Preprint ЕФИ-943(94)-86

ԵՐԵՎԱՆԻ ՖԻԶԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ
ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

A.A.OHANJANYAN

A MAGNETIC LENS FOR PRECISION BEAM FOCUSING

ЦНИИатоминформ

ЕРЕВАН-1986

Նախնատիպ ЕФИ-943(94)-86

Ա. Ա. ՕՀԱՆՋԱՆՅԱՆ

ՄԱԳՆԻՍԱԿԱՆ ՈՍՊՆՅԱԿ՝ ՃՇԳՐԻՏ ՓՆՋԵՐԻ ՖՈԿՈՒՍԱՑՄԱՆ ՀԱՄԱՐ

Առաջարկված է մագնիսական ոսպնյակի փաթույթը կազմող հաղորդիչների այնպիսի փոխտեղադրվածություն, որի դաշտի մեջ փունջը ֆոկուսացվում է երկու լայնակի ուղղություններով՝ խիստ ֆոկուսացման սկզբունքով: Ոսպնյակի տարածա-պարբերական մագնիսական դաշտը ներկայացված է Ֆուրյեի շարքով: Բերված են հարմոնիկների ամպլիտուդները, որոնք կախված են փաթույթը կազմող հաղորդիչների կորությունից: Ազատ բնութագրերը՝ Ֆուրյե-հարմոնիկների ամպլիտուդները և դաշտի պարբերությունը, կարող են ընտրվել ըստ փնջի դինամիկայի՝ հաղորդիչների կորության ընտրությամբ: Ոսպնյակի բացվածքի փոքրեցման ժամանակ մեծանում է ֆոկուսավորող ուժը, և միաժամանակ հնարավոր է դառնում մագնիսական դաշտի պարբերության փոքրացումը առանց ամպլիտուդի փոքրացման: Այդ հանգամանքը կարող է օգտագործվել ճշգրիտ փնջերի ֆոկուսացման համար:

Երևանի ֆիզիկայի ինստիտուտ

ԵՐԵՎԱՆ 1986

Препринт ЕФИ-943(94)-86

А.А.ОГАНДЖАНЯН

МАГНИТНАЯ ЛИНЗА ДЛЯ ФОКУСИРОВКИ ПРЕЦИЗИОННЫХ ПУЧКОВ

Рассмотрена конфигурация обмотки магнитной линзы, которая реализует поле, фокусирующее пучок сразу с обоих поперечных направлений по принципу жесткой фокусировки. Пространственно-периодическое магнитное поле линзы представлено рядом Фурье, выписаны амплитуды гармоник, которые являются функционалами формы изгиба проводников в обмотке. Свободные параметры - амплитуды фурье-гармоник и период поля, могут быть оптимизированы по динамике пучка за счет выбора формы изгиба проводников в обмотке. При уменьшении апертуры линзы увеличивается ее фокусирующая сила и одновременно появляется возможность укорочения периода магнитного поля без уменьшения амплитуды поля. Это обстоятельство может быть использовано при применении предложенной линзы для фокусировки прецизионных пучков.

Ереванский физический институт

Ереван 1986

Preprint ЕФИ-943(94)-86

A.A. OHANJANYAN

A MAGNETIC LENS FOR PRECISION BEAM FOCUSING

The winding configuration of the magnetic lens which realizes the field focusing the beam simultaneously in both transverse directions according to the alternating-gradient focusing principle is considered. The space-periodic magnetic field of the lens is presented as a Fourier series; the amplitudes of harmonics that are functionals of the bend shape of conductors in the winding are written out. The free parameters, i.e. the Fourier-harmonic amplitudes and the field period, can be optimized with respect to the beam dynamics, owing to the choice of the conductors bend shape in the winding. At decreasing the lens aperture its focusing strength enhances and simultaneously there appears a possibility of shortening the magnetic field period without reduction of the field amplitude. This circumstance can be used with the lens proposed for the precision beam focusing.

