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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

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LAGRANGIAN OF THE SELF-DUALITY EQUATION AND d = 10, N = 2b SUPERGRAVITY

ЦНИИатоминформ

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Ան.Ռ. ԿԱՎԱԼՈՎ Ռ.Լ. ՄԿՐՏՉՅԱՆ

ԻՆՔՆԱԴՈՒԱԼՈՒԹՅԱՆ ՀԱՎԱՍԱՐՄԱՆ ԵՎ $d = 10$ $N = 2в$ ՍՈՒՊԵՐԳՐԱՎԻՏԱՑԻԱՅԻ ԼԱՆԳՐԱՆԺԻԱՆԸ

Կառուցված է տասչափանի ընդլայնված կիրալ սուպերգրավիտացիայի ինվարիանտ լանգրանժիանը՝ մինչ ֆերմիոն դաշտերի քառակուսինները ներառյալ: Մտցված է նոր դաշտ՝ $\theta_{a_1\cdots a_5 в_1\cdots в_5}$, որի շարժման հավասարումը համարժեք է ֆիզիկական դաշտի $A_{\mu\nu\rho\lambda}$ լարվածության ինքնադուալության հավասարմանը: $\theta_{a_1\cdots a_5 в_1\cdots в_5}$-ը նոր տրամաչափային համաչափությամբ տրամաչափ դաշտ է՝ ազդեցություն ունեցող միայն զանգվային մակերևույթից դուրս:

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An.R. KAVALOV, R.L. MKRTCHYAN

LAGRANGIAN OF THE SELF-DUALITY EQUATION AND d = 10, N = 2b SUPERGRAVITY

An invariant (supersymmetric, Lorentz-invariant) Lagrangian of a ten-dimensional chiral extended supergravity is constructed up to bilinear over the Fermi fields terms inclusively. A new element is the introduction of a Lagrangian multiplier $\theta_{a_1 \ldots a_5 b_1 \ldots b_5}$, the variation over which yields an equation equivalent to that of the self-duality of the supercovariant strength tensor of the physical field $A_{\mu\nu\lambda\rho}$. The field $\theta_{a_1 \ldots a_5 b_1 \ldots b_5}$ is a gauge field of a gauge symmetry, nontrivially acting only off the mass shell, which closes the formerly open supersymmetry algebra on all the boson fields.

Yerevan Physics Institute

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ЛАГРАНЖИАН УРАВНЕНИЯ САМОДУАЛЬНОСТИ И $d = 10$, $N = 2b'$ СУПЕРГРАВИТАЦИИ

Построен инвариантный лагранжиан десятимерной киральной расширенной супергравитации с точностью до билинейных по ферми-полям членов включительно. Новым элементом является введение лагранжевого множителя $\theta_{a_1\ldots a_5 b_1\ldots b_5}$, вариация по которому дает уравнение, эквивалентное уравнению самодуальности суперковариантного тензора напряженности физического поля $A_{\mu\nu\rho\lambda}$. Поле $\theta_{a_1\ldots a_5 b_1\ldots b_5}$ является калибровочным полем калибровочной симметрии, нетривиально действующей лишь вне массовой поверхности, которая замыкает ранее незамкнутую алгебру суперсимметрии на всех бозонных полях.

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The aim of the present paper is to develop the Lagrangian of the ten-dimensional chiral supergravity N = 2b. It has been discovered in the works by Schwarz and Green to be the zero slope limit ($\alpha' \to 0$) of the II-B type superstring theory [1,2], i.e. the supermultiplet of this supergravity constitute zero-mass particles of the mentioned superstring theory. The considered supergravity has an extended supersymmetry, the both supercharges being Majorana-Weyl spinors of the same chirality. The latter property differing it from the other ten-dimensional supergravity of N = 2a where two Majorana-Weyl supercharges have different chirality. This theory, which is the limit of $\alpha' \to 0$ of the II-A type superstring theory, appears to be a trivial reduction of the eleven-dimensional N = 1 supergravity to a ten-dimensional one, while the N = 2b supergravity can not be derived from the reduction of any other supergravity.

No Lagrangian invariant under the known symmetries of the theory is known up to now for the N = 2b supergravity. Symmetries and covariant equations of motion are obtained in a component approach in an approximation linear over the Fermi fields in the works by West and Schwarz [3] and Schwarz [4], and in the superfield approach - in the work by West and Howe [5].

