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NONLINEAR EFFECTS IN PLASMA WAKE FIELD ACCELERATION

ЦНИИатоминформ

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ՈՉ ԳԾԱՅԻՆ ԷՖԵԿՏՆԵՐ ՊԼԱԶՄԱՅՈՒՄ ԿԻԼՎԱՏԵՐԱՅԻՆ ԱԼԻՔՆԵՐՈՎ ԱՐԱԳԱՑՄԱՆ ԺԱՄԱՆԱԿ

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Աշխատանքում հետազոտվում է լիցքավորված մասնիկների արագացման համար պլազմայում ռելյատիվիստական էլեկտրոնների փնջերով գրգռված՝ երկայնակի ալիքների օգտագործման հնարավորության մասին։ Գտնված են արտահայտություններ մասնիկների փնջի ներսում և նրանից դուրս ալիքների դաշտերի և երկարության համար։ Ցույց է տրված, որ երբ փնջի խտությունը մոտենում է պլազմայի էլեկտրոնների հավասարակշիռ խտության կեսին՝ դաշտի առավելագույն լարվածությունը կիլվատերային ալիքում համեմատական է $\gamma^{1/2}$ կամ γ /կախված՝ χ - փնջի երկարությունից/, և կարող է կիրառվել լիցքավորված մասնիկների արագացման համար։

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A.Ts.AMATUNI, S.S. ELBAKIAN,
E.V. SEKHPOSSIAN

NONLINEAR EFFECTS IN PLASMA WAKE FIELD ACCELERATION

The possible use of the nonlinear wake waves excited by electron bunches passing through plasma for acceleration of charged particles is investigated. The expressions for non - linear fields and wave lengths inside and behind the particle bunch are obtained. It is shown that when the bunch density approaches the half of the plasma electron equilibrium density the maximal wake field is proportional to $\gamma^{1/2}$ or γ depending on the bunch length, and one can use it for charged particle acceleration.

Yerevan Physics Institute

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НЕЛИНЕЙНЫЕ ЭФФЕКТЫ ПРИ УСКОРЕНИИ КИЛЬВАТЕРНЫМИ ВОЛНАМИ В ПЛАЗМЕ

Рассматривается возможность использования для ускорения заряженных частиц нелинейной кильватерной волны, возбуждаемой в плазме электронным сгустком. Получены выражения для полей и длин волн внутри и за сгустком. Показано, что когда плотность электронов сгустка приближается к половине равновесной плотности электронов плазмы, максимальная напряженность поля нелинейной кильватерной волны в плазме зависит от γ_0 - фактора электронного сгустка, что может быть использовано для ускорения заряженных частиц.

Ереванский физический институт

Ереван 1896

Introduction

The idea to accelerate particles by high gradient fields, using longitudinal wake waves in plasma, excited by moving electron bunches, was initially introduced by Y.B.Fainberg in 1956 [1].

Later on some experimental results were obtained by the Kharkov group [2,3], where a considerable amount of electrons with energy up to 24 MeV was detected, when electrons with initial energy 20 MeV passed the plasma column with the density $n_0 = 10^{15}-10^{17}cm^{-3}$.

In 1984-85 the interest to plasma wake field acceleration (PWFA) was renewed as an alternative to the plasma beat wave acceleration-PBWA [4] in a series of works carried out at SLAC and UCLA[5-13,15]. It was shown [8] , in particular, that for a given accelerating gradient, the plasma wake field accelerator has a higher efficiency and a lower total energy for the driving beam, compared to PBWA.

There is a considerable amount of publications devoted to accelerator physics issues, such as the transformation ratio, efficiency, energy spread, focusing etc.[7-15].. It was shown, in particular, that by a specialized shaping of the driving beams charge distribution, one can increase the transformation ratio and the efficiency, and decrease the energy spread.

Plasma wake field acceleration will be stadied experimentally at Argonne National Laboratory [16-18] in collaboration with the University of Wisconsin team, using the existing intense beam (10^{10}e/pulse) of the short pulse (10-100ps), electron linac (energy 22 MeV). At a specially constructed new facility the secondary driven beam with lower intensity and energy (15 MeV) will be obtained and then accelerated in a 10-20 cm long plasma column, with plasma electron density $n_0 \sim 10^{13} cm^{-3}$

The plasma wake field acceleration scheme is more or less completely reviewed in a series of excellent papers [19-23,8].

