from the case with S=0 baryons is that the minimum scheme requires to introduce two states with $J^P = (I \pm 1/2)^+$ into each isospin.

The following interesting peculiarity can be seen from formulae (4.26) and (4.27). If $\Delta I = \Delta J$, where $\Delta I = I' - I$ and $\Delta J = J' - J$, then couplings between these states tend to zero at $I \to \infty$. The picture is as if the states with $J = I \pm 1/2$ split into two chains - Λ, Σ^*, S_E^*, ..., ($J = I + 1/2$) and Σ, S_E, ..., ($J = I - 1/2$). The interaction between the particles from different chains vanishes at large I.

In the limit of small decay momenta the formulae for pion decays of $a(I, J = I \pm 1/2, +)$ states have the following form (the formula for $b \to a\pi$ decay, where a and b are baryons with arbitrary spins, is obtained in Appendix of Ref. [12]):

$$\Gamma_{\begin{pmatrix} I \\ J = I + 1/2 \end{pmatrix} \to \pi + \begin{pmatrix} I-1 \\ J = I - 1/2 \end{pmatrix}} = \left(G^{\Sigma^*\pi\Lambda}\right)^2 \frac{|\vec{P}|^3 (2I-1)}{2\pi(2I+1)} \tag{4.28}$$

Of this type is $\Sigma^* \to \Lambda\pi$ decay:

$$\Gamma_{\begin{pmatrix} I \\ J = I + 1/2 \end{pmatrix} \to \pi + \begin{pmatrix} I \\ J = I - 1/2 \end{pmatrix}} = \left(G^{\Sigma^*\pi\Lambda}\right)^2 \frac{|\vec{P}|^3}{2\pi(I+1)(2I+1)} \tag{4.29}$$

To this category belongs the $\Sigma^* \to \Sigma\pi$ decay:

$$\Gamma_{\begin{pmatrix} I \\ J = I - 1/2 \end{pmatrix} \to \pi + \begin{pmatrix} I-1 \\ J = I - 3/2 \end{pmatrix}} = \left(G^{\Sigma^*\pi\Lambda}\right)^2 \frac{|\vec{P}|^3 (I-1)}{2\pi I} \tag{4.30}$$

Note in conclusion that substitution of the found solution of SSR in FMSR results in vanishing of all latters. Some FMSR vanish due to the cancellation of contribution of possible states, others - due to zero contribution of each state separately. From the viewpoint of duality this implies the smallness of contribution of t-channel singularities to considered FMSR, at least in the masses region which corresponds to low-lying states.

5. SSR and FMSR. Derivation of Saturation Scheme from Sum Rules. Prediction of Δ_{33}, $\Sigma(1193)$ and $\Sigma^{*}(1385)$ Resonances.

To ground the saturation scheme of crossing-antisymmetric sum rules, there was used hitherto the information based on experimental data.

A question arises whether it is possible to derive saturation scheme from the sum rules themselves. The answer turns to be positive. Namely, we'll show that if we accept a scheme where the right-hand sides of all crossing-antisymmetric sum rules for scattering of I=1 reggeons on baryons are zero (the Regge contribution is small), then starting, for example, from the state with $J = I = 1/2$ and positive parity - $a(1/2, 1/2, +)$ (nucleon) we will necessarily come to a minimal scheme where the state with $I_{S(u)} = 1/2$ is saturated by the nucleon contribution, while the state with $I_{S(u)} = 3/2$ by contribution of resonance with $J^P = 3/2^+$. This resonance in its properties is identified with Δ_{33}-isobar.

In case of scattering of I=1 reggeons on baryon with $J^P = 1/2^+$ and $I = 0$ - $a(0, 1/2, +)$ (Λ-hyperon) a minimal set which saturates $I_{S(u)} = 1$ must consist of two resonances with positive parity and spins $J = 1 \pm 1/2$, which can be identified with $\Sigma(1193)$ and $\Sigma^{*}(1385)$ hyperons.

Consider sum rules for scattering on baryon $a(1/2, 1/2, +)$-nucleon:

$$\alpha_i N \rightarrow \alpha_K N \qquad (i, K = \rho, \pi, A_2) \tag{5.1}$$

and accept that the right-hand (high-energy) part of these sum rules is zero.

