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ON THE THREE -DIMENSIONAL ISING MODEL

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Ա.Ռ.ԿԱՎԱԼՈՎ, Ա.Գ.ՍԵԴՐԱԿՅԱՆ

Տրված է Իզինգի եռաչափ մոդելի ձևակերպումը որպես ֆերմիոնային կառուցվածքով մակերևույթների տեսություն: Գտնված գործողության անընդհատ սահմանը որոշվում է երկչափ մակերևույթի վրա սահմանված Դիրակի գործողությամբ:

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А.Р.КАВАЛОВ, А.Г.СЕДРАКЯН

О ТРЕХМЕРНОЙ МОДЕЛИ ИЗИНГА

Найдена формулировка трехмерной модели Изинга как теории поверхностей с фермионной структурой на них. Непрерывный предел найденного действия определяется индуцированным на двухмерной поверхности действием Дирака.

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ON THE THREE-DIMENSIONAL ISING MODEL

The three-dimensional Ising gauge model is reformulated as the theory of surfaces with fermionic structure on them. In the naive continuum limit the action of the resulting string theory is defined by the Dirac action induced on two-dimensional surface.

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It is well known that two-dimensional Ising model may be reduced to the two-dimensional fermion theory (a standard majorana fermion action is obtained near the critical point) [1]. For the three-dimensional Ising model (3dIM) the analogous describtion, using the fermions in three-dimensional space, seems not to exist. However, the three-dimensional Ising model is dual to three-dimensional Ising gauge model. A.M.Polyakov and V.S.Dotsenko have shown [2] that for the latter the problem reduces to the problem of summing over the surfaces with some fermionic structure on them (i.e. to a fermionic string theory). A hypothesis has been made about the equivalence of 3dIM near the critical point to Neveu-Schwarz-Ramond string theory [3].

In the present work the lattice fermionic string theory is formulated, which is equivalent (at arbitrary temperature) to the 3dIM . In the naive continuum limit which corresponds to the critical temperature the action of reduces to the sum of number action and the Dirac action of three-dimensional fermions induced on the world surface.

The partition function of 3dIM may be represented as the sum over closed surfaces on the lattice [2,4,5]

$$Z \sim \sum \lambda^{A} \qquad (1)$$

where A is the area of the surface measured in the units of lattice spacing, $\lambda = \mathrm{th}\,\frac{J}{KT}$, T is the temperature. In (1) one may pass to summing over parametrized surfaces. In this case the contribution each surface must be provided by the sign factor $(-1)^{\ell}$ in order to avoid the overcoming. Here ℓ is the number of links of selfintersection of the surface[2,4]

$$Z \sim \sum_{\{\vec{x}(\xi^1,\xi^2)\}} \lambda^{A} \, (-1)^{\ell} \qquad (2)$$

The sign factor $(-1)^{\ell}$ is analogous to Kac- Ward factor of Two-dimensional Ising model. Its presence in the partition function shows that the string must have some fermionic structure on it [2] . It admits the following expression in terms of the geometry of the surface [5] . Let us draw two parallel lines connecting the middles of neighbouring links on each plaquette of the surface. These are two ways of drawing this lines on each plaquette (see Fig.1)

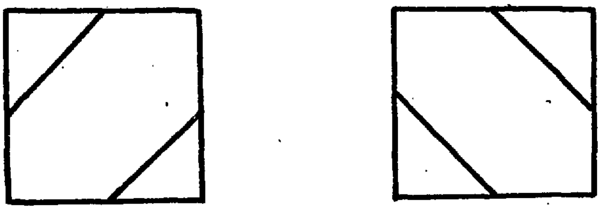

Fig.1

The whole surface will get covered (in one of 2^{A} possible ways) by some number of closed non-selfintersecting contours crossing once each link of the surface. On each part of the contour the unit tangent vectors of the surface $\vec{e}_{\parallel}$ and $\vec{e}_{\perp}$ and the normal vector $\vec{n}$ are defined. We choose $\vec{e}_{\parallel}$ to be tangent to the contour, $\vec{e}_{\perp}$ normal to the contour. Consider the matrix $S_{i,i+1}$ rotating (in spinor representation) the basis in point i to the basis in point $i+1$. Then [5]

$$(-1)^{\ell} = \prod_{\text{over the contours}} \Phi(c) = \prod_{\text{over the countours}} \left(-\frac{1}{2}\,\mathrm{tr}\prod S_{i,i+1}\right) \quad (3)$$