To the development of the Lagrangian obstructs the real antisymmetric tensor of fourth rank in the theory, the equation of motion of which in linear approximation is the condition of self-duality of its strength tensor. This equation can not be obtained from the quadratic over the fields invariant Lagrangian without introduction of additional physical degrees of freedom [6] . The same situation is also in the two-dimensional analogue of the theory under consideration - in the heterotic string, where there also are chiral boson and fermion fields-coordinates.

In Ref.[7] by Ziegel is shown that the difficulty can be overcome in both theories by introduction of a Lagrangian multiplier with a nonquadratic Lagrangian.

In the present work we use Ziegel's method to derive the Lagrangian of the ten-dimensional N = 2b supergravity, using the known equation of motion , in the component approach up to terms bilinear over the fermions, inclusively, the supersymmetry transformations being obtained precisely.

The basic problem on our way is that about the supersymmetry of our Lagrangian. Generally, the action giving the equation of motion with a certain symmetry does not owe to be invariant under this symmetry off the mass shell. It has been required to find such a transformation of the supersymmetry of fields involved in the Lagrangian, which would be reduced to the known ones on mass shell and relative to which the Lagrangian would be invariant already off the mass shell.

It turned out, that it is possible to find such transformation, if the Lagrangian multiplier $\theta_{\alpha\beta}$ proposed by Ziegel is substituted by $\theta_{\alpha_1 \ldots \alpha_5 \beta_1 \ldots \beta_5}$ which plays the same role, and adequately choose its supersymmetry transformations.

A very interesting consequence of the appearance of additional field $\theta_{a_1 \ldots a_5 b_1 \ldots b_5}$ is the fact that now the commutator of two transformations of the supersymmetry gives a combination of gauge symmetries of the theory plus terms proportional to equations of motion of only fermion fields, like it is in all the other supergravities. The fact is , as is shown in Ref.[4], that the considered supergravity is the only one where the gauge symmetry algebra is closed by using the boson--field equations of motion - the earlier mentioned equation of self-duality. Now the algebra is closed without the use of this equation and this takes place due to the appearance, together with $\theta_{a_1 \ldots a_5 b_1 \ldots b_5}$, of the new gauge symmetry, the gauge field of which is the very Lagrangian multiplier $\theta_{a_1 \ldots a_5 b_1 \ldots b_5}$ This symmetry is a nonphysical one, which nontrivially acts only on the fields off the mass shell.

As the considered supergravity is the $\alpha' \to 0$ limit of the II B type string theory, then one can naturally assume that in the corresponding superstring-field theory an important role will play the generalization of that gauge symmetry. The discussion of this and other aspects, together with a brief formulation of the results of the work are presented in the Conclusion. In the first section the Lagrangian for the self-duality equation is developed and its symmetries are found. In the second section the full supermultiplet is introduced and new transformations of the supersymmetry are presented, and also their commutator calculated. In the third section the supergravity Lagrangian is obtained. In the Appendix the properties of (anti)self-dual tensors are given.

1. Lagrangian for the Self-Duality Equation

In the 4k + 2 dimensional Minkovski space the little co--group of massless representation of the Poincare group is the group SO(4k) having an irreducible unitary representation in the space of real antisymmetric self-dual tensors of rank 2k. In terms of field equations this representation of Poincare group can be described by means of antisymmetric tensor field $A_{\mu_1 \ldots \mu_{2k}}$ satisfying the self-duality equation of its strength $F_{\mu_1 \ldots \mu_{2k+1}}$ (further k = 2)

$$F^{-}_{\mu_1 \ldots \mu_5} = 0$$

$$F_{\mu_1 \ldots \mu_5} = 5\, \partial_{[\mu_1} A_{\mu_2 \ldots \mu_5]}$$

Notations see in the Appendix. This equation is equivalent to the equations of motion of the following Lagrangian, where we have introduced for generality also the external gravitation field with orthogonal frame e^{a}_{μ} :

$$\mathcal{L} = e\,\big(F^{+}_{\mu_1 \ldots \mu_5} F^{-\,\mu_1 \ldots \mu_5} +$$

$$+ \theta_{a_1 \ldots a_5 b_1 \ldots b_5} F^{-\,a_1 \ldots a_5} F^{-\,b_1 \ldots b_5}$$

$$F_{a_1 \ldots a_5} = e^{\mu_1}_{a_1} \ldots e^{\mu_5}_{a_5} F_{\mu_1 \ldots \mu_5}$$

Here $\theta_{a_1 \ldots a_5 b_1 \ldots b_5}$ is the Lagrangian multiplier which can be thought to be arbitrary, but it is more convenient to apply to it the conditions of symmetry:

$$\theta_{a_1 \ldots a_5 b_1 \ldots b_5} = \theta_{[a_1 \ldots a_5][b_1 \ldots b_5]}$$

$$\theta_{a_1 \ldots a_5 b_1 \ldots b_5} = \theta_{b_1 \ldots b_5 a_1 \ldots a_5}$$

It is also convenient to hold it to be self-dual over two sets of five indices

$$\theta_{a_1\dots a_5 b_1\dots b_5} = \theta^{++}_{a_1\dots a_5 b_1\dots b_5}$$

$\theta_{a_1\dots a_5 b_1\dots b_5}$ will satisfy all these conditions further in this work.