Reactions (5.1) proceed via two $S(u)$-channel isospin states $I_{S(u)} =$ = 1/2 and 3/2, therefore in the general case the low-energy part of sum rules is determined by contributions of resonances: N , $a(1/2, J_\ell, P_\ell) \equiv N^*_\ell$ and $a(3/2, J_\ell, P_\ell) \equiv \Delta^*_\ell$.

Consider sum rules for (2.1) at helicity transitions $1/2 \rightarrow 1/2$ and $i = K$. They represent equations for spin-nonflip residues:

$$\left(G^{N\alpha_i N}_{1/2\ 1/2}\right)^2 + \sum_{N^*_\ell}\left(G^{N\alpha_i N^*_\ell}_{1/2\ 1/2}\right)^2 - \sum_{\Delta^*_\ell}\left(G^{N\alpha_i \Delta^*_\ell}_{1/2\ 1/2}\right)^2 = 0 \tag{5.2}$$

and for residues with helicity flip by ±1:

$$\left(G^{N\alpha_i N}_{1/2\,-1/2}\right)^2 + \sum_{N^*_\ell}\left\{\left(G^{N\alpha_i N^*_\ell}_{1/2\,-1/2}\right)^2 - \left(G^{N\alpha_i N^*_\ell}_{1/2\ 3/2}\right)^2\right\} +$$

$$+ \sum_{\Delta^*_\ell}\left\{\left(G^{N\alpha_i \Delta^*_\ell}_{1/2\,-1/2}\right)^2 - \left(G^{N\alpha_i \Delta^*_\ell}_{1/2\ 3/2}\right)^2\right\} = 0 \tag{5.3}$$

Equations (5.2) and (5.3) are in t-channel isospin states $I_t = 1$ and 0, respectively.

From Eq.(5.3) there first of all follows that at least one state with $J \geqslant 3/2$ must be among N^*_ℓ and Δ^*_ℓ-states. Otherwise the residues $G^{N\alpha_i N}_{1/2\,-1/2}$ vanish, the $N\pi N$-interaction constant vanishing too.

The analysis shows that if together with N we saturate $I_{S(u)} = 1/2$ by contribution of N^* state with $J_{N^*} = 3/2$, then self-consistent is the scheme when we should simultaneously saturate $I_{s(u)} = 3/2$ by contributions of three states with spins 1/2, 3/2 and 5/2, i.e. the set which saturates the sum rules consists of five particles.

The situation turns out more simple under assumption that the state with spin $J \geqslant 3/2$ is in the set Δ^*_e . Consider the sum rules (5.2) and (5.3) and saturate $I_{s(u)} = 1/2$ by contribution of nucleon, while $I_{s(u)} = 3/2$ by contribution of one state $\Delta^*_x \equiv a(3/2, J_x, P_x)$. Then from (5.2) it follows that

$$G^{N \alpha_I \Delta^*_x}_{1/2 \quad 1/2} = 0 \tag{5.4}$$

and since residues with helicity single-flip are nonzero, and $J_x \geqslant 3/2$, then from (4.14) and (4.15) it follows immediately that

$$J_x = 3/2 \qquad \text{and} \qquad P_x = +1 \ ,$$

i.e. the Δ^*_x state has quantum numbers of Δ_{33}-resonance. Further, from (4.16) we have $M_{\Delta^*_x} = M_N$.

Hence, proceeding from saturation minimality condition we come to the prediction of state $a(3/2, 3/2, +)$.

Further, the scheme of saturation of $I_{s(u)} = 1/2$, and 3/2 by contributions of N and $a(3/2, 3/2, +)$ satisfies all sum rules for the processes (2.1) and $\alpha_i N \longrightarrow a_k(3/2, 3/2, +)$ [4], while predicted interaction properties of state $a(3/2, 3/2, +)$ [4] allow one to identify it with Δ_{33}-resonance.

Turn now to sum rules for scattering of I=1 reggeons on $a(0, 1/2, +)$ state:

$$\alpha_i \Lambda \rightarrow \alpha_\kappa \Lambda \tag{5.5}$$

Since these processes proceed via isospin state of t-channel $I_t = 0$, then for them there takes place only the sum rule (2.6) which is saturated by the set of states

$$\left\{ a(1, J_\ell, P_\ell) \right\} \equiv \left\{ \Sigma^*_\ell \right\}$$

In accordance with the logics described in Introduction, consider together with reactions (5.5) the processes

$$\alpha_i \Lambda \rightarrow \alpha_\kappa \Sigma^*_\ell \tag{5.6}$$

and

$$\alpha_i \Sigma^*_\ell \rightarrow \alpha_\kappa \Sigma^*_m \tag{5.7}$$

where $\Sigma^*_{\ell,m}$ are resonances from the $\left\{ \Sigma^*_\ell \right\}$ set that saturates the sum rules for (5.5).

