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1/N-EXPANSION IN TWO-DIMENSIONAL SUPERSYMMETRIC O(N)xO(K)-INVARIANT O(N)/O(N-K) MODEL AND DYNAMIC PRODUCTION OF MASSIVE VECTOR PARTICLES

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ԵՐԿՉԱՓ ԳԵՐՀԱՄԱՉԱՓ $O(N)\times O(K)$-ԻՆՎԱՐԻԱՆՏ ՄՈԴԵԼԻ 1/N ՎԵՐԼՈՒԾՈՒՄԸ ԵՎ ԶԱՆԳՎԱԾԵՂ ՎԵԿՏՈՐԱՅԻՆ ՄԱՍՆԻԿՆԵՐԻ ԴԻՆԱՄԻԿ ԾՆՈՒՄԸ

$1/N$ վերլուծման շրջանակներում ուսումնասիրված է գերհամաչափ $O(N)\times O(K)$-ինվարիանտ $O(N)/O(N-K)$ մոդելը։ Ցույց է տրված, որ ըստ $1/N$-ի հիմնական կարգում դինամիկորեն ծնվում են զանգվածեղ դաշտերի գերմուլտիպլետներ, որոնց մեջ առկա են նաև վեկտորային դաշտեր։

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А.А.ДЕРИГЛАЗОВ**

$1/N$ – РАЗЛОЖЕНИЕ В ДВУХМЕРНОЙ СУПЕРСИММЕТРИЧНОЙ $O(N)\times O(K)$ – ИНВАРИАНТНОЙ $O(N)/O(N-K)$ МОДЕЛИ И ДИНАМИЧЕСКОЕ ОБРАЗОВАНИЕ МАССИВНЫХ ВЕКТОРНЫХ ЧАСТИЦ

В рамках $1/N$ – разложения исследована суперсимметричная $O(N)\times O(K)$ – инвариантная $O(N)/O(N-K)$ модель. Показано, что в основном по $1/N$ порядке динамически образуются массивные супермультиплеты полей, среди которых присутствуют и векторные поля.

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1/N-EXPANSION IN TWO-DIMENSIONAL SUPERSYMMETRIC O(N)×O(K)-INVARIANT O(N)/O(N-K) MODEL AND DYNAMIC PRODUCTION OF MASSIVE VECTOR PARTICLES

The supersymmetric O(N)×O(K)-invariant O(N)/O(N-K) model is investigated in the framework of 1/N-expansion. It is shown that the massive supermultiplets of fields (vector fields among them) are dynamically produced in the leading over 1/N order.

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1. A dynamic production of vector fields is known to be characteristic of chiral models. This phenomenon has been investigated in detail in symmetric CP^{N-1} [1] and $O(N)/O(N-K)\times O(K)$ [2] models. The vector fields emerging in these models appear to be massless. In $O(N)\times O(K)$-invariant models on Stiefel's manifolds $O(N)/O(N-K)$, as shown in [3], the production of massive vector fields is possible. These models are not symmetric, but belong to a larger class of invariant models on homogeneous spaces. Having lower symmetry than the symmetric theories have, they can include several coupling constants. This fact leads to new phenomena lacking in symmetric theories. The method for construction of such models is suggested in [4].

In the present work, in the framework of the 1/N-expansion, a model is investigated which is the supersymmetric generalization of $O(N)\times O(K)$-invariant $O(N)/O(N-K)$ model. It is shown that the massive supermultiplets of fields (vector fields among them) are dynamically produced in the leading over 1/N order. The $O(N)\times O(K)$-symmetric phase of the theory is homogeneous in contrast to the usual case [3] when it is inhomogeneous and, in its turn, consists of two phases differing from each other by the presence of massive vector particles.

The paper is organized as follows. In the section 2 the effective potential of the model is calculated in the leading over 1/N order; in section 3 the spectrum of the particles present in the theory is investigated for its $O(N)\times O(K)$-symmetric phase which turns to be supersymmetric and in which the absolute minimum of effective potential is realized.