Equations of motion of the considered Lagrangian

$$\frac{\delta\mathcal{L}}{\delta\theta} = F^{-a_1\dots a_5} F^{-b_1\dots b_5} = 0$$

$$\frac{\delta\mathcal{L}}{\delta A} = D^{\nu}\left(F_{\nu\mu_1\dots\mu_4} + 2S_{\nu\mu_1\dots\mu_4}\right) = 0$$

$$S_{\mu_1\dots\mu_5} = \theta_{\mu_1\dots\mu_5 b_1\dots b_5} F^{-b_1\dots b_5}$$

$$D_\nu = D_\nu(\omega(e))$$

are not independent - the second one follows from the first one from which the self-duality equation results. The dependency of equations leads to invariance of the Lagrangian under the gauge transformations with vector parameter α^μ , the analogue of which is pointed out in Ref.[7] :

$$\delta(\alpha)A_{\mu_1\dots\mu_4} = \alpha^\nu\left(F^{-}_{\mu_1\dots\mu_4\nu} - S_{\mu_1\dots\mu_4\nu}\right)$$

$$\delta(\alpha)\theta_{a_1\dots a_5 b_1\dots b_5} = \Big\{-5D_{(a_5}\alpha_{b_5)}\, g_{a_1 b_1}\cdots g_{a_4 b_4} +$$

$$+\alpha^\mu D_\mu \theta_{a_1\dots a_5 b_1\dots b_5} + 5D^{(e}\alpha^{d)}\theta_{c k_1\dots k_4 a_1\dots a_5}\theta_{d k_1\dots k_4 b_1\dots b_5} -$$

$$-5\left[\theta_{a_1\dots a_5 b_1\dots b_4 c}\, g_{b_5 d} D^{[c}\alpha^{d]} + (a \longleftrightarrow b)\right]\Big\}^{++}$$

This α -symmetry (as we shall call it further) or more exactly its supercovariant generalization, will be preserved also in the full Lagrangian of supergravity and play an important role in the closure of the supertransformations commutator on boson fields. When checking the above (below) given formulae, it is necessary to widely use the special properties of self-dual tensors presented in the Appendix.

2. Fields and Symmetries of d = 10, N = 2b Supergravity

We shall use indications and agreements of Ref.[4]*. The boson fields of theory except for the orthogonal frame e_μ^a include two antisymmetric tensors - the real $A_{\mu_1 \dots \mu_4}$ and the complex $A^\alpha_{\mu\nu}$ (α = 1,2)ones with the condition $(A^1_{\mu\nu})^* = A^2_{\mu\nu}$ and also scalar fields which parametrize the coset space SU(1,1)/U(1). It is convenient to describe them by 2 x 2 matrices $V^\alpha_\pm$, which belong to the group SU(1,1), and the factorization over U(1) will effectively take place due to local U(1)-invariance of the theory. Besides, we have also the tensor $\theta_{a_1 \dots a_5 b_1 \dots b_5}$ which does not carry physical degrees of freedom. The fermion fields are Weyl's complex spinors Ψ_μ and λ with opposite chirality: $\gamma_{11}\Psi_\mu = -\Psi_\mu$, $\gamma_{11}\lambda = \lambda$.

Beside the obvious general covariance and the local Lorentz-invariance, the theory also has the following symmetries [1-5]. The global SU(1,1) symmetry, the small-transformation parameter of which is the 2 x 2 matrix m^α_β

$$m^\alpha_\beta = \begin{pmatrix} i\gamma, & \alpha \\ \alpha^*, & -i\gamma \end{pmatrix} , \quad \gamma^* = \gamma$$