Note that, in principle, the states with $I_{S(u)} = 2$ contribute to processes (5.7). Using, however, the fact that the spin structure of these contributions is identical at $I_t = 0$ and 2, one can pass to sum rules which are determined by contributions of states with $I_{S(u)} = 0$ and 1. We shall give only the result of saturation analysis which consists in the fact that self-consistent is a minimal scheme where $I_{S(u)} = 0$ is saturated by Λ - resonance contribution, while $I_{S(u)} = 1$ by contributions of two states - a(1, 1/2, +) and a(1, 3/2, +), whose quantum numbers are unambiguously determined from sum rules. Just as in case with N and Δ_{33}, the solution resulting from the sum rules is self-consistent in the limit when masses of these states are equal to the mass of Λ -resonance.

The states $\alpha(0, 1/2, +)$, $\alpha(1, 1/2, +)$ and $\alpha(1, 3/2, +)$ coincide in quantum numbers with Λ, $\Sigma(1193)$ and $\Sigma^*(1385)$ hyperons. Moreover, the sum rule predictions describe correctly experimentally studied properties of interaction of $\Lambda(1115)$, $\Sigma(1193)$ and $\Sigma^*(1385)$ hyperons.

Notice that the notion of strangeness was absent in our analysis. In each of considered cases the minimal saturation scheme is a consequence of spin and isospin algebraic structure of sum rules. This fact is rather intriguing, since experimental schemes of saturation of amplitudes $B^{(+)}_{\pi N}$ (by nucleon and Δ_{33}-isobar) and $B^{(+)}_{\pi\Lambda}$ (by Σ and Σ^*-hyperon) represent, at first sight, the SU(3)-scheme of saturation by octet and decouplet.

6. Scattering of I=1 Reggeons on Hyperons with S = -2 and -3 Strangeness.

6.1. Scattering on S=-2 Hyperons.

Since isospin of I_{Ξ} is equal to I_N, then the minimal self-consistent saturation scheme in this case is the same as for the scattering on nucleons, i.e. the state with $I_{S(u)} = 1/2$ is saturated by Ξ hyperon contribution, while $I_{S(u)} = 3/2$ by contribution of resonance with $J = I = 3/2$ (since $S = -2$, this resonance is exotic). Thus there is the sum rule solution which coincides with one found in Ref. [8].

At the same time it seems likely that in the sum rules in $I_{S(u)} = 1/2$ a contribution comparable in magnitude to that of Ξ is made by $\Xi^*(1530)$ ($J^P = 3/2^+$) - isobar. This possibility is indicated, in particular, by calculation of ratio of constants $\Xi\pi\Xi$ and $\Xi^*\pi\Xi$ in the framework of SU(6). Absence of sufficient information does not allow one to carry out a correct analysis of resonance contribution to sum rules for amplitude

$B^{(+)}_{\pi\Xi}$ of $\pi\Xi \to \pi\Xi$ scattering in order to check the saturation scheme. In this connection, we'll consider here a case when to the sum rules for processes

$$\alpha_i \Xi \to \alpha_K \Xi \tag{6.1}$$

Ξ and Ξ^* hyperons contribute simultaneously.

Without going into details of the analysis, note that at chosen input (when to $I_{S(u)} = 1/2$ Ξ and Ξ^* contribute) the minimal saturation scheme requires to take into account in $I_{S(u)} = 3/2$ the contributions of three states with spin-parity $1/2^+$, $3/2^+$ and $5/2^+$ - $a(3/2, 1/2, +)$, $a(3/2, 3/2, +)$ and $a(3/2, 5/2, +)$. The nontrivial solution of the set of equations for helicity residues $G^{c\alpha_i d}_{\lambda_c \lambda_d}$ (c, d = $a(1/2, 1/2, +)$, $a(1/2, 3/2, +)$, $a(3/2, 1/2, +)$, $a(3/2, 3/2, +)$, $a(3/2, 5/2, +)$; $i = \pi, \rho, A_2$) has the form analogous to the previous cases:

a) Residues with helicity non-flip are zero.

b) Residues with single-flip obey the relations (4.17) and (4.18).

c) For invariant functions $G^{c\pi d}/G^{\Xi\pi\Xi}$ two solutions are predicted. These solutions are given in Table 3. Proceeding with the procedure of self-consistent consideration of sum rules for scattering of I=1 reggeons on states with I = 3/2 etc. we are necessitated to introduce states with $I \geq 5/2$. In this case the number of states at the given isospin depends on the choice of the solutions given in Table 3. In case of the first solution the necessary number of states is three at any I . Parities of these states are positive, while spins are equal to I - 1, I and I + 1.

