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L.S.DULYAN

**THE CALCULATION OF QCD TRIANGULAR FEYNMAN GRAPHS
IN EXTERNAL GLUONIC FIELD USING THE REDUCE-2 SYSTEM**

ЦНИИатоминформ

ЕРЕВАН-1986

Լ. Ս. ԳՈՒԼՅԱՆ

ԱՐՏԱՔԻՆ ԳԼՅՈՒՈՆԱՑԻՆ ԴԱՇՏՈՒՄ ՔԻՄ ՖԵՅՄԱՆԻ
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REDUCE -2 ՀԱՄԱԿԱՐԳԻ ՕԳՆՈՒՓՅԱՄԲ

Նկարագրված են այն եղանակները, որ կիրառվել են REDUCE -2
համակարգի օգնությամբ Քիմ ֆեյմանի եռանկյունի դիագրամների հաշվար-
կման ժամանակ: Զննարկված են REDUCE -2 համակարգի առանձնահատկու-
թյունների հետ առնչվող դժվարությունները:

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Л.С.ДУЛЬЯН

ВЫЧИСЛЕНИЕ ТРЕУГОЛЬНЫХ ФЕЙНМАНОВСКИХ ДИАГРАММ КХД
ВО ВНЕШНЕМ ГЛУБОКОМ ПОЛЕ ПРИ ПОМОЩИ СИСТЕМЫ REDUCE-2

Описаны методы и приемы, применявшиеся при вычислении треугольных фейнмановских диаграмм КХД при помощи системы REDUCE-2. Обсуждаются возникшие при этом трудности, связанные со специфическими особенностями системы REDUCE-2.

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THE CALCULATION OF QCD TRIANGULAR FEYNMAN GRAPHS
IN EXTERNAL GLUONIC FIELD USING THE REDUCE-2 SYSTEM

Methods and ways applied in the calculation of the QCD triangular Feynman graphs with the use of the REDUCE-2 system are described. The encountered difficulties connected with the specific features of the REDUCE-2 system are discussed.

Yerevan Physics Institute

Yerevan 1986

1. Introduction.

The system of analytical calculations, REDUCE-2 [1] was successfully used by us in the calculation of widths of radiative transitions between different levels of quarkonium - the particle consisting of heavy quarks.

The physical essence of the problem as well as results obtained are presented in Ref. [2], while here we describe the computer algebra abilities. Our aim was to calculate contributions to the process amplitude from the set of Feynman graphs that describe the interaction of quarks with external gluonic field (see Fig.1). Recipes for such kind of calculations are cited in manuals of quantum electrodynamics. The obstacle encountered here is that at intermediate steps of calculations one has to deal with highly extended expressions, the hand-made calculation taking too much time and being fraught with errors. Therefore we have made use of the REDUCE-2 system - the most powerful and universal of those available at our disposal.

2. Calculation Steps.

The calculation of contribution of the Fig.1 diagrams to the process amplitude comes to the calculation of several integrals over 4-dimensional

momentum space as follows:

$$I = \int d^4\ell \frac{Sp_{\mu\nu\dots}(p, \kappa, \ell)}{p^a K^b L^c} \quad (1)$$

where p and κ are external momenta; $P = (p - \ell)^2 - m^2$, $K = (\kappa + \ell)^2 - m^2$, $L = \ell^2 - m^2$ are denominators of fermion propagators entering the amplitude; m is the quark mass, and $Sp_{\mu\nu\dots}(p, \kappa, \ell)$ is a trace of product of 12 and more γ -matrices (with respect to total angular momentum of decaying particle).

The calculation of traces is done in a standard way using the high energy physics package built in the REDUCE. It should be noted that the calculation of traces of the form of $Sp\{\gamma_\mu(\hat{r}_1 + m)\dots\gamma_\nu(\hat{r}_n + m)\}$ with further substitution $r_1 = \kappa + \ell, \dots, r_n = p - \ell$ is performed more quickly than the direct calculation of traces $Sp\{\gamma_\mu(\hat{K} + \hat{\ell} + m)\dots\gamma_\nu(\hat{p} - \hat{\ell} + m)\}$.