2. So, let us consider the theory described by the action

$$S=\int d^2x\,d^2\theta\,\frac{1}{2}\left\{(\bar{D}^{\alpha}G^{ia})(D_{\alpha}G^{ia})-\frac{\lambda_2}{4N}(G^{ia}\overleftrightarrow{\bar{D}^{\alpha}}G^{ja})(G^{i\beta}\overleftrightarrow{\bar{D}_{\alpha}}G^{j\beta})\right\}, \tag{1}$$
$$G^{ia}\overleftrightarrow{\bar{D}_{\alpha}}G^{ja}=G^{ia}D_{\alpha}G^{ja}-(D_{\alpha}G^{ia})G^{ja},$$

where $G^{ia}(x,\theta)=g^{ia}(x)+\bar{\theta}\psi^{ia}(x)+\delta(\theta)F^{ia}(x)$ is the $K\times N$-matrix real superfield on which the following constraint is imposed

$$G^{ia}(x,\theta)G^{ja}(x,\theta)=\frac{N}{\lambda_1}\delta^{ij},\quad a,b,\ldots=1,\ldots,N;\ i,j,\ldots=1,\ldots,K \tag{2}$$

(summation over repeating indices is supposed), θ_{α} is a two-component Majorana spinor, D_{α} is a covariant derivative, λ_1 and λ_2 are coupling constants ($\lambda_1>0$, $\lambda_1>\lambda_2$; the meaning of the second condition will become clearer later).

Note, that the case of $K=1$ corresponds to the model $O(N)\times O(N-1)$ and is considered in [5].

For more convenience in further calculations, let us use the standard procedure of introduction of auxiliary fields, namely, the auxiliary Majorana superfield $\phi_{\alpha}^{ia}(x,\theta)$ which allows to get rid of the quadruple interaction in the action (1), and also the superfield $\omega^{ij}(x,\theta)$ which is the Lagrange multiplier for the constraint (2), and rewrite the action (1) as $S=\int d^2x\,d^2\theta\,\frac{1}{2}\left\{(\bar{D}^{\alpha}G^{ia})(D_{\alpha}G^{ia})+\bar{\phi}^{\alpha,ij}(G^{ia}\overleftrightarrow{\bar{D}_{\alpha}}G^{ja})+\right.$

$$+\frac{N}{\lambda_2}\bar{\Phi}^{\alpha,ij}\Phi_\alpha^{ij}-2\omega^{ij}\left(G^{ia}G^{ja}-\frac{N}{\lambda_1}\delta^{ij}\right)\Big\} \cdot \tag{3}$$

The generating functional of Green's function of the theory is

$$Z(J)=\int dG^{ia}d\Phi d\omega \exp\left\{i\left[S+\int d^2x d^2\theta\, J^{ia}G^{ia}\right]\right\}, \tag{4}$$

where S is given by the expression (3).

Let us investigate the possibility for spontaneous breaking of the symmetry of the theory by the nonzero vacuum expectation values of $G^{ia}(x,\theta)$ and $\omega^{ij}(x,\theta)$ superfields (the vacuum expectation value of the superfield $\Phi_\alpha^{ij}(x,\theta)$ is assumed to be zero). For this purpose let us reduce the functional (4) to a form convenient for apllication of the stationary-phase method at large N. Then, choose boundary conditions for the superfield $G^{ia}(x,\theta)$ in (4) as: $G^{ia}(x,\theta)\longrightarrow(G^{ij}(\theta),0)$ at $|x|\to\infty$, where G^{ij}, as distinguished from G^{ia}, is a square matrix of dimension K. It, obviously, does not restrict the generality, since this choice of "coordinates" in the space of matrices G^{ia} when $G^{ia}(x,\theta)\Big|_{|x|\to\infty}=0$ for $a=K+1,\dots,N$, may always be realized by $O(N)$ rotations. Integrating in (4) over $G^{ia}(x,\theta)$, $a=K+1,\dots,N$ one obtains