We shall define now the fermionic structure producing the described family of contours and providing each contours by the trace of ordered product of rotation operators. Let us describe the unfolded surface by the poligon with appropriate identifications on the boundary. In the middle of each link of the surface we define a pair of two-dimensional vectors $\tau_a^{\alpha}(\xi)$ (the index $a=1,2$ numerates the vectors $\alpha=1,2$ is the vector index, ξ^{α} are the coordinates of the point on the surface). The vectors τ_a form a "chess-like" structure (Fig.2)

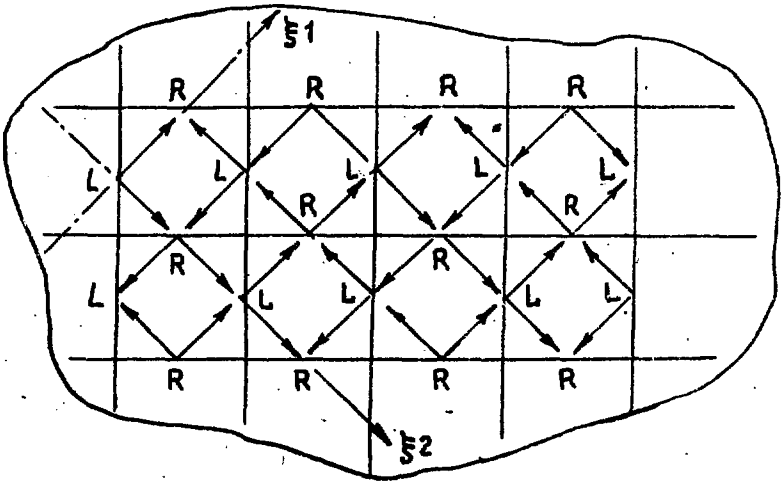

Fig.2

They are equal to the distance in parameter space between the neighbouring points of the lattice surface

$$g_{\alpha\beta}(\xi)\,\tau_a^\alpha(\xi)\,\tau_b^\beta(\xi) = \frac{\varepsilon^2}{2}\,\delta_{ab}\,, \tag{4}$$

where $g_{\alpha\beta}$ is the induced metric of the surface, ε is the lattice spacing. In conformal gauge $g_{\alpha\beta} = \rho\,\delta_{\alpha\beta}$, so that

$$\tau_a^\alpha\;\tau_b^\alpha = \frac{\varepsilon^2}{2}\;\frac{1}{\rho}\,\delta_{ab}\,. \tag{5}$$

Each pair of the vectors τ_a may be used to choose one of the two plaquettes containing the corresponding link. Now let us define a chiral fermion field in the middle point of each link of the surface; we put the left-handed fermions in the even points and right-handed fermions in the odd points*. The chirality is defined locally with respect to a normal vector of the surface, taken on the plaquette chosen by the corresponding vectors τ_a. On the boundary of the poligon define the following identification rule: if the identified links are both even or both odd the fields are considered equal if the parties of links are different the fields are connected by the equation $\Psi_L = \hat{e}_\perp \Psi_R$ where $\hat{e}_\perp = \vec{e}\,\vec{\sigma}$, $\vec{\sigma}$ are Pauly matsices, $\vec{e}_\perp$ is unit tangent vector of the surface which is normal to the identified links and has the direction corresponding to that chosen by vectors τ_a on the identified links. The action of the

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[*] An analogous idea leading to a hipothesis of topologically invariant formulation of the sign factor has been considered by ... orikov. The ferromagnetic-antiferromagnetic structure in 3DIM has been also considered by A.M.Polyakov and V.S.Dorenko.

model reads $S=S_1+S_2$, where S_1 is the area of the surface (Nambu action) and S_2 is given by

$$S_2 = i \sum_{\xi} \sum_{a=1}^{2} \sqrt{\rho}(\xi)\bar{\Psi}_{L,R}(\xi)\tau_a^{\alpha}\gamma_{\alpha}\Omega(\xi,\xi+\tau_a(\xi))\Psi_{R,L}(\xi+\tau_a(\xi)) \qquad (6)$$