All possible scalar products of the 4-vectors are expressed through invariants $m^2, S_1 = (\kappa + p)^2, S_2 = p^2, K, L, P$ using the LET substitution ability.

In order to cancel K, L, P arisen in numerator and those in denominator we switch on the DIV flag. The resultant integrand in (1) comes out as a sum of great number (100 + 300) of terms:

$$\sum_i \frac{D_i}{p^{a_i} K^{b_i} L^{c_i}}$$

where coefficients D_i are constructed from m^2, S_1, S_2 and components of 4-vectors p, κ, ℓ . Note that some of a_i, b_i, c_i may be zero.

Further, in order to carry out integration over $d^4\ell$, one should first realize Feynman parametrization of integrand. Since the denominator's terms each having three factors $p^{a_i}, K^{b_i}, L^{c_i}$, are parametrized otherwise against those with two factors, the whole sum is to be divided into groups

which subsequently are to be transformed separately. Such division can readily be done by equalizing P^{-1} , K^{-1} and L^{-1} to zero one by one, this being possible at the RAT flag switched on.

The REDUCE-2 system does not allow direct substitutions $XYZ \rightarrow F$, and especially $X^\alpha Y^\beta Z^c \rightarrow F(\alpha, \beta, c)$; therefore, for the realization of formulae

$$\frac{1}{P^\alpha K^\beta L^c} = \frac{(\alpha + \beta + c - 1)!}{(\alpha - 1)! (\beta - 1)! (c - 1)!} \int_0^1 dx \int_0^x dy \frac{(1-x)^{\alpha-1} (x-y)^{\beta-1} y^{c-1}}{[P(1-x) + K(x-y) + Ly]^{\alpha+\beta+c}},$$

$$\frac{1}{P^\alpha K^\beta} = \frac{(\alpha + \beta - 1)!}{(\alpha - 1)! (\beta - 1)!} \int_0^1 dx \frac{(1-x)^{\alpha-1} x^{\beta-1}}{[P(1-x) + Kx]^{\alpha+\beta}}$$

a special program is written. This program, via successive use of $P^\alpha \rightarrow F(\alpha)$ type substitutions, performs these transformations, then makes a relevant shift of the integration variable ℓ and finally carries out integration over 4-momentum space according to the formula:

$$\int \frac{d^4 \ell}{(\ell^2 - \alpha^2)^n} = \frac{i\pi^2 (-1)^n}{(n-1)(n-2)\alpha^{2(n-2)}} \quad (2)$$

Here account is taken of the fact that integrals containing odd power of ℓ in numerator are zero, while those with even powers of ℓ are expressed through (2).

After integration over $d^4 \ell$ one should extract from the amplitude the kinematical factors constructed from the external momenta components, the metric tensor $g_{\mu\nu}$ and antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$. In the pseudoscalar case it is $P_\alpha K_\beta \epsilon_{\mu\nu\alpha\beta}$, in the scalar case $g_{\mu\nu}$ and $P_\mu K_\nu / (K P)$, while in the axial case $S_1 = K_\alpha \epsilon_{\alpha\mu\nu\sigma}$, $S_2 = P_\alpha \epsilon_{\alpha\mu\nu\sigma}$ and $S_3 = P_\mu P_\alpha K_\beta \epsilon_{\alpha\beta\nu\sigma} / (K P)$. Each of them should be factorized. It is known that in virtue of gradient invariance the amplitude must be proportional to

$g_{\mu\nu} = P_{\mu} K_{\nu} / (KP)$ in the scalar case, while in the axial case there exist two independent structures: S_1 and $S_2 - S_3$. The mutual equality of the expressions standing before the relevant structures serves as a good criterion for correctness of the program run.