$$Z(J)=\int dG^{ij}d\Phi d\omega \exp\left\{i\left[NS_1+S_2\right]\right\},$$

$$S_1=\frac{i}{2}\operatorname{str}\ln M^{ij}(1,2)+\int\left[\frac{1}{2}G^{iK}(1)M^{ij}(1,2)G^{jK}(2)+\frac{1}{2\lambda_2}\bar{\Phi}^{\alpha,ij}(1)\Phi_\alpha^{ij}(1)+\frac{1}{\lambda_1}\omega^{ii}(1)\right],$$

$$S_2=-\frac{iK}{2}\operatorname{str}\ln M^{ij}(1,2)+\int\left[-\frac{1}{2}\sum_{a=K+1}^{N}J^{ia}(1)(M^{-1})^{ij}(1,2)J^{ja}(2)+N^{\frac{1}{2}}G^{iK}(1)J^{iK}(1)\right], \tag{5}$$

$$M^{ij}(1,2)=\left[-\delta^{ij}\bar{D}^{\alpha}D_{\alpha}+\bar{\Phi}^{\alpha,ij}(1)D_{\alpha}(1)+\bar{\Phi}^{\alpha,ji}(2)D_{\alpha}(2)-2\omega^{ij}(1)\right]\delta(1,2),$$

where $1\longleftrightarrow(x_{1\mu},\ \theta_{1\alpha})$, $2\longleftrightarrow(x_{2\mu},\theta_{2\alpha})$ and G^{ij} is substituted by $\sqrt{N}G^{ij}$. The system stability points G_s^{ij} and ω_s^{ij} will be found from the condition of minimum of the effective potential V . Restricting to their constant values, let us represent G_s^{ij} and ω_s^{ij} in the form of $G_s^{ij}=g_s^{ij}+\delta(\theta)F_s^{ij}$, $\omega_s^{ij}=\rho_s^{ij}+\delta(\theta)\sigma_s^{ij}$ (the vacuum expectation values of the fermion components of G^{ij} and ω^{ij} superfields are equal to zero under requirements for Lorentz-invariance). Following the technique used in ref.[5], we obtain the expression for the effective potential of the system in the leading over 1/N order

$$V(G_s,\omega_s)\int d^2x=-\frac{1}{2}\,\mathrm{str}\,\ln M_s^{ij}(1,2)+$$

$$+\int d^2\theta\left[\frac{1}{2}G_s^{ik}(\theta)\left(\delta^{ij}\frac{\partial}{\partial\theta_\alpha}\ \frac{\partial}{\partial\theta^\alpha}+2\omega_s^{ij}(\theta)\right)G_s^{jk}(\theta)-\frac{1}{\lambda_1}\omega_s^{ii}(\theta)\right]\int d^2x \quad (6)$$

For simplicity assume, that even if the initial O(N)×O(K)-symmetry is broken by the vacuum state, the O(N-K)×O(K)-symmetry is preserved. As O(K)-subgroup let us choose the diagonal O(K)-subgroup of the group O(K)×O(K) which, in its turn, is the subgroup of O(N)×O(K). Then the values of G_s^{ij} and ω_s^{ij} will be tensors of second rank, invariant under the subgroup O(K) and they can be written as $G_s^{ij}=\delta^{ij}(g_s+\delta(\theta)F_s)$, $\omega_s^{ij}=\delta^{ij}(\rho_s+\delta(\theta)\sigma_s)$. Substituting these expansions into (6) and integrating over θ , we obtain the following expression for V .

$$V(G_s,\omega_s)=K\left[-\frac{i}{2}\int\frac{d^2K}{(2\pi)^2}\ell n\left(1-\frac{\sigma_s}{K^2-\rho_s^2}\right)-F_s^2+2\rho_s F_s g_s+g_s^2\sigma_s-\frac{1}{\lambda_1}\sigma_s\right].$$

The F_s and σ_s components can be excluded using the stationary conditions $\partial V/\partial F_s=0$ and $\partial V/\partial\sigma_s=0$ Then, integrating over the momenta we finally find that

$$V(G_s,\omega_s)=K\left\{-\frac{\rho_s^2}{8\pi}\left(1-\ell n\frac{\rho_s^2}{\mu^2}\right)+\frac{\rho_s^2}{\lambda_{1R}}+\frac{\mu^2}{8\pi}\exp\left[8\pi\left(g_s^2-\frac{1}{\lambda_{1R}}\right)\right]\right\} \qquad (7)$$

where λ_{1R} is the renormalized charge introduced by the relation

$$\frac{1}{\lambda_1}=\frac{1}{\lambda_{1R}}+\frac{i}{2}\int\frac{d^2K}{(2\pi)^2}\,\frac{1}{K^2-\mu^2}\,.$$