The situation is different if we start from the second solution. In this case self-consistency of the scheme requires introduction of (2I + 3)/2 resonances in the given isospin state I . Spins of these resonances take the values 1/2, 3/2, 5/2,..., I + 1.

Relations for coupling constants predicted from sum rules in case of the first solution are given in Table 4.

6.2. Scattering on S=-3 Hyperons.

In the framework of hadron systematics the states which contribute to $S(u)$-channel of processes

$$\alpha_i \Omega \to \alpha_K \Omega \qquad (6.2)$$

are exotic with quark composition $SSSq\bar{q}$. From the viewpoint of our sum rules these are states with isospin $I_{S(u)} = 1$, contributing to scattering of I=1 reggeons on baryons with I=0 and $J^P = 3/2^+$. The analysis shows that a minimal self-consistent saturation scheme requires the account of contribution of three states with spins 1/2, 3/2 and 5/2 and positive parity in (6.2) processes. Helicity vertices of interaction of these states have a solution analogous to the previous cases (nonflip residues are zero, and residues with $\Delta\lambda = \pm 1$ obey the relations (4.17) and (4.18). The solution for invariant vertices is given in Table 5.

7. Conclusion.

The present paper completes the analysis of application of supercovergent sum rules for αa -amplitudes to scattering of nonstrange boson reggeons with I = 0.1 ($\mathbb{P}, f, \omega, \rho, \pi, A_2$) on baryons. The main results of the analysis are as follows:

a) In considering SSR for amplitudes of reggeon scattering on low-lying states - N, Δ_{33}, Λ, Σ, $\Sigma^*(1385)$, there works a logically closed bootstrap scheme of sum rules saturation by contributions of the same states.

Predictions of such scheme agree well with all experimentally measured quantities (there are correctly described the helicity structure of residues, the relations between vertices, widths of pion decays).

A simple classification of helicity residues of Regge poles arises. So for I=0 reggeons the SSR solution has the form $G_{\lambda_a\ \lambda_b}^{a\alpha_j^{I=0}b} = A^j\delta_{ab}\,\delta_{\lambda_a\lambda_b}$ [5]. In case of I=1 reggeons the nonflip residues are zero. All residues

$$G_{\lambda\ \lambda\pm1}^{a\alpha_j^{I=1}\Delta_{33}}\ (a = N, \Delta_{33}) \quad \text{and} \quad G_{\lambda\ \lambda\pm1}^{a\alpha_j^{I=1}b}\ (a, b = \Lambda, \Sigma, \Sigma^*)$$

are expressed through $G_{1/2\ -1/2}^{N\alpha_j^{I=1}N}$ and $G_{1/2\ -1/2}^{\Sigma\alpha_j^{I=1}\Lambda}$ residues, respectively; the residues of ρ and A_2 -reggeons with $\Delta\lambda = \pm1$ are connected via simple phase relations with the π -pole residues. The relations between helicity transitions with $\Delta\lambda = \pm1$ in vertices $a\alpha_\pi b$ are the same as in the Rarita-Schwinger formalism (see [8] and this paper).

b) The sets of equations for residues following from SSR for αa -amplitudes turn out strongly overconstrained. This is connected, first, with the inclusion of high-spin particles and, second, with simultaneous consideration of $\mathbb{P}$, ω, f, ρ, A_2 and π -trajectory scattering, this resulting in sharp increase in number of superconvergent relations in comparison, e.g. with the case of real pion scattering.

Despite the overconstrainedness, there is a nonzero solution in each of the considered cases, this being a rather intriguing circumstance. Most likely, this indicates the presence of algebra to which the Regge residues obey (see item (d) of this Section).

c) Successive application of SSR for I=1 reggeon-baryon scattering together with the saturation scheme self-consistency condition leads to the prediction of the series of baryons with increasing spins and isospins.