Each of expressions obtained is now a sum of large number of single and double integrals over Feynman parameters:

$$\sum_i \int_0^1 d\alpha \int_0^x dy \frac{d_i (1-\alpha)^{\alpha_i} y^{\beta_i} (x-y)^{\delta_i}}{\alpha^{2R_i}} + \sum_{j=1}^3 \sum_i \int_0^1 \frac{d_{ij} \alpha^{\alpha_{ij}} (1-\alpha)^{\beta_{ij}}}{\alpha_j^{2R_{j1}}} \quad (3)$$

where $\alpha^2 = m^2 - S_1(1-\alpha)(x-y) - S_2(x-y)y$, $\alpha_1^2 = m^2 - S_2(1-\alpha)\alpha$, $\alpha_2^2 = m^2 - S_1(1-\alpha)\alpha$, $\alpha_3^2 = m^2$, and d_i and d_{ij} contain m^2 , S_1 and S_2 only.

One can, in principle, subject this expression to numerical integration at various values of S_1 and S_2 and thus obtain solution of the problem.

Another approach is to calculate all derivatives over S_1 and S_2 from (3) at some point; in particular, we considered derivatives at $S_1 = S_2 = 0$.

The differentiation and integration being exchanged for convenience, the result can be obtained in the analytical form.

In order to differentiate the integrands in (3) N times with respect to S_1 and K times with respect to S_2 and then to put $S_1 = S_2 = 0$, we have introduced a new operator that possesses all properties of the usual differentiation operator. Evidently, the DF operator built in the REDUCE does not suit, for it cannot help with calculation of N -th order derivative until N gets a definite integer value.

Such an operator

$$D(N, K, X) = \frac{1}{N! K!} \frac{\partial^N}{\partial S_1^N} \frac{\partial^K}{\partial S_2^K} X \Big|_{S_1 = S_2 = 0}$$

possessing necessary linearity properties is introduced by a set of substitu-

tion rules of the type:

$$\text{FORALL } N, K, X, Y \quad \text{LET } D(N, K, X+Y) = D(N, K, X) + D(N, K, Y), \\ D(N, K, S1 * X) = D(N-1, K, X)$$

and so on. Note that in the versions of the REDUCE-3 system [3] this can be realized far more easily by using the DEPEND declaration.

Due to the action of $D(N, K, X)$ operator on (3), the integrands acquire the form:

$$\sum_i \tilde{d}_i (1-x)^{\tilde{\alpha}_i} y^{\tilde{\beta}_i} (x-y)^{\tilde{\gamma}_i} \quad \text{and} \quad \sum_{j=1}^3 \sum_i \tilde{d}_{ji} (1-x)^{\tilde{\beta}_{ji}} x^{\tilde{\alpha}_{ji}}$$

where $\tilde{\alpha}_i, \tilde{\beta}_i, \tilde{\gamma}_i, \tilde{\alpha}_{ji}, \tilde{\beta}_{ji}$ are sums of N, K and integers; \tilde{d}_i and \tilde{d}_{ji} are numerical multipliers (m to the appropriate power is already factored out).

It is left to make use of the known formulae:

$$\int_0^1 dx \int_0^x dy (1-x)^\alpha y^\beta (x-y)^\delta = \frac{\alpha! \beta! \gamma!}{(\alpha + \beta + \gamma - 2)!}, \\ \int_0^1 dx x^\alpha (1-x)^\beta = \frac{\alpha! \beta!}{(\alpha + \beta - 1)!}$$

but their realization within the REDUCE system encounters some troubles. For example, the expression X^{N+2} is written by the REDUCE as $X^N * X^2$, i.e. it separates symbolic and numerical exponents, this being caused by intrinsic representation of expressions in the system. Therefore the substitution $X^N \rightarrow N!$ brings to a wrong result when X^{N+2} is replaced by $N! 2!$ instead of $(N+2)$.

This difficulty is managed to avoid by transforming all numerical exponents to symbolic ones via the replacement $X^2 \rightarrow X^{F(2)}$ with a fictitious

operator F , and only at the very end replacing $F(2)$ by 2.

This way requires large memory and time resources; therefore we used in the particular cases other, more economic though less generalized ways.

To our mind, the solution of the problem of "non-separation" of symbolic and numerical exponents would enrich and beautify the REDUCE system.