Here the operator $\Omega(\xi,\xi+\tau_a(\xi))$ rotates the basic unit tangent vectors of the surface $\frac{\gamma_\alpha}{\sqrt{\rho}}$ $\frac{\vec{x}_\alpha\vec{\sigma}}{\sqrt{\rho}}$ ($\alpha=1,2$, $\vec{\sigma}$ are Pauly matrices) taken in the point ξ to those taken in the point $\xi+\tau_a$: $\frac{\gamma_\alpha}{\sqrt{\rho}}(\xi)\Omega(\xi,\xi+\tau_a(\xi)) = \Omega(\xi,\xi+\tau_a(\xi))\frac{\gamma_\alpha}{\sqrt{\rho}}(\xi+\tau_a(\xi))$. The vectors $\vec{x}_\alpha$ are defined through the finite differencies between the points lying on the plaquette chosen by the vectors τ_a . When performing the integration over fermion fields in the partition function

$$Z = \sum_{\{\vec{x}(\xi^1,\xi^2)\}} \prod_{\xi} \int d\bar{\Psi} d\Psi e^{S} \qquad (7)$$

one obtains the contours described above. The integration measure in (7) is defined by the equations

$$\int d\bar{\Psi}_{L,R} d\Psi_{L,R} \quad \Psi_{L,R}\ \bar{\Psi}_{L,R} = \frac{1}{\sqrt{\rho}} P_{L,R} , \qquad (8)$$

where $P_{L,R}=\frac{1}{2}(1\pm\vec{n}\,\vec{\sigma})$ are the chiral projectors. For each contour one obtains the factor $\mathrm{tr}\,P_L(1)\prod_i \hat{e}_{\parallel}(i)\Omega_{i,i+1}$, where $\hat{e}_{\parallel}(i)$ is the unit tangent vector of the contour between the point i and $i+1$. This factor may be shown to coinside with the factor $\Phi(C)$ (3). The partirion function thus takes the form (2) and coinsides with the partition function of 3d IM .

In the zero-lattice-spacing limit the action (6) takes the form

$$S_2 = i\int d^2\xi \sqrt{\rho}\ \bar{\Psi}\ \gamma^{\alpha}(\partial_\alpha + \Omega^{-1}\partial_\alpha\Omega)\Psi , \qquad (9)$$

where Ω is the operator rotating the local coordinate basis of the surface into some fixed basis of σ-matrices: $\frac{\gamma_\alpha}{\sqrt{\rho}}(\xi) = \Omega^{-1}(\xi)\sigma_\alpha\,\Omega(\xi)$. Let us make the field redefinition $\Psi \to \rho^{1/4}\Psi$. Such rescoling is necessary for the restoration of reparametrization symmetry. In terms of the new fields the action S_2 coinsides with the induced Dirac action

$$S_2 = i\int d^2\xi \sqrt{g}\, g^{\alpha\beta}\,\bar{\Psi}\,\gamma_\alpha\left(\partial_\beta + \Omega^{-1}\partial_\beta\Omega - \frac{i}{2}n\omega_\beta\right)\Psi, \qquad (10)$$

$$\omega_\beta = \frac{1}{2}\,\frac{\varepsilon^{\alpha\gamma}}{\sqrt{g}}\,e^{a}_{\alpha}\,\nabla_\beta\, e_{a\gamma}$$

taken in the conformal gauge. The problem of investigation of the critical behaviour of the 3dIM is thus reduced to the problem of investigation of the properties of Dirac operator induces on two dimensional surface, having singularities (the ends of the lines of selfintersection). For nonsingular surfaces such an operator is investigated in the work [6].

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О ТРЕХМЕРНОЙ МОДЕЛИ ИЗИНГА

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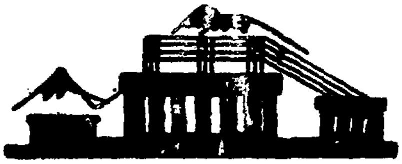

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