Nevertheless, it is necessary to mention that in almost all the previously cited calculations and computer simulations the estimates have been obtained in the linear approximation when the plasma density perturbations caused by the driving beam are small $|\Delta n| \ll n_0$, or $n_B \ll n_0$, where n_B is the density of the driving beam. Starting from 1977 some attempts to take into account nonlinear effects of wake field generation by electron bunches have been taken by the Yerevan group[24-28]. Later on in [7] the model of an infinitely thin, charged plane moving in plasma was also considered (cp[24]) in nonlinear approximation. In publication [24-28] the problem of the excitation of nonlinear longitudinal stationary waves in plasma by a charged plane and electron bunches with the infinite transverse dimensions and finite thickness has been considered, and it has been shown that for certain conditions, namely, when the electron density n_B in the bunch approaches the half of the plasma electron density, $n_0/2$, the excited longitudinal wake field appears to be proportional to the square root of bunch's γ-factor and may provide a sufficiently high acceleration rate.

So the qualitative and quantitative importance of the nonlinear treatment of PWFA was demonstrated. However, it was carried out within a model which needs some improvements.

In [25-28] it has been assumed that the electric field inside the bunch is equal to zero, i.e. that a complete charge and current neutralization of the bunch is established.

In present work the interaction of an electron bunch with plasma is considered without the above mentioned assumption. The expressions for the longitudinal fields inside and behind the particle bunch are obtained for different conditions. These expressions may be used in planning the experiments on plasma wake field acceleration (see, for exp. [16-18]).

Exact Nonlinear Analytical Solution of the Model Problem

Let us consider a monoenergetic bunch with constant electron density n_B, finite length d and relativistic velocity v_0 moving along the z axis in a cold, homogeneous, collisionless plasma with motionless ions. It is assumed that the transverse dimensions of the bunch are infinite.

We shall further consider longitudinal waves ($E_x=E_y=0, E_z\neq 0$) and look for stationary solutions when all the unknown quantities are functions of a single variable $\tilde{z}=z-v_0t$, were v_0 equal to the phase velocity of the excited wave. Then the complete system of hydrodynamical and Maxwell's equations describing the interaction of the bunch with plasma reduces to an equation for the z-component of the dimensionless momentum $\rho_z = P_z/mc$ of plasma electrons [26].

$$\frac{d^2}{d\tilde{z}^2}\left(\beta\rho_z - \sqrt{1+\rho_z^2}\right) + \frac{\omega_p^2 \rho_z}{c^2\left(\beta\sqrt{1+\rho_z^2} - \rho_z\right)} + \tag{1}$$

$$+ \frac{\omega_b^2}{c^2}\left[\vartheta(\tilde{z}) - \vartheta(\tilde{z}-d)\right] = 0 ,$$

where $\beta = v_0/c$, $\omega_p^2 = 4\pi e^2 n_0/m$, $\omega_b^2 = 4\pi e^2 n_b/m$ and ϑ is the Heaviside's step function. The dimensionless momentum ρ_z is related to the longitudinal field E_z through the equation of motion

$$\left(\beta - \frac{\rho_z}{\sqrt{1+\rho_z^2}}\right)\frac{d\rho_z}{d\tilde{z}} = \frac{eE_z}{mc^2} \tag{2}$$

The plasma electron density $n(\tilde{z})$ satisfies the continuity equation

$$\frac{d}{d\tilde{z}}\left(nv - nv_0\right) = 0 \; ; \quad \left(v = \frac{\rho_z c}{\sqrt{1+\rho_z^2}}\right) . \tag{2a}$$

First let us obtain the solution of the problem inside the bunch assuming continuity of the z-components of momentum $\rho_z(\tilde{z})$ and the field $E_z(\tilde{z})$ on the front boundary of the bunch $\tilde{z} = d$ (further the index z will be omitted). Since there is no plasma excitation in front of the bunch the boundary conditions take the form: $E(d) = 0$, $\rho(d) = 0$, $n(d) = n_0$ These conditions ensure the zero solution of the pr⁻ ıem in the re - gion $\tilde{z} \geqslant d$ - in front of the bunch. Inte ating equation (1) with the same boundary conditions we obtain the following expressions for $\rho(\tilde{z})$, $E^{6}(\tilde{z})$ and $n(\tilde{z})$ inside the bunch:

$$\frac{\omega_p}{c}\sqrt{2}(d-\tilde{z}) = \pm\int_{\rho}^{0}\frac{(\beta\sqrt{1+\rho^2}-\rho)\,d\rho}{\sqrt{1+\rho^2}\left[\left(1-\frac{n_6}{n_0}\right)\left(1-\sqrt{1+\rho^2}\right)-\frac{n_6}{n_0}\beta\rho\right]^{1/2}}, \qquad (3)$$

$$E^{6}(\tilde{z}) = \pm\frac{m\omega_p c}{e}\sqrt{2}\left[\left(1-\frac{n_6}{n_0}\right)\left(1-\sqrt{1+\rho^2}\right)-\frac{n_6}{n_0}\beta\rho\right]^{1/2}, \qquad (4)$$

$$n(\tilde{z}) = \frac{n_0\beta\sqrt{1+\rho^2}}{\beta\sqrt{1+\beta^2}-\rho} \geqslant 0, \qquad (5)$$

where $0 \leqslant z \leqslant d$, $-\rho_0 \leqslant \rho(\tilde{z}) \leqslant 0$ and ρ_0 is the maximum allowed value of the absolute magnitude of momentum inside the bunch, equal to

$$\rho_0 = \frac{2a\beta}{1-a^2\beta^2}, \qquad (6)$$

where $a = \frac{n_6/n_0}{1-n_6/n_0} < 1$, $\frac{n_6}{n_0} \leqslant 1/2$. In (3) and (4) it is necessary to take the positive sign if $\rho(\tilde{z})$ grows with the increase of $\tilde{z}$, otherwise the negative sign must be chosen. The integration of (3) leads to the following implicit dependence of the longitudinal momentum ρ on $\tilde{z}$ inside the bunch:

$$d-\tilde{z}(\rho) = \pm\frac{c\sqrt{2}}{\omega_p}\,\frac{\sqrt{1+\sqrt{1-a^2\beta^2}}}{(1-n_6/n_0)^{1/2}(1-a^2\beta^2)}\cdot$$

$$\cdot\Big\{\beta(1+a)\Big[E\Big(\mathrm{arctg}\frac{\mathrm{sh}\,\Psi}{\sqrt{1-K^2\mathrm{ch}^2\Psi}},\sqrt{1-K^2}\Big)+E\Big(\frac{\pi}{2},\sqrt{1-K^2}\Big)- \qquad (7)$$

$$-\frac{K^2}{2}F\Big(\mathrm{arctg}\frac{\mathrm{sh}\,\Psi}{\sqrt{1-K^2\mathrm{ch}^2\Psi}},\sqrt{1-K^2}\Big)-\frac{K^2}{2}F\Big(\frac{\pi}{2},\sqrt{1-K^2}\Big)-$$

$$-\mathrm{th}\,\Psi\cdot\sqrt{1-K^2\mathrm{ch}^2\Psi}\Big]-(1+a\beta^2)\sqrt{1-K^2\mathrm{ch}^2\Psi}\Big\} \equiv I(-\rho),$$

where $\Psi=(\lambda-\alpha)/2$, $\lambda=\mathrm{arsh}(-\rho)$, $\alpha=\mathrm{arch}(1/\sqrt{1-a^2\beta^2})$, F and E are the elliptic integrals of the first and second kinds respectively and $K=(2\sqrt{1-a^2\beta^2}/(1+\sqrt{1-a^2\beta^2}))^{1/2}$. As it is seen from (4) the field $E(\tilde{z})$ becomes zero for $\rho(\tilde{z})=0$ and

$\rho(\tilde{z}) = -\rho_o$ and reaches an extremum value equal to

$$E^{\beta}_{max} = -E^{\beta}_{min} = \frac{m\omega_p c}{e}\sqrt{2}\left(1-\frac{n_\beta}{n_o}\right)^{1/2}\left[1-\sqrt{1-a^2\beta^2}\,\right]^{1/2} \tag{8}$$

at

$$\rho(\tilde{z}) = -\rho_m = \frac{a\beta}{\sqrt{1-a^2\beta^2}} \,. \tag{9}$$

If one chooses the bunch such that $\rho(\tilde{z})$ is equal to $-\rho_o$ or $-\rho_m$ on the rear boundary $(\tilde{z}=0)$ then the bunch length will be determined by the following expressions:

$$d = (2\tau_1 + 1)\frac{\tilde{z}^{\beta}_{\lambda}}{2} \qquad \text{for} \quad \rho(\tilde{z}=0) = -\rho_o \tag{10a}$$

$$d = \begin{cases} I(|\rho_m|)\,, \\ \tau_2 \tilde{z}^{\beta}_{\lambda} \pm I(|\rho_m|) \end{cases} \qquad \text{for} \quad \rho(\tilde{z}=0) = -\rho_m \tag{10b}$$

where $\tilde{z}^{\beta}_{\lambda} = 2I(|\rho_o|)$ and τ_1 and τ_2 are positive integers beginning from zero and one, respectively. For $n_\beta \approx n_o/2\,(a \to 1)$ the bunch length depends on the γ - factor since in this case in (10a) and (10B) $z^{\beta}_{\lambda} \approx 16 v_o \gamma^2/\omega_p$, $I(|\rho_m|) = 2 v_o \gamma/\omega_p$.
For $n_\beta/n_o \ll 1$ one has $\tilde{z}^{\beta}_{\lambda} = 2\pi v_o/\omega_p$ and $I(|\rho_m|) = \pi v_o/2\omega_p$
The maximum field inside the bunch is $|E^{\beta}_{max}| \approx m\omega_p c/e$ and $|E^{\beta}_{max}| \approx \frac{m\omega_p c}{e}\frac{n_\beta}{n_o}$ for $n_\beta \approx n_o/2$ and $n_\beta/n_o \ll 1$ respectively.

In order to obtain the field $E(\tilde{z})$ behind the electron bunch $(\tilde{z} \leqslant 0)$ it is necessary to integrate equation (1) for $n_\beta = 0$. One obtains the following expression for the field $E(\tilde{z})$:

$$E(\tilde{z}) = \pm\frac{m\omega_p c}{e}\sqrt{2}\left[A - \sqrt{1+\rho^2}\,\right]^{1/2}, \tag{11}$$

where the constant A is determined from the continuity conditions for momentum ρ and field E at the rear boundary of the bunch $\tilde{z}=0$, while $-\sqrt{A^2-1} \leqslant \rho \leqslant \sqrt{A^2-1}$. The values $\rho=\pm\sqrt{A^2-1}$ are extremal, and the field becomes equal to zero for these values. The field amplitude achieves its maximum value at $\rho=0$. The wave length of the stationary oscillations excited by the bunch is determined by the expression

$$\frac{\omega_p}{c}\sqrt{2}\,\frac{\tilde{z}_\lambda}{2}=\int_{-\sqrt{A^2-1}}^{\sqrt{A^2-1}}\frac{(\beta\sqrt{1+\rho^2}-\rho)\,d\rho}{\sqrt{1+\rho^2}\left[A-\sqrt{1+\rho^2}\right]^{1/2}}=$$

$$=4\beta\sqrt{A+1}\left\{E\left(\frac{\pi}{2},K\right)-\frac{1-K^2}{2}F\left(\frac{\pi}{2},K\right)\right\}, \qquad (12)$$

where $K=\sqrt{(A-1)/(A+1)}$. The electron density behind the bunch is determined by eq. (5).

In the cases when the rear boundary of the bunch is at a place where ρ has values $-\rho_0\ (E^{\beta}=0)$ and $-\rho_m\ (E=\pm E^{\beta}_{max})$ the constant A is given by the following expressions

$$A=A_1=\sqrt{1+\rho_0^2},$$
$$A=A_2=\left(1-\frac{n_\beta}{n_0}\right)+\frac{n_\beta}{n_0}\sqrt{1+\rho_m^2}-\frac{n_\beta}{n_0}\beta\rho_m. \qquad (13)$$

then the conditions

$$1\ll\gamma^2\ll\frac{n_\beta^2}{n_0^2(1-2n_\beta/n_0)}, \quad n_\beta\approx\frac{n_0}{2} \qquad (14)$$

are fulfilled, from (6), (9) and (13) follows that

$$A_1 = \frac{1 - 2\frac{n_b}{n_o} + (1+\beta^2)\frac{n_b^2}{n_o^2}}{1 - 2\frac{n_b}{n_o} + (1-\beta^2)\frac{n_b^2}{n_o^2}} \approx 2\gamma^2 ,$$

$$A_2 = \frac{\frac{n_b}{n_o}\left(1 - \frac{n_b}{n_o} + \frac{n_b}{n_o}\beta^2\right) + \left(1 - \frac{n_b}{n_o}\right)\sqrt{1 - 2\frac{n_b}{n_o} + (1-\beta^2)\frac{n_b^2}{n_o^2}}}{\sqrt{1 - 2\frac{n_b}{n_o} + (1-\beta^2)\frac{n_b^2}{n_o^2}} ,} \approx \gamma \tag{15}$$