After integration over Feynman parameters, the transformed expressions acquire a form of a sum of still large number of fractions constructed of factorials $(N+i_1)!$, $(K+i_2)!$, $(N+K+i_3)!$, $(2N+2K+i_4)!$, where $i_{1,2,3,4}$ are integers. Note that while N and K have no definite integer value, the factorial is not defined being perceived by the system as a function without any properties. Having equipped this function with the property $(N+M)! = N!(N+1)(N+2)\dots(N+M)$ using the FORALL M LET ... substitution, one can attain some cancellations in numerators and denominators of the fractions. The final result is obtained by reduction of fractions to a common denominator with the GCD flag switched on. Although the system had to divide by each other high-power polynomials of two variables, it coped with this successfully, so in all cases under consideration there came out rather compact expressions of the form:

$$\frac{(N+K)!(N+K+i_1)!}{(2N+2K+i_2)!} P(N,K)$$

where $i_{1,2}$ are integers, and $P(N,K)$ is a polynomial over N and K of power not higher than four with integer coefficients. These formulae are just a solution of the problem stated.

3. Conclusion.

Our designed program complex for the REDUCE system is intended for the calculation of Feynman diagrams that describe the interaction of the quarks

with the gluonic condensate. The programs of this complex not only allow one to escape from cumbersome calculations at some steps, but also automatize the whole process from the beginning to the end ensuring us from errors and give an answer in the form of a compact formula.

The complex possesses certain universality in the sense that with its help the diagrams for particles with various quantum numbers can be calculated immediately. As a test we have reproduced the results of Refs. [4,5] where similar calculations were carried out without a computer. At present, these programs are used to calculate processes with tensor and hybrid states.

The experience accumulated during creation and exploitation of the cited programs allows us to make the following remark: when processing very cumbersome expressions we often encountered the computer storage deficiency. This problem can be avoided via artificial division of the cumbersome expressions into pieces. If one considers these pieces as elements of array $A(I)$ and, introducing some substitution rule by means of the LET operator, then requires scanning of the array's elements by the instruction `FOR I: = M:N DO A(I): = A(I)`, the substitution will not be performed; whereas, by the explicit instruction `A(1):=A(1); A(2):=A(2);etc.` the substitution is performed.

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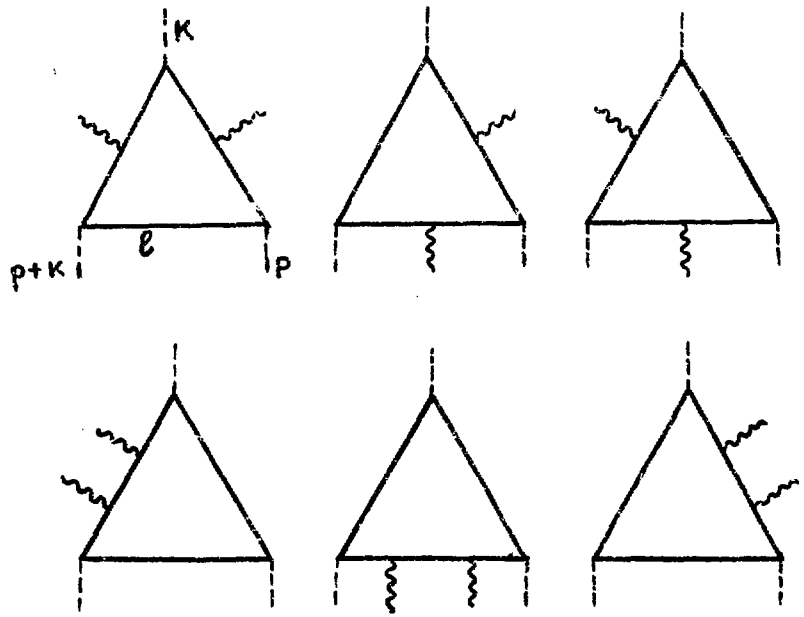


Fig. 1. Set of graphs describing the interaction of quarks with external gluonic field.

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