Yerevan Physics Institute

Yerevan 1986

In Ref. [1] there was considered the dynamics of electrons in the fields of periodically bent current-carrying conductors. It was shown that the magnetic field of two such conductors, parallel-connected to the power source and disposed in the same plane at equal distances from the longitudinal axis, focuses the paraxial beam in one direction and defocuses it in the other. When adding extra two current-carrying conductors (whose field defocuses in the first direction and focuses in the second one), disposed so that to produce alternating along the axis sections at which, first, the first-pair field is stronger, then, the second-pair one is and vice versa, then there is formed a simplest configuration of alternating-gradient (AG) focusing field sources. The shape and disposition of the second pair of conductors as well as the current intensity and direction in it (all these determined by the zero-equality condition of the Fourier-series steady component of the focusing field) may be preset in a non-unique way. These can be a pair of rectilinear or periodically bent conductors with reverse-directed and intensity-selected current disposed in the same plane or a pair of conductors located in perpendicular plane, and so on. Note that the magnetic field period is preset by the conductor bend period and is equal to it.

In order to increase the focusing magnetic field amplitude, one can arrange several such simplest configurations of sources on the cylindrical surface in radial planes. In this case the relative directions of currents in them are chosen such that to minimize the focusing field quenching. As a result, such AG focusing lens acquires the same form as a superconductive magnetic quadrupole with double-layer winding (see, e.g. [2]) in which the conductors of at least one layer are periodically bent along the lens axis in radial planes. The current direction in the layers is mutually reverse, while the ratio of current intensities is chosen from the absence condition of zero Fourier-harmonics of focusing field.

In this paper the fields of such lens are written out as well as the stability of paraxial trajectories in them is shown.

Let the z-axis be aligned with the lens axis, and 2N conductors of each layer of the winding are located on the cylindrical surface in radial planes which make an angle α_i, $i = 1,\dots, 2N$ with the plane $y = 0$ (N is the number of simplest configurations of the AG focusing field sources). The field $\vec{H}$ of such a lens, linearized by small deviations from the axis, is defined by Biot-Savart's law and is written in the form:

$$\vec{H} = -\frac{J}{c}\,\vec{e}\cdot\int_{-\infty}^{\infty} dz_1\cdot\left\{\left[(z-z_1)^2+f^2(z_1)\right]^{-3/2} - \delta_0\cdot\left[(z-z_1)^2+\varphi^2(z_1)\right]^{-3/2}\right\}, \quad (1)$$

where

$$\vec{e} = \sum_{i=1}^{2N}\delta_i\cdot\left[\vec{e}_1\cdot(y\cos 2\alpha_i + x\sin 2\alpha_i) + \vec{e}_2\cdot(x\cos 2\alpha_i - y\sin 2\alpha_i)\right], \quad (2)$$

$\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ are unit vectors along the x, y, z axes, respectively; $\delta_i = \pm 1$ determines the current relative direction in a i-th conductor; c is velocity of light; J is current intensity in the winding first layer, $J\cdot\delta_0$ - in the second layer; the periodic with λ -period $f(z)$

and $\varphi(z)$ functions preset the shape of the first- and second-layer conductors, respectively (in the general case this shape may be different for different conductors and then the integration in (1) is carried out under the summation sign in (2)).

Let the conductors be connected to the power supply so that $\delta_i = 1$ at $-\frac{\pi}{4} < \alpha_i < \frac{\pi}{4}$ and $\frac{3}{4}\pi < \alpha_i < \frac{5}{4}\pi$, and $\delta_i = -1$ at $\frac{\pi}{4} < \alpha_i < \frac{3}{4}\pi$ and $\frac{5}{4}\pi < \alpha_i < \frac{7}{4}\pi$. Then $\vec{e} = 4\cdot(y\vec{e}_1 + x\vec{e}_2)\cdot ctg\frac{\pi}{N}$ ($|\vec{e}| \sim N$ at large N). Preset the periodic $f(z)$ and $\varphi(z)$ functions as the Fourier series:

$$f(z) = f_0 + \sum_{n=1}^{\infty} f_n \cdot \cos n\varkappa z \quad ; \quad \varphi(z) = \varphi_0 + \sum_{n=1}^{\infty} \varphi_n \cdot \cos n\varkappa z \,, \qquad (3)$$

where $\varkappa = \frac{2\pi}{\lambda}$, and $f(z)$ and $\varphi(z)$ are taken even for simplicity. In this case, for field (1) we have:

$$\vec{H} = \frac{2J}{cS_f^2} \cdot \vec{e} \cdot \sum_{k=1}^{\infty} h_k \cdot \cos k\varkappa z \,, \qquad (4)$$