For the case of nonstrange baryons (half-integer isospins) this series consists of $a(I, J=I, P=+1)$ states with $I \geqslant 5/2$, while for $S = -1$ hyperons (integer isospins) there arise two chains of states - $a(I, J=I \pm 1/2, P=+1)$, where $I \geqslant 2$. In the quark model these states are exotic. In our approach these states are close relatives of the corresponding nonexotic baryons $N, \Delta_{33}, \Lambda, \Sigma, \Sigma^*$, i.e. the lowest states in the world with given isospin. Correspondingly, it is expected that at each isospin there exists a rich spectrum of excited exotic baryons, in particular, the Regge trajectories on which the predicted states lie.

The question on whether exotic baryons exist or they are an artefact of the considered approach remains open so far and requires experimental solution. At the same time note that exotic baryons appear in many theoretical schemes - strong coupling theory [14-16], Chew-Low static model [9, 17-19], dual [20,21] and string [22] models, bag model [23-26], soliton baryon model [27].

d) In Ref. [9] the strong coupling limit in Chew-Low static model as applied to pion-baryon scattering was considered. It was shown that at $\lambda \to \infty$ (λ is coupling constant) the commutative relations take place:

$$[A_{i\alpha}, A_{j\beta}] = 0 \tag{7.1}$$

where $A_{\kappa\gamma}$ are meson sources that represent the tensors in space of joined representations of spin and isospin groups ($i, j; \alpha, \beta = 1, 2, 3$). The relation (4.1) together with relations

$$\begin{aligned} &[I_\alpha, I_\beta] = i\varepsilon_{\alpha\beta\gamma} I_\gamma ; \quad [J_i, J_j] = i\varepsilon_{ijk} J_k ; \\ &[I_\alpha, A_{i\beta}] = i\varepsilon_{\alpha\beta\gamma} A_{i\gamma} ; \quad [J_i, A_{\alpha j}] = i\varepsilon_{ijk} A_{\alpha k} \end{aligned} \tag{7.2}$$

where I_ρ and J_ϱ are generators of spin and isospin groups, determine the

Lie algebra of group

$$G = SU_I(2) \otimes SU_J(2) \times T_g \tag{7.3}$$

where T_g is the group of translations.

So long as G is noncompact group, then its unitary representations by which hadrons are classified are infinite-dimensional. Among irreducible representations of G -group, there are two representations - with $J = I = \frac{2n+1}{2}$ [5] and $J = I \pm 1/2$ (I is integer) [19] , i.e. series of baryons coinciding with the series predicted in 8 and this paper. In Refs. [18,19] were studied predictions of the group for the interaction of baryons from these representations: decay widths, isovector electromagnetic coupling constants, etc. These predictions also coincide with our results. At the same time our sum rules and the bootstrap nonrelativistic model are based on different dynamical approaches; therefore the algebra (if it exists) our vertices obey may differ from the algebra of group G which corresponds to the static model. This question needs a separate study.

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Table 1

$\begin{Bmatrix} 1 & 1 & 2 \\ I_\alpha & I_\beta & I_s \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 & 1 \\ I_\alpha & I_\beta & I_s \end{Bmatrix}$	$\begin{Bmatrix} 1 & 1 & 1 \\ I_\alpha & I_\beta & I_s \end{Bmatrix}$
$\begin{Bmatrix} 1 & 1 & 1 \\ I & I & I \end{Bmatrix} = \sqrt{\frac{(2I+3)(2I-1)}{5}}\,K$	$\begin{Bmatrix} 1 & 1 & 1 \\ I & I & I-1 \end{Bmatrix} = -(I+1)K$	$-\begin{Bmatrix} 1 & 1 & 0 \\ I & I & I \end{Bmatrix} =$
$\begin{Bmatrix} 1 & 1 & 2 \\ I & I & I+1 \end{Bmatrix} = I\sqrt{\frac{2I-1}{5(2I+3)}}\,K$	$\begin{Bmatrix} 1 & 1 & 1 \\ I & I & I+1 \end{Bmatrix} = IK$	$= \begin{Bmatrix} 1 & 1 & 0 \\ I & I & I+1 \end{Bmatrix} =$
$\begin{Bmatrix} 1 & 1 & 2 \\ I & I & I-1 \end{Bmatrix} = (I+1)\sqrt{\frac{2I+3}{5(2I-1)}}\,K$	$\begin{Bmatrix} 1 & 1 & 1 \\ I & I+1 & I+1 \end{Bmatrix} = -\sqrt{\frac{I(2I+1)(I+2)}{2I+3}}\,K$	$= \begin{Bmatrix} 1 & 1 & 0 \\ I & I & I-1 \end{Bmatrix} =$
$\begin{Bmatrix} 1 & 1 & 2 \\ I & I+1 & I+1 \end{Bmatrix} = -I\sqrt{\frac{3(2I+1)}{5(2I+3)}}\,K$		$= \sqrt{2I(I+1)}\,K$
$\begin{Bmatrix} 1 & 1 & 2 \\ I & I+1 & I \end{Bmatrix} = -\sqrt{\frac{3I(I+2)}{5}}\,K$	$K = \begin{Bmatrix} 1 & 1 & 1 \\ I & I & I \end{Bmatrix}$	