*The metrics $\eta^{ab} = (+ - - \dots -)$. Dirac matrices in Majorana representation: $\{\gamma^a \gamma^b\} = 2\eta^{ab}$, $\gamma_{11} = \gamma^0 \cdots \gamma^9$,
$\gamma^{[a_1} \cdots \gamma^{a_n]} = \gamma^{a_1 \dots a_n}$, $(\gamma^0)^* = (\gamma^0)^T = -\gamma^0$,
$(\gamma^i)^* = (-\gamma^i)^T = -\gamma^i$, $(i = 1, \dots, 9)$, $\gamma_{11}^* = \gamma_{11}^T = \gamma_{11}$

and the action on the fields looks:

$$\delta V_\pm^\alpha = m^\alpha{}_\beta V_\pm^\beta \ , \ \delta A_{\mu\nu}^\alpha = m^\alpha{}_\beta A_{\mu\nu}^\beta$$

The local U(1) symmetry with a parameter $\Sigma(x)$ and action

$$\delta(\Sigma) V_\pm^\alpha = \pm i \Sigma V_\pm^\alpha, \quad \delta(\Sigma)\Psi_\mu = \frac{i}{2}\Sigma\Psi_\mu, \quad \delta(\Sigma)\lambda = i\frac{3}{2}\Sigma\lambda$$

It is convenient to use SU(1,1)-invariant combinations

$$Q_\mu = -i\varepsilon_{\alpha\beta} V_-^\alpha \partial_\mu V_+^\beta \ , \qquad \delta(\Sigma) Q_\mu = \partial_\mu \Sigma$$

transforming like the U(1) gauge field, and $P_\mu = +\varepsilon_{\alpha\beta} V_+^\beta \partial_\mu V_+^\alpha$,

$\delta(\Sigma) P_\mu = 2i\Sigma P_\mu$ its U(1) charge being two.

The theory is invariant also under the local symmetry with a parameter $\Lambda_{\mu\nu\lambda}$ and action on the fields

$$\delta(\Lambda\ldots) A_{\mu_1\ldots\mu_4} = 4\partial_{[\mu_1}\Lambda_{\mu_2\mu_3\mu_4]}$$

and under the local symmetry with a parameter of Λ_μ^α (α = 1,2) and action on the fields

$$\delta(\Lambda.) A_{\mu\nu}^\alpha = 2\partial_{[\mu}\Lambda_{\nu]}^\alpha \ , \ \delta(\Lambda.) A_{\mu_1\ldots\mu_4} = -\frac{i}{4}\kappa\varepsilon_{\alpha\beta}\Lambda^\alpha_{[\mu_1} F^\beta_{\mu_2\mu_3\mu_4]}$$

The corresponding strengths, invariant under the last two transformations, have the following form:

$$F^\alpha_{\mu_1\mu_2\mu_3} = 3\partial_{[\mu_1} A^\alpha_{\mu_2\mu_3]}$$

$$F_{\mu_1\ldots\mu_5} = 5\partial_{[\mu_1} A_{\mu_2\ldots\mu_5]} + \frac{5}{8} i\kappa\varepsilon_{\alpha\beta} A^\alpha_{[\mu_1\mu_2} F^\beta_{\mu_3\mu_4\mu_5]}$$

The SU(1,1)-invariant combination is introduced in a standard way

$$G_{\mu\nu\lambda} = -\varepsilon_{\alpha\beta} V_+^\alpha F^\beta_{\mu\nu\rho}$$

with its charge being unity.

Before turning to transformations of the supesymmetry and α -symmetry, we shall present formulae for the supercovariant strengths and connection:

$$\hat{P}_\mu = P_\mu - \kappa^2 \overline{\Psi}_\mu^{\,*}\lambda\ , \quad \hat{G}_{\mu\nu\rho} = G_{\mu\nu\rho} - 3\kappa\overline{\Psi}_{[\mu}\gamma_{\nu\rho]}\lambda - 6i\kappa\overline{\Psi}^{*}_{[\mu}\gamma_\nu\Psi_{\rho]}\ ,$$

$$\hat{F}_{\mu_1\ldots\mu_5} = F_{\mu_1\ldots\mu_5} - 5\kappa\,\overline{\Psi}_{[\mu_1}\gamma_{\mu_2\mu_3\mu_4}\Psi_{\mu_5]} - \frac{1}{16}\kappa\bar{\lambda}\gamma_{\mu_1\ldots\mu_5}\lambda\ ,$$

$$\hat{S}_{\mu_1\ldots\mu_5} = \Theta_{\mu_1\ldots\mu_5}{}^{\nu_1\ldots\nu_5}\,\hat{F}^{-}_{\nu_1\ldots\nu_5}\ ,$$

$$\hat{\omega}_{\mu\nu\rho} = \omega_{\mu\nu\rho}(e) + \kappa^2\,\mathrm{Im}\left(\overline{\Psi}_\nu\gamma_\mu\Psi_\rho + \overline{\Psi}_\nu\gamma_\rho\Psi_\mu + \overline{\Psi}_\mu\gamma_\nu\Psi_\rho\right)$$