The potential (7) has a minimum at the following values of g_s and ρ_s :

$$g_s=0\,,\quad \rho_s^2=m^2=\mu^2\exp\left(-\frac{8\pi}{\lambda_{1R}}\right), \qquad (8)$$

and $F_s=\sigma_s=0$. The solution (8) corresponds to the explicit supersymmetric phase, and, besides, has initial $O(N)\times O(K)$ invariance. The further calculations will be carried out only for that phase.

3. Now let us consider a new superfield $\omega'^{ij}(x,\theta)=\omega^{ij}(x,\theta)-m\delta^{ij}$, the vacuum expectation value of which is zero, and integrate over the superfields $G^{ij}(x,\theta)$ in the functional integral (5). We'll obtain

$$Z(J)=\int d\Phi\, d\omega'\exp\left[-\frac{i}{2}\int d1d2\, J^{ia}(1)(M'^{-1})^{ij}(1,2)J^{ja}(2)+iS(\Phi,\omega')\right], \qquad (9)$$

where

$$S(\Phi,\omega') = \frac{iN}{2}\,\mathrm{str}\,\ell n\, M'^{ij}(1,2) + \int d1 \left[\frac{N}{2\lambda_2}\bar{\Phi}^{\alpha,ij}(1)\,\bar{\Phi}^{ij}_{\alpha}(1) + \frac{N}{\lambda_1}\omega'^{ii}(1)\right],$$

$$M'^{ij}(1,2) = \left[\delta^{ij}(-2m - \bar{D}D) + \bar{\Phi}^{\alpha,ij}(1)D_\alpha(1) + \bar{\Phi}^{\alpha,ji}(2)D_\alpha(2) - 2\omega'^{ij}(1)\right]\delta(1,2).$$

To obtain the 1/N expansion, one must expand the exponent in (9) in powers of superfields ϕ^{ij}_α and ω'^{ij}. In the expansion of $S(\phi,\omega')$ the coefficient of the first power of the superfield ω'^{ij} turns to be zero owing to the stability equation $\partial V/\partial\sigma_s = 0$. As to linear over the superfield ϕ^{ij}_α terms, they will also lack due to the entisymmetry of ϕ^{ij}_α. The next terms of the expansion give the free propagators of these superfields. Since in this case we are interested in the structure of free propagators, we'll consider only the terms quadratic over Φ and ω' in the expansion of $S(\Phi,\omega')$. The quadratic part of $S(\Phi,\omega')$ (let us express it by $S_o(\Phi,\omega')$) has the form

$$S_0(\Phi,\omega') = \int d1d2 \left[\bar{\Phi}^{\alpha,ij}(1)(\Pi^{-1})^{\beta}_{\alpha}(1,2)\,\Phi^{ij}_{\beta}(2) + \omega'^{ij}(1)\,\mathcal{D}^{-1}(1,2)\,\omega'^{ij}(2)\right],$$

where Π^{-1} and $\mathcal{D}^{-1}$ are given by the expressions

$$(\Pi^{-1})^{\beta}_{\alpha}(1,2) = \frac{iN}{2}\Big[D_\alpha(1)M_0^{-1}(1,2)\cdot\bar{D}^{\beta}(2)M_0^{-1}(1,2) -$$

$$- M_0^{-1}(1,2)\cdot D_\alpha(1)\bar{D}^{\beta}(2)M_0^{-1}(1,2) - \frac{i}{\lambda_2}\delta^{\beta}_{\alpha}\,\delta(1,2)\Big],$$

$$\mathcal{D}^{-1}(1,2) = -iN M_0^{-1}(1,2)\,M_0^{-1}(1,2), \qquad M_0(1,2) = M'(1,2)\Big|_{\Phi=\omega'=0}.$$