Table 2

REACTIONS	I_t	EQUATIONS
$\alpha_\pi a(I,J,+) \to \alpha_\pi a(I',J,+)$	0, 2	$\sum_{I_d} X_{I_t I_d}\left[\begin{pmatrix} I & I_d \\ J & J-1 \end{pmatrix}\begin{pmatrix} I_d & I' \\ J-1 & J \end{pmatrix} + \frac{1}{4J}\begin{pmatrix} I & I_d \\ J & J \end{pmatrix}\begin{pmatrix} I_d & I' \\ J & J \end{pmatrix} + \frac{J(2J+3)}{(J+1)(2J+1)}\begin{pmatrix} I & I_d \\ J & J+1 \end{pmatrix}\begin{pmatrix} I_d & I' \\ J+1 & J \end{pmatrix}\right]=0$
	1	$\sum_{I_d} X_{1 I_d}\left[\begin{pmatrix} I & I_d \\ J & J-1 \end{pmatrix}\begin{pmatrix} I_d & I' \\ J-1 & J \end{pmatrix} - \frac{(2J-1)}{4J}\begin{pmatrix} I & I_d \\ J & J \end{pmatrix}\begin{pmatrix} I_d & I' \\ J & J \end{pmatrix} + \frac{J(2J-1)}{(J+1)(2J+1)}\begin{pmatrix} I & I_d \\ J & J+1 \end{pmatrix}\begin{pmatrix} I_d & I' \\ J+1 & J \end{pmatrix}\right]=0$
$\alpha_\pi a(I,J,+) \to \alpha_\pi a(I',J+1,+)$	0, 2	$\sum_{I_d} X_{I_t I_d}\left[\begin{pmatrix} I & I_d \\ J & J \end{pmatrix}\begin{pmatrix} I_d & I' \\ J & J+1 \end{pmatrix} + \frac{J+2}{J+1}\begin{pmatrix} I & I_d \\ J & J+1 \end{pmatrix}\begin{pmatrix} I_d & I' \\ J+1 & J+1 \end{pmatrix}\right] = 0$
	1	$\sum_{I_d} X_{1 I_d}\left[\begin{pmatrix} I & I_d \\ J & J \end{pmatrix}\begin{pmatrix} I_d & I' \\ J & J+1 \end{pmatrix} - \frac{J}{J+1}\begin{pmatrix} I & I_d \\ J & J+1 \end{pmatrix}\begin{pmatrix} I_d & I' \\ J+1 & J+1 \end{pmatrix}\right] = 0$
$\alpha_\pi a(I,J,+) \to \alpha_\pi a(I',J+2,+)$	1	$\sum_{I_d} X_{1 I_d}\begin{pmatrix} I & I_d \\ J & J+1 \end{pmatrix}\begin{pmatrix} I_d & I' \\ J+1 & J+2 \end{pmatrix} = 0$