Transformations of the supersymmetry of the fields e_μ^a , $V_\pm^\alpha$, $A_{\mu\nu}^\alpha$, $A_{\mu\nu\rho\lambda}$, λ are the same as before [4] :

$$\delta(\varepsilon)e_\mu^a = -2\kappa\,\mathrm{Im}\,(\bar{\varepsilon}\gamma^a\Psi_\mu)$$

$$\delta(\varepsilon)V_+^\alpha = \kappa V_-^\alpha\,\overline{\varepsilon^*}\lambda\ , \quad \delta(\varepsilon)V_-^\alpha = \kappa V_+^\alpha\,\bar{\varepsilon}\lambda^*\ ,$$

$$\delta(\varepsilon)A_{\mu\nu}^\alpha = V_+^\alpha\,\overline{\varepsilon^*}\gamma_{\mu\nu}\lambda^* + V_-^\alpha\,\bar{\varepsilon}\gamma_{\mu\nu}\lambda +$$

$$+ 4i\left(V_+^\alpha\,\bar{\varepsilon}\gamma_{[\mu}\Psi^*_{\nu]} + V_-^\alpha\,\overline{\varepsilon^*}\gamma_{[\mu}\Psi_{\nu]}\right),$$

$$\delta(\varepsilon)A_{\mu\nu\rho\lambda} = 2\,\mathrm{Re}\left(\bar{\varepsilon}\gamma_{[\mu\nu\rho}\Psi_{\lambda]} - \frac{3}{8}i\kappa\,\varepsilon_{\alpha\beta}A^\alpha_{[\mu\nu}\,\delta A^\beta_{\rho\lambda]}\right)$$

$$\delta(\varepsilon)\lambda = \frac{i}{\kappa}\gamma^\mu\varepsilon^*\hat{P}_\mu - \frac{1}{24}i\gamma^{\mu\nu\rho}\varepsilon\,\hat{G}_{\mu\nu\rho}$$

The supertransformation of gravitino Ψ_μ includes additional as compared with Ref.[4], terms proportional to $\Theta_{a_1\ldots a_5 b_1\ldots b_5}$ and to the equation of motion of the field $A_{\mu\nu\rho\lambda}$. The latter has the following form in the full theory:

$$\hat{F}^{-}_{\mu_1\ldots\mu_5} = 0$$

Due to this circumstance it is obvious that the equations of motion [4] are invariant under the new transformation of Ψ_μ, which reads:

$$\delta(\varepsilon)\Psi_\mu = \frac{1}{K} D_\mu(\hat{\omega})\varepsilon + \frac{1}{480} i\gamma^{\nu_1\ldots\nu_5}\gamma_\mu \varepsilon(\hat{F}_{\nu_1\ldots\nu_5} + \hat{S}_{\nu_1\ldots\nu_5}) +$$

$$+ (\lambda\Psi\varepsilon,\ \lambda\lambda\varepsilon - [4])$$

$$D_\mu(\hat{\omega})\varepsilon = (\partial_\mu + \frac{1}{4}\hat{\omega}_\mu^{\ a b}\gamma_{ab} - \frac{1}{2} i Q_\mu)\varepsilon$$

Finally, the transformation of the supersymmetry of $\Theta_{a_1\ldots a_5 b_1\ldots b_5}$ has the form:

$$\delta(\varepsilon)\Theta_{a_1\ldots a_5 b_1\ldots b_5} = \Big[-\frac{5}{2}\Theta_{a_1\ldots a_5 K_1\ldots K_4 m}\Theta_{b_1\ldots b_5 K_1\ldots K_4 n} e^{\mu m} e^{\nu n}\delta(\varepsilon) g_{\mu\nu} +$$

$$+ \frac{5}{2}\eta_{a_1 b_1}\ldots\eta_{a_4 b_4} e^\mu_{a_5} e^\nu_{b_5}\delta(\varepsilon) g_{\mu\nu} + 5(\Theta_{\mu a_2\ldots a_5 b_1\ldots b_5}\delta(\varepsilon) e^\mu_{a_1} + (a \leftrightarrow b)) +$$

$$+ \frac{5}{2}(\Theta_{a_1\ldots a_5 b_1\ldots b_4 m} e^{\mu m} e^\nu_{b_5}\delta(\varepsilon) g_{\mu\nu} + (a \leftrightarrow b))\Big]^{++}$$

$$\delta(\varepsilon) g_{\mu\nu} = -4K\,\mathrm{Im}(\bar{\varepsilon}\gamma_{(\mu}\Psi_{\nu)})$$