where

$$h_k = \sum_{n=1}^{\infty} \frac{(-1)^n}{2^n \cdot n!} \cdot \Big\{ (k\varkappa S_f)^{n+1} \cdot K_{n+1}(k\varkappa S_f) \cdot \sum_{k_2=1}^{\infty} \varepsilon_{k_2} \cdots \sum_{k_n=1}^{\infty} \varepsilon_{k_n} \cdot \sum_{m=2}^{n} C_{m-1}^{n-1} \cdot a_m -$$

$$- \delta_0 \cdot \left(\frac{S_f}{S_\varphi}\right)^2 \cdot (k\varkappa S_\varphi)^{n+1} \cdot K_{n+1}(k\varkappa S_\varphi) \cdot \sum_{k_2=1}^{\infty} \tilde{\varepsilon}_{k_2} \cdots \sum_{k_n=1}^{\infty} \tilde{\varepsilon}_{k_n} \cdot \sum_{m=2}^{n} C_{m-1}^{n-1} \cdot b_m \Big\}$$

$$S_f^2 = f_0^2 + \frac{1}{2} \cdot \sum_{n=1}^{\infty} f_n^2 \quad ; \quad S_\varphi^2 = \varphi_0^2 + \frac{1}{2} \cdot \sum_{n=1}^{\infty} \varphi_n^2$$

$$C_m^n = \frac{n!}{(n-m)!\, m!} \qquad (5)$$

$$\varepsilon_k = \frac{1}{S_f^2}\cdot\left\{2f_0 f_k + \sum_{n=1}^{\infty} f_{n+k} f_n + \frac{1}{2}\sum_{n=1}^{k-1} f_{k-n} f_n\right\}$$

$$\tilde{\varepsilon}_k = \frac{1}{S_\varphi^2}\cdot\left\{2\varphi_0\varphi_k + \sum_{n=1}^{\infty} \varphi_{n+k}\varphi_n + \frac{1}{2}\sum_{n=1}^{k-1} \varphi_{k-n}\varphi_n\right\}$$

$$a_m = \begin{cases} 0 & ;\ K^+ \le 0 \\ \varepsilon_{K^+} & ;\ K^+ > 0, K^- \le 0 \\ \varepsilon_{K^+} + \varepsilon_{K^-} & ;\ K^\pm > 0 \end{cases} \quad ; \quad b_m = \begin{cases} 0 & ;\ K^+ \le 0 \\ \tilde{\varepsilon}_{K^+} & ;\ K^+ > 0, K^- \le 0 \\ \tilde{\varepsilon}_{K^+} + \tilde{\varepsilon}_{K^-} & ;\ K^\pm > 0 \end{cases}$$

$$K^\pm = -(K_1 + \ldots + K_m - \ldots - K_n) \pm K$$

$K_n(x)$ are McDonald's functions. When deriving (4), the vanish condition for the direct field component is taken into account:

$$2\cdot\left(\frac{1}{S_f^2} - \frac{\delta_0}{S_\varphi^2}\right) + \lim_{K\to 0} h_k = 0 \qquad (6)$$

which determines the parameter δ_0 .

From formulae (4), (5) it follows that essential are only those field harmonics whose k numbers are small as compared to $2\pi S_{f,\varphi}/\lambda$; at larger values of K , the harmonic amplitude decreases exponentially due to McDonald's function.

The electron trajectories in field (4) are described by Hill's equations of the form:

$$\begin{cases} \dfrac{d^2x}{dz^2} + K_0\cdot\sum\limits_{K=1}^{\infty} h_k\cdot\cos k\varkappa z \cdot x = 0 \\ \dfrac{d^2y}{dz^2} - K_0\cdot\sum\limits_{K=1}^{\infty} h_k\cdot\cos k\varkappa z \cdot y = 0 \end{cases} \qquad (7)$$

where $K_0 = \dfrac{8Je}{mc^3\beta\gamma S_f^2}\cdot ctg\dfrac{\pi}{N}$, e is electron charge, β is velocity in units of light velocity, γ is Lorentz factor.

If in expansion of (4) field the constant term had been preserved, it would have entered Eqs. (7) with different signs, and then one of the equations would have given unstable solutions. Therefore the condition (6) is necessary for the stability of paraxial trajectories in both directions. On the other hand, different signs standing before the alternating part of field in (7) do not prevent from coming into stability region both in x and y directions (see, e.g. [3]). If, in particular, $2\pi S_{f,\varphi}/\lambda \gtrsim 1$ and one can neglect in (4) all harmonics except the first one, then the Hill equations transform into Mathieu's equations, and at $K_0 < 1$ solutions of (7) are in the first stability region of Mathieu's equations. The calculation of betatron oscillations by Eqs.(7) can be done by one of the methods cited, e.g., in [4] .