Table 3

Vertex	I Solution	II Solution
$\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 3/2 \end{pmatrix}'$	$\sqrt{3}$	$\frac{\sqrt{3}}{2}$
$\begin{pmatrix} 1/2 & 3/2 \\ 1/2 & 1/2 \end{pmatrix}'$	$\alpha 2\sqrt{2}$	$\mu\sqrt{2}$
$\begin{pmatrix} 1/2 & 3/2 \\ 1/2 & 3/2 \end{pmatrix}'$	$-\beta\frac{\sqrt{15}}{2}$	$\nu\sqrt{\frac{3}{2}}$
$\begin{pmatrix} 1/2 & 1/2 \\ 3/2 & 3/2 \end{pmatrix}'$	3	$-\frac{3}{5}$
$\begin{pmatrix} 1/2 & 3/2 \\ 3/2 & 1/2 \end{pmatrix}'$	$-\alpha\frac{\sqrt{3}}{2\sqrt{2}}$	$\mu\sqrt{\frac{3}{2}}$
$\begin{pmatrix} 1/2 & 3/2 \\ 3/2 & 3/2 \end{pmatrix}'$	$\beta\frac{6}{\sqrt{5}}$	$-\nu\frac{3\sqrt{2}}{5}$
$\begin{pmatrix} 3/2 & 3/2 \\ 1/2 & 1/2 \end{pmatrix}'$	$\sqrt{10}$	$-\sqrt{\frac{2}{5}}$
$\begin{pmatrix} 3/2 & 3/2 \\ 3/2 & 3/2 \end{pmatrix}'$	$-\frac{33}{5}\sqrt{\frac{2}{5}}$	$\frac{3}{5}\sqrt{\frac{2}{5}}$
$\begin{pmatrix} 3/2 & 3/2 \\ 1/2 & 3/2 \end{pmatrix}'$	$\alpha\beta\sqrt{3}$	$-\mu\nu\sqrt{\frac{3}{10}}$

Table 4

$\begin{pmatrix} I & I \\ I & I+1 \end{pmatrix}' = \frac{3}{2}\sqrt{\frac{3}{2}}\ \frac{\sqrt{2I+1}}{I+1}$
$\begin{pmatrix} I & I+1 \\ I & I \end{pmatrix}' = \alpha 3\sqrt{\frac{3}{2}}\sqrt{\frac{I(2I+3)}{(I+1)^3}}$
$\begin{pmatrix} I & I+1 \\ I & I+1 \end{pmatrix}' = -\beta\frac{3}{2}\sqrt{\frac{3}{2}}\ \frac{\sqrt{I(I+2)(2I+1)}}{I+1}$
$\begin{pmatrix} I & I \\ I+1 & I+1 \end{pmatrix}' = 3\sqrt{\frac{3}{2}}\sqrt{\frac{I(2I+1)}{I+1}}$
$\begin{pmatrix} I & I+1 \\ I+1 & I \end{pmatrix}' = -\alpha\frac{3}{2}\sqrt{\frac{3}{2}}\ \frac{1}{(I+1)\sqrt{2I+3}}$
$\begin{pmatrix} I & I+1 \\ I+1 & I+1 \end{pmatrix}' = \beta 3\sqrt{\frac{3}{2}}\sqrt{\frac{2I+1}{(I+1)(I+2)}}$
$\begin{pmatrix} I+1 & I+1 \\ I & I \end{pmatrix}' = 3\sqrt{\frac{3}{2}}\ \frac{I}{I+1}\sqrt{\frac{(I+2)(2I+3)}{I+1}}$
$\begin{pmatrix} I & I \\ I & I \end{pmatrix}' = 3\sqrt{\frac{3}{2}}\ \frac{1-I-I^2}{I+1}\sqrt{\frac{2I+1}{I(I+1)}}$
$\begin{pmatrix} I+1 & I+1 \\ I & I+1 \end{pmatrix}' = \alpha\beta\frac{3}{2}\sqrt{\frac{3}{2}}\ \frac{\sqrt{2I+1}}{I+1}$
$\begin{pmatrix} I & I+1 \\ I-1 & I \end{pmatrix}' = \beta\frac{3}{2}\sqrt{\frac{3}{2}}\sqrt{\frac{(2I-1)(2I+3)}{2I+1}}$

Table 5

$G^{\Omega\pi a(1,3/2,+,S=-3)} = \alpha\, 2\sqrt{\frac{6}{5}}\, g$
$G^{\Omega\pi a(1,5/2,+,S=-3)} = -\beta\sqrt{2}\, g$
$G^{a(1,1/2,+,S=-3)\pi a(1,1/2,+,S=-3)} = \alpha\, 2\sqrt{\frac{1}{3}}\, g$
$G^{a(1,3/2,+,S=-3)\pi a(1,3/2,+,S=-3)} = \alpha\,\frac{4}{5}\sqrt{3}\, g$
$G^{a(1,5/2,+,S=-3)\pi a(1,5/2,+,S=-3)} = \alpha\, 2\sqrt{3}\, g$
$G^{a(1,1/2,+,S=-3)\pi a(1,3/2,+,S=-3)} = \sqrt{\frac{5}{2}}\, g$
$G^{a(1,3/2,+,S=-3)\pi a(1,5/2,+,S=-3)} = \beta\, 3\sqrt{\frac{1}{5}}\, g$
$g \equiv G^{\Omega\pi a(1,1/2,+,S=-3)}\ ; \quad \alpha\beta = \pm 1$