Then the field behind the bunch will depend on the γ -factor and is equal to

$$E(\tilde{z}) \approx \pm \frac{m\omega_p c}{e}\sqrt{2}\left[2\gamma^2 - \sqrt{1+\rho^2}\right]^{1/2} ,$$

$$|E_{max}| \approx 2\frac{m\omega_p c}{e}\gamma , \text{ for } \rho(\tilde{z}=0) = -\rho_o \tag{16}$$

and

$$E(\tilde{z}) \approx \pm \frac{m\omega_p c}{e}\sqrt{2}\left[\gamma - \sqrt{1+\rho^2}\right]^{1/2} ,$$

$$|E_{max}| \approx \frac{m\omega_p c}{e}\sqrt{2}\,\gamma^{1/2} \quad \text{for } \rho(\tilde{z}=0) = -\rho_m \tag{17}$$

Note that with the increase of γ the condition $n_b/n_o \approx 1/2$ must be fulfilled with increasing accuracy, $(1/2 - n_b/n_o) \ll 1/8\gamma^2$. The wave length behind the bunch is determined as:

$$\tilde{z}_\lambda \approx \frac{8 v_o}{\omega_p}\gamma , \quad \text{for} \quad \rho(\tilde{z}=0) = -\rho_o \tag{18a}$$

$$\tilde{z}_\lambda \approx \frac{4\sqrt{2}\, v_o}{\omega_p}\gamma^{1/2} , \quad \text{for} \quad \rho(\tilde{z}=0) = -\rho_m \tag{18b}$$

Note also that the value $n_b = n_o/2$ for the bunch electron density is critical, and for this value a turnover of the wave and "break" of its amplitude takes place. This is also seen from Fig. 1,2 a), b), c) in which the schematic graphs of the relative magnitudes of $\rho(\tilde{z})$, $eE(\tilde{z})/m\omega_p c$, $n(\tilde{z})/n_0$ and $N(\tilde{z})/n_0$ are shown ($N(\tilde{z})$ is the total charge equal to $N(\tilde{z})=n_0-n(\tilde{z})-n_b$ inside the bunch and $N(\tilde{z})=n_0-n(\tilde{z})$ outside the bunch) for the case when on the rear boundary $\rho(\tilde{z}=0)=-\rho_0, -\rho_m$ and $n_b=n_0/2$, $\gamma=10^2$. It is necessary to note that the plasma electron density inside the bunch varies considerably only at the front of the bunch and then remains practically constant $n(\tilde{z})\approx n_0/2$ as it is seen from Figs 1,2 c). This corresponds to the above made assumption on the constancy of the bunch electron density n_b . Let us also note that the electric field inside the bunch is relatively low which confirms in some extent the assumption made in ref-s [25-28] that the field inside the bunch is equal to zero.

For $n_b/n_0 \ll 1$ the field is independent of the γ - factor and its maximum value is given by the expressions:

$$|E_{max}| \approx \frac{2m\omega_p c}{e} \frac{n_b}{n_0}\beta , \qquad \text{for} \quad \rho(\tilde{z}=0)=-\rho_0 \qquad (19)$$
$$|E_{max}| \approx \frac{m\omega_p c}{e} \frac{n_b}{n_0}\beta , \qquad \text{for} \quad \rho(\tilde{z}=0)=-\rho_m .$$