The new gauge symmetry which appeared in our approach, is the α -symmetry with transformations which generalize the formulae in the section 1: $\delta(\alpha)\Theta_{a_1\ldots a_5 b_1\ldots b_5}$ remains unchanged, while $\delta(\alpha) A_{\mu\nu\rho\lambda}$ is equal to

$$\delta(\alpha) A_{\mu\nu\rho\lambda} = \alpha^\varphi (\hat{F}^-_{\mu\nu\rho\lambda\varphi} - \hat{S}_{\mu\nu\rho\lambda\varphi})$$

Let us consider the commutator of two transformations of the supersymmetry with parameters ε_1 and ε_α. Generally speaking, the commutator of two transformations of the symmetry of the theory gives a combination of all symmetries of the theory with definite parameters plus terms proportional to the equations of motion. In our case

$$[\delta(\varepsilon_1), \delta(\varepsilon_2)] = \delta(\xi) + \delta(\ell) + \delta(\varepsilon) + \delta(\Lambda.) + \delta(\Lambda\cdots) +$$

$$\delta(\Sigma) + \delta(\alpha) + \text{(equations of motion)}.$$

Here $\delta(\xi)$ is general coordinate transformation, $\delta(\ell)$ is the Lorentz local transformation.

Our statement is, that at the action of the commutator on boson fields (including $\theta_{a_1 \ldots a_5 b_1 \ldots b_5}$) no equations of motion appear in the right hand side. Before presenting expressions for the transformation parameters, we shall make a comparison with the results from Ref.[4] obtained in the absence of α -symmetry. In this case the commutator has the form shown above, but with the term $\delta(\alpha)$ lacking, and in Ref.[4] was found that when the commutator acts on boson fields, in the right hand side appears the only equation of motion, namely the equation of motion of the field $A_{\mu\nu\rho\lambda}$.
In our case, instead of this equation of motion, the transformation of α -symmetry has appeared.

The right hand side transformation parameters coincide with those from Ref.[4] in case of supersymmetry, U(1), $\Lambda.$, $\Lambda\ldots$ and general coordinates transformations. The Lorentz parameter of transformation is derived from that in Ref.[4] by the substitution $\hat{F}_{\mu_1\ldots\mu_5} \to \hat{F}_{\mu_1\ldots\mu_5} + \hat{S}_{\mu_1\ldots\mu_5}$, and the α -symmetry parameter turns out to be equal to the parameter of coordinate transformation:

$$\alpha^\mu = \xi^\mu = 2\,\mathrm{Im}\,(\bar{\varepsilon}_1 \gamma^\mu \varepsilon_2)$$

Thus, the additional field $\theta_{a_1 \dots a_s b_1 \dots b_s}$ plays a double role: first, it allows to develop a Lagrangian for the cosidered supergravity, as will be seen in the section 3, and second, partially solves the problem arising in any theory with a supersymmetry - to build a set of auxiliary fields which close the algebra of the theory symmetries. In our case, at introduction of the field $\theta_{a_1 \dots a_s b_1 \dots b_s}$ the commutator of the supersymmetry transformations is closed on all boson fields of the theory, in particular, on the very $\theta_{a_1 \dots a_s b_1 \dots b_s}$ too.

Now we want to give a qualitative explanation to the fact of the commutator closure on boson fields. First of all, in all theories, except for the considered one, the commutator is closed on boson fields due to the fact that there appear only the first derivatives from the fields, while in the equations of motion of boson fields always the second derivatives are present. The only exception makes the self-duality equation, the very one appearing in the considered theory before introduction of additional field $\theta_{a_1 \dots a_s b_1 \dots b_s}$. After its introduction all boson equations of motion turn into though not second-order ones, but into those quadratic over the derivatives (see sections 1 and 3), and the commutator is closed for the above mentioned reason. Most likely, the same will take place in the formulation of heterotic string with the additional field λ_{++} [8,9] .

3. Invariant Lagrangian of d = 10, N = 2b Supergravity

In this section will be discussed the Lagrangian of the mentioned supergravity with that property, that its equations of motion are equivalent to those obtained in Refs.[3-5]. Like in Ref.[4], we shall restrict ourselves to the first non-trivial order over fermion fields (though it should be noted that transformations of the supersymmetry and other symmetries and all the statements made in the preceding section concerning the commutators, are valid in all the orders over the Fermi fields).