There is divergence in $(\Pi^{-1})_\alpha^\beta$ which is removed by renormalizing the charge λ_2 :

$$\frac{1}{\lambda_2} = \frac{1}{\lambda_{2R}} + \frac{i}{2}\int \frac{d^2\kappa}{(2\pi)^2}\,\frac{1}{\kappa^2-\mu^2} .$$

After simple transformations one obtains the following expressions for $(\Pi^{-1})_\alpha^\beta$ and $\mathcal{D}^{-1}$:

$$(\Pi^{-1})_\alpha^\beta(1,2) = \frac{N}{2}\left(\frac{1}{\lambda_{2R}} - \frac{1}{\lambda_{1R}}\right)\delta_\alpha^\beta\,\delta(1,2) + \tag{10}$$

$$+ \frac{N}{16}\int \frac{d^2\kappa}{(2\pi)^2}\, e^{-i\kappa(x_1-x_2)} \Sigma(\kappa^2)\left\{(2m-\hat{\kappa})\hat{\kappa}\left[\delta(\theta_1-\theta_2) - \frac{\hat{\kappa}}{\kappa^2}\, e^{\bar{\theta}_1\hat{\kappa}\theta_2}\right]\right\}_\alpha^\beta ,$$

$$\mathcal{D}^{-1}(1,2) = \frac{N}{4}\int \frac{d^2\kappa}{(2\pi)^2}\, e^{-i\kappa(x_1-x_2)} \Sigma(\kappa^2)\left[2m\delta(\theta_1-\theta_2) + e^{\bar{\theta}_1\hat{\kappa}\theta_2}\right] , \tag{11}$$

$$\Sigma(\kappa^2) = \frac{1}{2\pi}\int_{4m^2}^{\infty} \frac{dt}{\sqrt{t(t-4m^2)}\,(t-\kappa^2)} . \tag{12}$$

Note, that there is no inverse value for $(\Pi^{-1})_\alpha^\beta$ at $\lambda_{1R} = \lambda_{2R}$ and the superfield Φ_α becomes a gauge one. This situation for the supersymmetric CP^{N-1} -model was considered in [6] .

We'll obtain the propagators of the superfields Φ_α^{ij} and ω'^{ij} by inversing the expressions (10) and (11), respectively:

$$<\Phi_\alpha^{ij}(1)\bar{\Phi}^{\beta,\kappa\ell}(2)> = \left(\delta^{i\kappa}\delta^{j\ell} - \delta^{i\ell}\delta^{j\kappa}\right)\int \frac{d^2\kappa}{(2\pi)^2}\, e^{-i\kappa(x_1-x_2)}\,\Pi_\alpha^\beta(\kappa) ,$$

$$\Pi_\alpha^\beta(\kappa) = \frac{i}{4N\nu\Delta(\kappa^2)}\left[4\nu - \Sigma(\kappa^2)\hat{\kappa}(2m+\hat{\kappa})\right]_\alpha^\gamma \times$$

$$\times\left[\left(2\nu + \frac{1}{4}\Sigma(\kappa^2)\hat{\kappa}(2m-\hat{\kappa})\right)\delta(\theta_1-\theta_2) + \frac{1}{4}\Sigma(\kappa^2)(2m-\hat{\kappa})\, e^{\bar{\theta}_1\hat{\kappa}\theta_2}\right]_\gamma^\beta , \tag{13}$$

$$\Delta(K^2)=4\nu^2-2\nu K^2\Sigma(K^2)-\frac{1}{4}K^2\Sigma^2(K^2)(4m^2-K^2)\,,\qquad \nu=\frac{1}{\lambda_{2R}}-\frac{1}{\lambda_{1R}}\,.$$

$$<\omega'^{ij}(1)\omega'^{ke}(2)>=\left(\delta^{ik}\delta^{je}+\delta^{ie}\delta^{jk}\right)\int\frac{d^2K}{(2\pi)^2}e^{-iK(x_1-x_2)}\mathcal{D}(K)\,, \tag{14}$$

$$\mathcal{D}(K)=\frac{i}{N}\Sigma^{-1}(K^2)\,\frac{2m\delta(\theta_1-\theta_2)-e^{\bar\theta_1\hat K\theta_2}}{4m^2-K^2}\,,$$