In both cases the wave length is equal to $\tilde{z}_\lambda \approx 2\pi \vartheta_0/\omega_p$ As it is seen from (19) $E_{max}\approx 2\pi^{1/2}(mc^2 n_b)^{1/2}(n_b/n_0)^{1/2}\approx 0{,}96\cdot 10^2 n_b^{1/2}(n_b/n_0)^{1/2}$ and if $n_b/n_0\approx 0{,}1$, $E_{max}\approx 10\,MV/m$ and $E_{max}\approx 1\,GV/m$ for $n_b=10^{11}cm^{-3}$, and $n_b=10^{15}cm^{-3}$ respectively. This means that in this case the production of high accelerating fields requires also a dense bunch and not only dense plasma as it is believed usually. Let us also note that if one chooses the bunch length equal to $d=\tau\tilde{z}_\lambda^b$, where τ are positive integers beginning from one, then no wake field behind the bunch is excited for zero boundary conditions at the rear boundary of the bunch, i.e. $E(\tilde{z}) = E(\tilde{z}=0) = 0$ and $\rho(z) = \rho(\tilde{z}=0) = 0$. The solutions for the longitudinal waves obtained in the present consideration are stable with respect to the longitudinal fluctuations. This may be shown using the techniques of generalized variables used in [29].

Numerical Examples and some Accelerator Physics Issues

As it follows from the considerations given above a stationary state with an equilibrium between the bunch, wake and proper fields of the bunch emerges as a result of the interaction between the electron bunch and the plasma. The minimum equilibrium bunch length is equal to $d_0 = I(|\rho_0|)$ for $\rho(\tilde{z}=0) = -\rho_0$ and $d_0 = I(|\rho_m|)$ for $\rho(\tilde{z}=0) = -\rho_m$. When the conditions (14) are fulfilled the wake field amplitude reaches high values, which may provide a high particle acceleration rate. For instance, when $2n_b \approx n_0 \approx 10^{13} cm^{-3}$, $\omega_p \approx 1.7 \cdot 10^{11} s^{-1}$ and $\gamma = 10^2$ the maximum value of the wake field and the acceleration rate are $E_{max} = 5.8 \cdot 10^{10}$ V/m and $eE_{max} \approx 58$ GeV/m, respectively, for the case $\rho(\tilde{z}) = -\rho_0$ However, in this case the bunch length d_0 is equal to 141m. In the case when $\rho(\tilde{z}=0) = -\rho_m$ on the rear boundary, $E_{max} = 4.1 \cdot 10^9$ V/m, the acceleration rate is $eE_{max} = 4.1$ GeV/m, and the bunch length $d_0 = 35$ cm (1.1ns). The maximum energy W acquired by an accelerated bunch with density n_1 is equal to $W = n_1 e E_{max} L$, where E_{max} is the maximum accelerating field of the wake wave and L is the acceleration length. Due to the stationary of the solutions one may use formulae obtained above when the energy acquired by the accelerated bunch is less than the energy of the accelerating bunch and the energy of the wake field i.e.

$$n_1 e E_{max} L \ll \min\left\{ mc^2 \gamma n_b ,\ \frac{E^2_{max}}{8\pi} \right\}.$$

Therefore the acceleration length

$$L \ll \min\left\{ \frac{mc^2 \gamma n_b}{eE_{max} n_1} ,\ \frac{E_{max}}{8\pi n_1 e} \right\}, \qquad n_1 \ll n_b .$$

When the conditions (14) are fulfilled the acceleration length

$L \ll \frac{c}{2\omega_p} \frac{n_b}{n_1}$ and $L \ll \frac{c}{\sqrt{2}\omega_p} \frac{n_b}{n_1} \gamma^{1/2}$ for $\rho(\tilde{z}=0)=-\rho_0$ and $\rho(\tilde{z}=0)=-\rho_m$, respectively. In the case $n_b/n_0 \ll 1$ the acceleration length $L \ll \frac{v_0}{\omega_p} \frac{n_b}{n_1}$. It is necessary to note that the acceleration length is also limited by the condition[30,31] of absence of the satelite nonstability $L \lesssim \frac{c}{\omega_p} \gamma \frac{2}{\sqrt{3}} \left(\frac{2n_0}{n_b}\right)^{1/3} \alpha$ where $\alpha \geqslant 1$. One has to take into account also the fact that in the result of acceleration the particles of the accelerated bunch may escape from the acceleration phase of the wake wave. However, such dephasing takes place at a length $L \sim \frac{c}{\omega_p} \gamma^2$ (see the estimates in [13]) which are significantly larger than that at which the bunch begins to break down due to the development of satelite nonstability in the system.