Our basic statement is that the Lagrangian which will follow is invariant, up to complete derivatives and terms of higher power over Fermi-fields, with respect to all symmetries mentioned in the preceding section.

The Lagrangian is equal to

$$\mathcal{L} = e\Big\{-\frac{1}{2K^2}R + \frac{1}{K^2}P^\mu P^*_\mu + 2i\bar{\Psi}_\mu \gamma^{\mu\nu\rho} D_\nu \Psi_\rho +$$

$$+(-i)\bar{\lambda}\gamma^\mu D_\mu \lambda + \frac{1}{4!}G^{\mu\nu\rho}G^*_{\mu\nu\rho} + \frac{1}{30}F^-_{\mu_1\dots\mu_5}\cdot$$

$$\cdot(F^{\mu_1\dots\mu_5} + S^{\mu_1\dots\mu_5}) - \bar{\lambda}\gamma^\mu\gamma^\rho\Psi^*_\mu P_\rho -$$

$$-\overline{\Psi^*_\mu}\gamma^\rho\gamma^\mu\lambda P^*_\rho - \frac{K}{2\cdot 5!}\bar{\lambda}\gamma^{\mu_1\dots\mu_5}\lambda(F_{\mu_1\dots\mu_5} + S_{\mu_1\dots\mu_5}) +$$

$$+\frac{K}{2\cdot 5!}\bar{\Psi}_\mu\gamma^{\mu\nu\rho}\gamma^{\lambda_1\dots\lambda_5}\gamma_\rho\Psi_\nu(F_{\lambda_1\dots\lambda_5} + S_{\lambda_1\dots\lambda_5}) +$$

$$+\frac{K}{4!}\bar{\lambda}\gamma^\mu\gamma^{\lambda_1\lambda_2\lambda_3}\Psi_\mu G_{\lambda_1\lambda_2\lambda_3} - \frac{K}{4!}\bar{\Psi}_\mu\gamma^{\lambda_1\lambda_2\lambda_3}\gamma^\mu\lambda G^*_{\lambda_1\lambda_2\lambda_3} -$$

$$-\frac{iK}{4\cdot 4!}\bar{\Psi}_\mu \gamma^{\mu\nu\rho}(\gamma_\rho{}^{\lambda\eta\sigma} - 9\delta_\rho^\lambda \gamma^{\eta\sigma})\Psi_\nu^* G_{\lambda\eta\sigma} -$$

$$-\frac{iK}{4\cdot 4!}\bar{\Psi}_\nu^* (\gamma_\rho{}^{\lambda\eta\sigma} + 9\delta_\rho^\lambda \gamma^{\eta\sigma})\gamma^{\mu\nu\rho}\Psi_\mu G^*_{\lambda\eta\sigma} +$$

$$+\frac{5iK}{6\cdot 4!}\frac{1}{60}\frac{1}{e}\varepsilon_{\mu_1\ldots\mu_{10}} A^{\mu_1\ldots\mu_4} G^{\mu_5\mu_6\mu_7} G^{*\mu_8\mu_9\mu_{10}} + O(\Psi^4)\Big\}$$

Here $R \equiv R(\omega(e))$ is scalar curvature, and the covariant derivatives have the form

$$D_\mu \Psi_\nu = (\partial_\mu + \omega_\mu^{ab}(e)\frac{1}{4}\gamma_{ab} - i\frac{1}{2}Q_\mu)\Psi_\nu$$

$$D_\mu \lambda = (\partial_\mu + \omega_\mu^{ab}(e)\frac{1}{4}\gamma_{ab} - i\frac{3}{2}Q_\mu)\lambda$$

The invariance is checked directly, using also the formulae given in the Appendix.

Conclusion

In the present work the Lagrangian of the ten-dimensional N = 2b supergravity is developed up to four-fermion terms, invariant under all transformations of the theory, in particular, under the modified supersymmetry transformations. Besides, it is invariant under one more gauge transformation which nontrivially acts only off the mass shell. The development of the Lagrangian and also of the new gauge symmetry (α - symmetry) became possible due to introduction of the Lagrangian multiplier $\Theta_{a_1\ldots a_5 b_1\ldots b_5}$ which serves as a gauge field of the new symmetry and the variation over which yields equations equivalent to the self-duality one. The field $\theta_{a_1 \dots a_s b_1 \dots b_s}$ and the α-symmetry lead to the closure of the commutator of supertransformations on boson fields (including $\theta_{a_1 \dots a_s b_1 \dots b_s}$).