$\Sigma^{-1}(K^2)$ is a value inverse to (12). The propagator of the basic superfield $G^{ia}(x,\theta)$ is found from (9):

$$<G^{ia}(1)G^{j\beta}(2)>=\frac{i}{2}\delta^{ij}\delta^{a\beta}\int\frac{d^2K}{(2\pi)^2}e^{-iK(x_1-x_2)}\,\frac{m\delta(\theta_1-\theta_2)+e^{\bar\theta_1\hat K\theta_2}}{K^2-m^2}\,. \tag{15}$$

Now let us thoroughly investigate the obtained expressions. The values of K^2 at which the propagator $\Pi_\alpha^\beta(K^2)$ of the superfield ϕ_α (the multipliers corresponding to isotropic structure will be omitted below) has a pole, is evidently determined from the condition $\Delta(K^2)=0$, which is equivalent to two equations

$$\frac{1}{\lambda_{2R}}-\frac{1}{\lambda_{1R}}=\frac{1}{4}\Sigma(K^2)(K^2+2m\sqrt{K^2}),\quad \frac{1}{\lambda_{2R}}-\frac{1}{\lambda_{1R}}=\frac{1}{4}\Sigma(K^2)(K^2-2m\sqrt{K^2}). \tag{16}$$

Let us consider the first one. The value of $\Sigma(K^2)$ in the range of $0\leqslant K^2<4m^2$ is real and positive which directly follows from the eq.(12); the $\Sigma'(K^2)$, derivative of $\Sigma(K^2)$ over K^2, is also positive. Besides, $\Sigma(0)=1/4\pi m^2$ and $\Sigma(K^2)\to+\infty$ at $K^2\to 4m^2$. The following properties of the function $y_+(K^2)=\Sigma(K^2)(K^2+2m\sqrt{K^2})$ result from it: $y_+(K^2)>0$ in the range of $0<K^2<4m^2$ and monotonously increases with K^2; $y_+(0)=0$, $y_+(K^2)\to+\infty$ at $K^2\to 4m^2$. Thus, the right side of the equation considered is a monotonously growing positively determined value. It is clear that at $\lambda_{1R}<\lambda_{2R}$ this equation has no solutions, while at $\lambda_{1R}>\lambda_{2R}$ it has but one solution. Let us designate the value of K^2, which satisfies this equation, by M^2 $(0<M^2<4m^2)$. Later we shall restrict ourselves to consider the case when $\lambda_{1R}>\lambda_{2R}$. It is easy to see, that the second equation in (16) has no solution, since the function $y_-(K^2)= =\Sigma(K^2)(K^2-2m\sqrt{K^2})$ satisfies the condition $y_-(K^2)<0$ in the investigated range of K^2.

Thus, the propagator $\Pi_\alpha^\beta(K)$ has a pole at $K^2=M^2$.

Let us write the expression $\Pi_\alpha^\beta(K)$ in the pole:

$$\Pi_\alpha^\beta(K)\Big|_{K^2\sim M^2}\approx\frac{16m\left(\frac{1}{\lambda_{2R}}-\frac{1}{\lambda_{1R}}\right)}{(K^2-M^2)\,\Delta'(M^2)(M+2m)}\left(1-\frac{\hat{K}}{M}\right)_\alpha^\gamma T_\gamma^\beta(K,\theta_1,\theta_2), \qquad (17)$$

where $T_\gamma^\beta(K,\theta_1,\theta_2)$ is the transverse projector in momentum representation (see ,e.g., ref.[6]) taken in the point $K^2=M^2$

$$T_\gamma^\beta(K,\theta_1,\theta_2)=\frac{1}{2}\left(\delta(\theta_1-\theta_2)-\frac{\hat{K}}{M^2}\right)_\gamma^\beta e^{\bar{\theta}_1\hat{K}\theta_2},$$

and the $\Delta'(M^2)$ is the derivative of $\Delta(K^2)$ taken in the point $K^2=M^2$, is expressed as

$$\Delta'(M^2)=-\left(\frac{1}{\lambda_{2R}}-\frac{1}{\lambda_{1R}}\right)\left[\frac{16m(M+m)}{M^2(M+2m)^2}\left(\frac{1}{\lambda_{2R}}-\frac{1}{\lambda_{1R}}\right)+4mM\Sigma'(M^2)\right]$$

Note, that $\Delta'(M^2)<0$ $(\Sigma'(M^2)>0)$.