Here the following should be noted. The transverse motion is determined by amplitudes h_n of magnetic field harmonics which by relation (5) are expressed through amplitudes f_n and φ_n of expansions (3) and may vary. This circumstance can be used to suppress certain harmonics (e.g. those close to resonance ones) in the electron oscillation spectrum. Besides, the magnetic field period in the lens can be optimized by the lens focusing strength.

The numerical values of the field gradient can be estimated according to the following model. Let $f(z)$ and $\varphi(z)$ functions, presetting the conductors bend shape, be step functions: $f(z) = r_0$ at $\lambda\cdot(n-\frac{1}{2}) < z < \lambda n$ and $f(z) = r_1$ at $\lambda n < z < \lambda\cdot(n+\frac{1}{2})$, $r_1 > r_0$, $n = 0,\pm1,\pm2,\ldots,$ and $\varphi(z) = f(z + \frac{\lambda}{2})$. If the lens winding is cooled by running water, then the maximal intensity of current transmitted through the winding can be found by equalizing the heat released in the conductors to that taken off by water. Let the interval between the i-th and (i + 1)-th conductors at a distance of r_0 from the axis be equal to the conductor diameter, and the convective heat transfer coefficient in Newton-Richmann's formula be equal to $2 \cdot 10^4 \cdot w$ (m/sec) [5] (w is the water speed). Then the gradient H' of the lens magnetic field can be evaluated by the formula:

$$\dot{H}' \simeq 8.1 \cdot \sqrt{w \text{ (m/sec)}} \quad \text{(kgauss/cm)}$$

Having a knowledge of the cooled region aperture, one can connect H' with the water flow rate; e.g., at r_0 = 3 cm, r_1 = 10 cm and 0.5 l/sec water rate $H' \simeq$ 1 kgauss/cm. One should bear in mind the approximateness of these estimates, which is due to inaccuracy in the determination of the heat transfer coefficient. At large currents the interaction forces between the conductors in winding increase, and hence the requirements for the conductors fixing increase too. The presence of the fixing elements, which are not considered here, results in worsening of heat transfer, so the real values of the gradient will be lower. The gradient will come out higher when using superconductors.

The proposed magnetic AG focusing lens can be used for the focusing of the precision beams, since the phase volume of such beams enables one to reduce the lens aperture, due to which the focusing strength of these lenses enhances. Simultaneously with the aperture reduction there appears a possibility to shorten the magnetic field period without decreasing its amplitude. The demagnetization factor does not allow one to use the ferromagnetic materials in this case.

REFERENCES

1. Оганджанян А.А. Жесткофокусирующая проводочная структура с током. Препринт ЕФИ-901(52)-86,Ереван,1986.
2. Моносзон Н.А. Особенности конструкций сверхпроводящих магнитов синхротронов высоких энергий. В кн.: Труды VI Всесоюзного совещания по ускорителям заряженных частиц, Дубна,1979, т.II,с.368.
3. Стретт М.Д.О. Функции Ляме, Матье и родственные им в физике и технике. Харьков, ГНТИ Украины, 1935.
4. Коломенский А.А.,Лебедев А.Н. Теория циклических ускорителей. М.: Физматгиз, 1962 .
5. Михеев М.А.,Михеева И.М. Основы теплопередачи.М.: Энергия, 1977.

The manuscript was received 4 November 1986

А.А.ОГАНДЖАНЯН

МАГНИТНАЯ ЛИНЗА ДЛЯ ФОКУСИРОВКИ ПРЕЦИЗИОННЫХ ПУЧКОВ

(на английском языке, перевод Асланян З.Н.)

Редактор Л.П.Мукаян

Технический редактор А.С.Абрамян

Подписано в печать 29/XII-86г. ВФ-06425 Формат 60х84/16
Офсетная печать. Уч.изд.л.0,5 Тираж 299 экз. Ц. 8 к.
Зак.тип. 697 Индекс 3624

Отпечатано в Ереванском физическом институте
Ереван 36, Маркаряна 2

индекс 3624

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