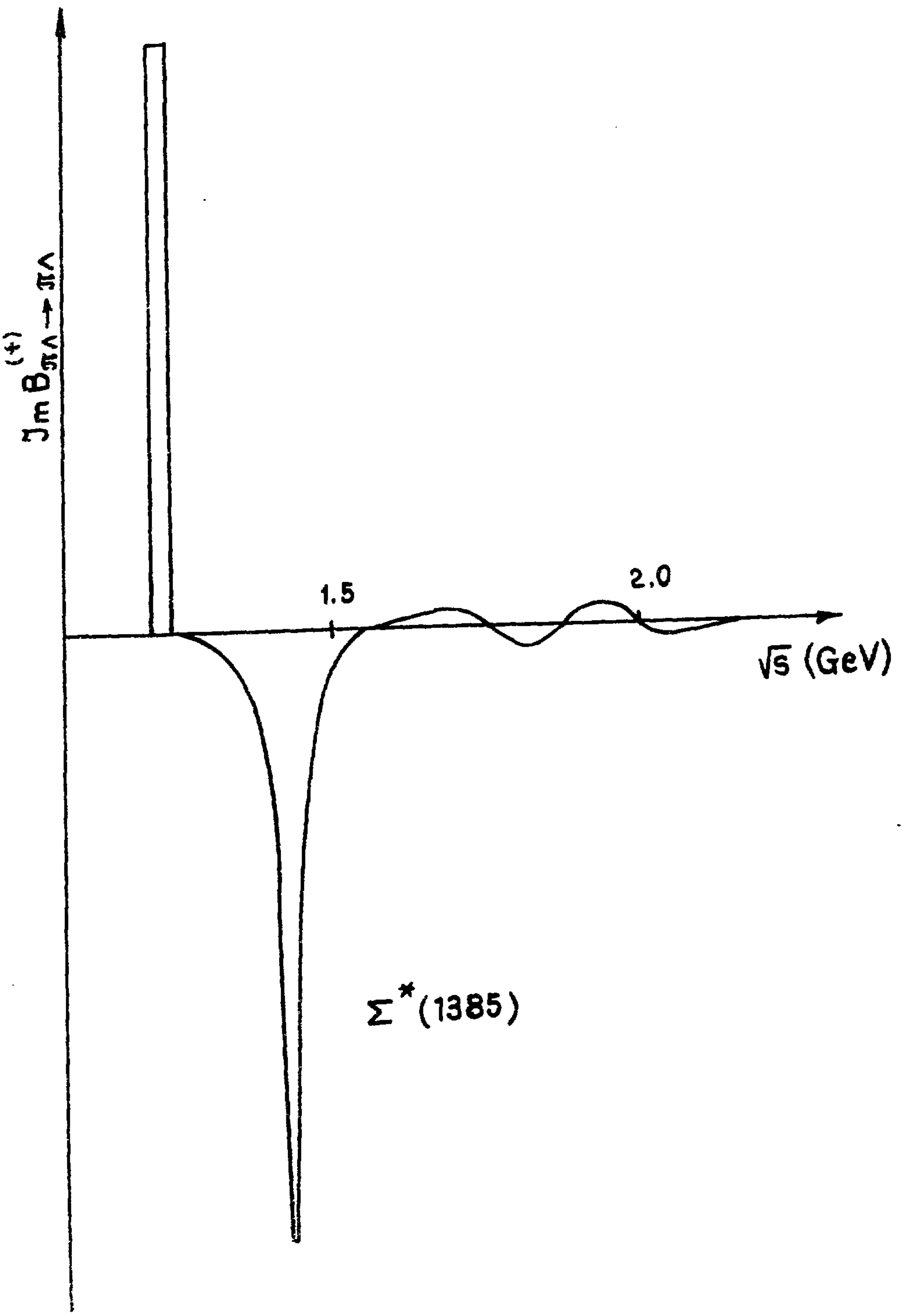

Fig. 1

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ОБЩЕЕ РЕШЕНИЕ СВЕРХСХОДЯЩИХСЯ ПРАВИЛ СУММ ДЛЯ РАССЕЯНИЯ

I = I РЕДЖЕОНОВ НА БАРИОНАХ

(на английском языке, перевод З.Н.Асланян)

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