The luminosity determined by the total number of the particles $N_1 = n_1 V$ where V is the volume of the bunch is another important characteristics for particle acceleration by plasma wake wave. One can estimate this number from the constraint on the acceleration bunch field which must be much less than the wake field of the accelerating bunch. Therefore the field of the accelerated bunch may be obtained from the linearized system of equations of motion and Maxwell's equations (1) (compare, for instance, with [13]), which corresponds to the case $n_1/n_0 \ll 1$. According to (19) the field amplitude in such case is equal to $\frac{\alpha m \omega_p c}{e} \frac{n_1}{n_0}$, where $\alpha \sim 1$ and the particle density n_1 of the accelerated beam, therefore, must be much less than the particle density n_b of the accelerating bunch $n_1 \ll n_b$. In the case when eqs (17) and (18b) are valid, the condition on n_1 is

$$\frac{n_0}{\gamma^{1/2}} \ll n_1 \ll n_0 \approx 2n_b$$

Conclusion

It is shown that the nonlinear treatment of the wake waves in plasma gives qualitatively and quantitatively new possibilities for the acceleration of particles by plasma wake fields. Especially interesting for laboratory experiments seems the case when the driving beam density

$$n_b \to n_0/2\,, \quad \left(\frac{1}{2} - \frac{n_b}{n_0}\right) \ll \frac{1}{8\gamma^2}\,;$$

and the bunch length $d \approx \frac{2v_0}{\omega_p}\gamma$. Then the maximum longitudinal electric field behind the bunch is $E_{max} = \frac{\sqrt{2}\,\omega_p mc}{e}\gamma^{1/2}$, wake wave length $\tilde{z}_\lambda = \frac{4\sqrt{2}\,v_0}{\omega_p}\gamma^{1/2}$, transformation ratio $R \equiv \frac{E_{max}}{E^b_{max}} = \sqrt{2}\,\gamma^{1/2}$ acceleration length $L^a \ll \frac{1}{4}\frac{c}{\omega_p}\gamma^{1/2}$ and the density of the driven beam to satisfy the condition $\frac{n_0}{\gamma^{1/2}} \ll n_1 \ll n_0 \simeq 2n_b$.

If for $n_b \to \frac{n_0}{2}$ the length of the driving bunch could be chosen as $d = \frac{8 v_0}{\omega_p} \gamma^2$ then eq.(16) for the maximum electric field is valid and more drastic increase in acceleration rate will be provided. However, this case, due to the large value of d, may have, as it seems to us, only astrophysical applications, connected with the mechanisms of the acceleration of Cosmic Rays in a relatively dense plasma of various astrophysical objects. Perhaps, one should take into account the presented nonlinear mechanisms of acceleration of charged particles for explaining the origin of high and ultra-high energy Cosmic Ray particles (see [32]).

The simple model which we have considered has an exact analytical solution, which was essential in finding the above mentioned new nonlinear effects. At the same time the simplicity of the model has the obvious drawbacks which, it is necessary to overcome by taking into account, for example the finite transverse dimensions of the driving beam, shaping the charge distribution inside the bunch and seeking for nonstationary solutions.