Let us discuss the possible directions of development of the results of this work. First of all, the construction of four-fermion terms in the Lagrangian will not, apparently, face serious difficulties. Next, there is one more chiral supergravity, among the equations of motion of which there is a self-duality equation - it is the six-dimensional N = 4 supergravity with its Weyl gravitinos having the same chirality [10]. It is obvious that one has the reason to think that it is possible to develop an invariant Lagrangian for it.*

The potentially important direction of progress involves the construction of the Lagrangian for the II B type superstring field theory. We assume that in a covariant formulation this Lagrangian will include string fields, in the expansion of which there appears the analogue of the field $\theta_{a_1 \dots a_s b_1 \dots b_s}$ and that it will be invariant under the corresponding generalization of the α-symmetry.

*This Lagrangian has been constructed by us recently.

Below $F^-_{\mu_1\dots\mu_5}$, $G^-_{\mu_1\dots\mu_5}$, $F^+_{\mu_1\dots\mu_5}$, $G^+_{\mu_1\dots\mu_5}$ indicate the (anti) self-dual parts of tensors $F_{\mu_1\dots\mu_5}$, $G_{\mu_1\dots\mu_5}$. $\theta_{\mu\nu}$ is a symmetric traceless tensor, $\Lambda_{\mu\nu}$ is an antisymmetric tensor. The (anti)symmetrization is always carried out with a unit weight, for example $\theta_{\mu\nu} = \theta_{(\mu\nu)} = \frac{1}{2}(\theta_{\mu\nu} + \theta_{\nu\mu})$.

The following equations are valid:

$$F_{\mu_1\dots\mu_5} = F^+_{\mu_1\dots\mu_5} + F^-_{\mu_1\dots\mu_5}$$

$$F^\pm_{\mu_1\dots\mu_5} = \frac{1}{2}\left(F_{\mu_1\dots\mu_5} \pm \frac{1}{5!}\,\frac{1}{e}\,\varepsilon_{\mu_1\dots\mu_5\nu_1\dots\nu_5} F^{\nu_1\dots\nu_5}\right)$$

$$F^-_{\mu_1\dots\mu_5} G^{+\mu_1\dots\mu_5} = F^+_{\mu_1\dots\mu_5} G^{-\mu_1\dots\mu_5} = 0$$

$$F^{-\alpha_1\alpha_2\alpha_3\mu_1\mu_2} F^-_{\alpha_1\alpha_2\alpha_3\nu_1\nu_2} = \frac{1}{2}\delta^{[\mu_1}_{[\nu_1} F^{-\mu_2]\alpha_1\dots\alpha_4} F^-_{\nu_2]\alpha_1\dots\alpha_4}$$

$$F^{-\mu_1\dots\mu_4\alpha} F^-_{\nu_1\dots\nu_4\alpha} = -\delta^{[\mu_1}_{[\nu_1}\,\delta^{\mu_2}_{\nu_2}\,\delta^{\mu_3}_{\nu_3} F^{-\mu_4]\alpha_1\dots\alpha_4} F^-_{\nu_4]\alpha_1\dots\alpha_4} +$$

$$+ 4\,\delta^{[\mu_1}_{[\nu_1} F^{-\mu_2\mu_3\mu_4]\alpha_1\alpha_2} F^-_{\nu_2\nu_3\nu_4]\alpha_1\alpha_2}$$

$$F^-_{\varkappa\alpha_1\dots\alpha_4} G_\lambda^{+\alpha_1\dots\alpha_4} + (\lambda \leftrightarrow \varkappa) = \frac{1}{5}\, g_{\varkappa\lambda} F^-_{\alpha_1\dots\alpha_5} G^{+\alpha_1\dots\alpha_5}$$

$$F^-_{\varkappa\alpha_1\dots\alpha_4} G_\lambda^{-\alpha_1\dots\alpha_4} - (\varkappa \leftrightarrow \lambda) = 0$$

$$F^\pm_{\alpha[\mu_1\dots\mu_4}\,\theta^\alpha_{\mu_5]} = \left(F^\pm_{\alpha[\mu_1\dots\mu_4}\,\theta^\alpha_{\mu_5}\right)^\mp$$

$$F^\pm_{\alpha[\mu_1\dots\mu_4}\,\Lambda^\alpha_{\mu_5]} = \left(F^\pm_{\alpha[\mu_1\dots\mu_4}\,\Lambda^\alpha_{\mu_5]}\right)^\pm$$

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СУПЕРГРАВИТАЦИИ

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