It follows from the expression (17), that the pole of the propagator $\Pi_{\alpha}^{\beta}(K)$ is in its transverse part. To definitely speak about the appearance of particles of mass M in the theory, one must be sure that the residue of the propagator $\Pi_{\alpha}^{\beta}(K)$ in the pole is positive. For this purpose let us use the expression for the propagator of the, say, $A_{\mu}^{ij}(x)$ -component of the superfield $\Phi_{\alpha}^{ij} = \varphi_{\alpha}^{ij} + \theta_{\alpha} d^{ij} + (\gamma^{5}\theta)_{\alpha}\lambda^{ij} + i(\gamma^{\mu}\theta)_{\alpha} A_{\mu}^{ij} + \xi_{\alpha}^{ij}\delta(\theta)$ which can be derived from the general expression (13):

$$\Pi_{\mu\nu}(K) = \frac{i}{4N}\left\{\Delta^{-1}(K^2)\left[4\left(\frac{1}{\lambda_{2R}} - \frac{1}{\lambda_{1R}}\right) - K^2\Sigma(K^2)\right]P_{\mu\nu} + \left(\frac{1}{\lambda_{2R}} - \frac{1}{\lambda_{1R}}\right)^{-1} L_{\mu\nu}\right\},$$

$$P_{\mu\nu} = g_{\mu\nu} - \frac{K_{\mu}K_{\nu}}{K^2}, \qquad L_{\mu\nu} = \frac{K_{\mu}K_{\nu}}{K^2}$$

and find its value in the pole:

$$\Pi_{\mu\nu}(K)\Big|_{K^2 \sim M^2} \approx \frac{iZ}{M^2 - K^2}\left(g_{\mu\nu} - \frac{K_{\mu}K_{\nu}}{K^2}\right), \qquad Z = -\frac{2m\left(\frac{1}{\lambda_{2R}} - \frac{1}{\lambda_{1R}}\right)}{N(M+2m)\Delta'(M^2)}.$$

It follows from the expression obtained, that Z , which is the residue of $\Pi_{\mu\nu}(K)$ in the pole, is positive ($\Delta'(M^2) < 0$). It should also be noted that at $K^2 = 0$ the propagator $\Pi_{\mu\nu}(K)$ has no singularities, which just follows from the expression (13) for $\Pi_{\alpha}^{\beta}(K)$.

Thus, the analysis of the expression for the propagator $\Pi_{\alpha}^{\beta}(K)$ shows that in the supersymmetric O(N)×O(K)-symmetric phase of the model considered at $\lambda_{1R} > \lambda_{2R}$, there dynamically appears a vector O(K)-supermultiplet of mass M . Besides, the model includes a scalar N × K-supermultiplet of mass m , witnessed of by the expression (15) of the propagator of the superfield $G^{ia}(x,\theta)$. The O(N)×O(K)- symmetric phase at $\lambda_{1R} > \lambda_{2R}$ turns to be homogeneous, while in case of the usual (not supersymmetric) model [3] it is inhomogeneous and, in its turn, consists of two phases differing from each other by the existence of massive vector fields. As to the propagator (14) of the superfield $\omega'^{ij}(x,\theta)$, it has no pole singularity in contrast to the expressions (13) and (15). Namely, at $K^2 \sim 4m^2$ $\mathcal{D}(K) \sim (K^2-4m^2)^{-1/2}$, since $\Sigma^{-1}(K^2) \sim (K^2-4m^2)^{1/2}$ at $K^2 \sim 4m^2$. That is why it is evidently impossible to speak about appearance of particles of mass 2m in the theory.

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