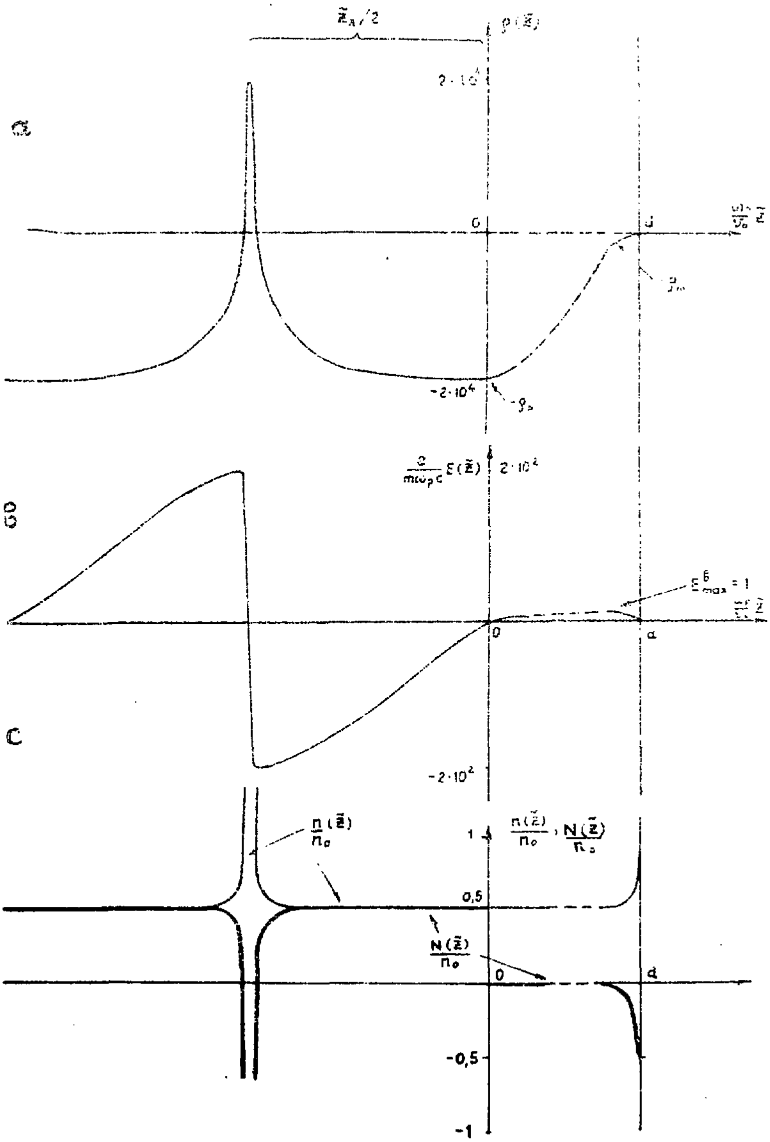

Fig. 1

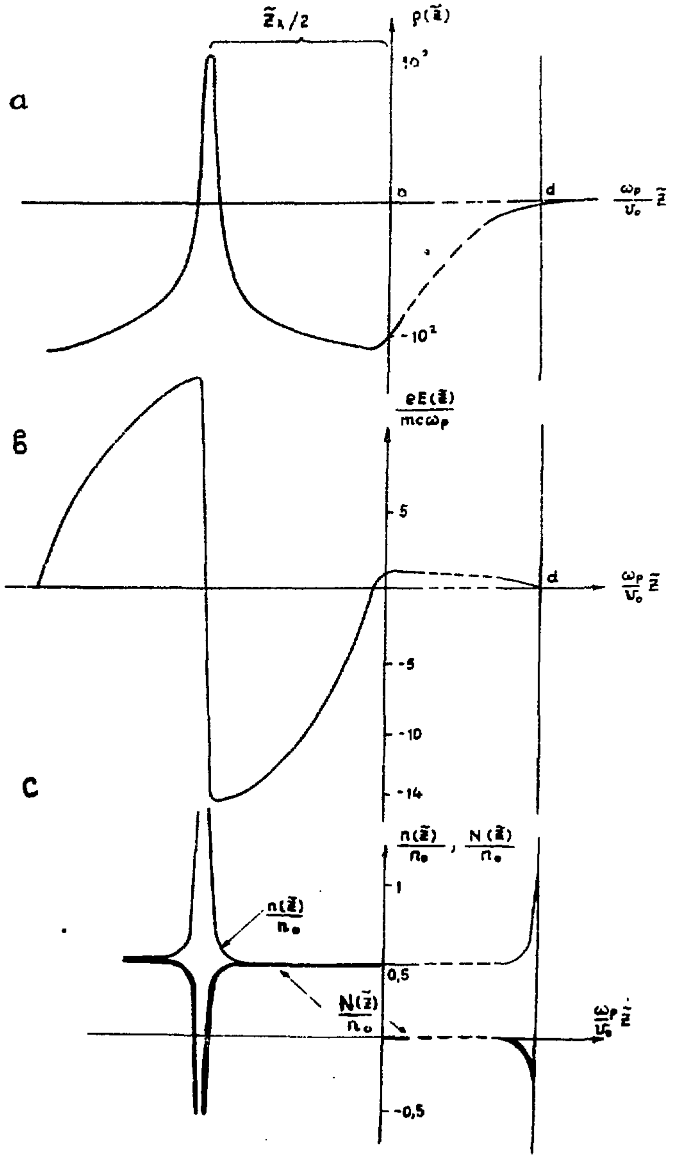

Fig.2

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