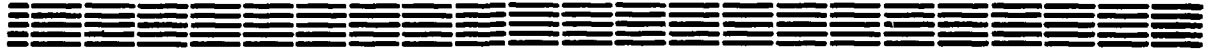


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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ
YEREVAN PHYSICS INSTITUTE



R.V.TEVIKIAN

TO THE THEORY OF MAGNETIC CHARGE

ЦНИИатоминформ
ЕРЕВАН — 1987

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Առաջարկված է էլեկտրական և մագնիսական լիցքերի համար Լագրանժի տեսութիւնը: Հետազոտութիւնը հիմնված է վեկտորական մասնիկների առավել Օշտված նկարագրի վրա, որի հետ կապված են՝ երբ $E \rightarrow \infty$ սահմանային դաշտեր $j = \pm 1, 0, -1, +1$: Ցույց է տրված, որ դաշտի $j = -1, +1$ թաղաղորչները սինգուլյար են : Լագրանժի Ֆունկցիայից ստացված է Դիրակի առնչութիւնը:

Երևանի Ֆիզիկայի ինստիտուտ

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R.V. TEVIKIAN

TO THE THEORY OF MAGNETIC CHARGE

A Lagrangian theory for the electric and magnetic charges is constructed. The study is based on a more correct description of the vector particles to which, at $E \rightarrow \infty$, the limiting fields $j = \pm 1, 0, -1, +1$ are connected. The components of the field $j = -1, +1$ are shown to be singular. The Dirac relation is obtained from the Lagrangian.

Yerevan Physics Institute

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Р.В. ТЕВИКЯН

К ТЕОРИИ МАГНИТНОГО ЗАРЯДА

Построена лагранжева теория для электрического и магнитного зарядов. Исследование основывается на более корректном описании векторных частиц, с которыми связаны, при $E \rightarrow \infty$, предельные поля $j = \pm I, 0, -I, +I$. Показано, что компоненты поля $j = -I, +I$ сингулярны. Из лагранжиана получено соотношение Дирака.

Ереванский физический институт

Ереван 1987

1. Introduction.

In the works of Dirac [1,2], there is studied the magnetic charge theory on the basis of the Maxwell equation, i.e. the fields with the helicity $j = \pm 1$. The $j = -1, +1$ field components are taken into account incompletely, due to which the well-known singular "filaments" occurred in the theory.

In the present work we are studying the electric and magnetic charges on the basis of a more correct definition of electromagnetic field as a field describing the set of $j = \pm 1, -1, +1$ states. A nonabelian generalization of the theory is also considered.

2. Total Description of Vector Field.

The field of the particle with the spin $s = 1$ describes the reducible representation of the Lorentz group:

$$\mathcal{D}(1,0) \oplus \mathcal{D}(0,1) \oplus \mathcal{D}(1/2, 1/2).$$

In the limit of infinite energy of particles $E \rightarrow \infty$ the reducible representation is decomposed and describes four limiting fields with the helicity

ties $j = \pm 1, 0, -1, +1$. The Lagrangian will describe the field $s=1$ completely only in the case if it describes also all possible limiting fields at E. The usual equation for the vector field, e.g. [3], describes the $s=1$ incompletely, since it describes only one limiting field, $j=\pm 1$. In the work of this author [4], two parameters were introduced to study the vector field. To describe four limiting fields, the Lagrangian must contain four arbitrary parameters. The requirement of complete description of particles determines the field Lagrangian unambiguously up to the divergence.

The Lagrangian describing the $s=1$ field completely is as follows:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} (\alpha_1 G_{\mu\nu}^- + \alpha_2 G_{\mu\nu}^+) (\partial^\mu \overset{*}{C}^\nu - \partial^\nu \overset{*}{C}^\mu) - \\ & -\frac{1}{2} (\alpha_1 \overset{*}{G}_{\mu\nu}^- + \alpha_2 \overset{*}{G}_{\mu\nu}^+) (\partial^\mu C^\nu - \partial^\nu C^\mu) + \\ & + \frac{1}{2} m_1 \overset{*}{G}_{\mu\nu} G^{\mu\nu} + m_2 \overset{*}{C}_\mu C^\mu, \end{aligned} \quad (2.1)$$

where

$$\begin{aligned} G_{\mu\nu} &= -G_{\nu\mu}, \quad G_{\mu\nu}^\pm = \frac{1}{2} (G_{\mu\nu} \pm G_{\mu\nu}^d), \\ G_{\mu\nu}^d &= \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}, \quad \overset{*}{G}_{\mu\nu}^\pm = (G_{\mu\nu}^\pm)^*, \quad g_{\mu\nu} = (+---). \end{aligned}$$

The Lagrangian contains four real parameters: $m_1, m_2, \alpha_1, \alpha_2$. To exclude any of the parameters by means of any transformations is impossible, since they take zero values as well.

If all the parameters differ from zero, then the Lagrangian describes the $s=1$ with squared mass:

$$m^2 = \frac{m_1 m_2}{\alpha_1 \alpha_2} \quad (2.2)$$

If $a_1, a_2 \neq 0$, then, in virtue of the identity

$$(a_1 G_{\mu\nu}^{*-} + a_2 G_{\mu\nu}^{*+}) (a_1 G^{-\mu\nu} + a_2 G^{+\mu\nu}) = a_1 a_2 G_{\mu\nu}^* G^{\mu\nu}$$

the parity is conserved.

The conserved energy-momentum tensor has the form:

$$\begin{aligned} T_{\mu\nu} = & -2m_1 \frac{a_1}{a_2} (G_{\mu\alpha}^- G_{\nu}^{*\alpha} + G_{\mu\alpha}^{*-} G_{\nu}^{-\alpha}) - 2m_1 \frac{a_2}{a_1} (G_{\mu\alpha}^+ G_{\nu}^{*\alpha} + G_{\mu\alpha}^{*+} G_{\nu}^{+\alpha}) + \\ & + m_2 (G_{\mu} C_{\nu} + C_{\mu} C_{\nu} - g_{\mu\nu} C_{\alpha} C^{\alpha}). \end{aligned} \quad (2.3)$$

At $m_2 = 0$ the Lagrangian describes the field $j = \pm 1$, while at $m_2 = 0$ - the field $j = 0$. The energy-momentum tensor has poles at $a_2 = 0$, and at $a_1 = 0$ this means that the states $j = -1, +1$ are singular.

Instead of $G_{\mu\nu}, C_{\mu}$ it is more convenient to introduce the real functions $F_{\mu\nu}, H_{\mu\nu}, A_{\mu}, B_{\mu}$ by the formulae:

$$G_{\mu\nu} = \frac{1}{2\sqrt{2}} (F_{\mu\nu} + F_{\mu\nu}^d + H_{\mu\nu} - H_{\mu\nu}^d) + \frac{i}{2\sqrt{2}} (F_{\mu\nu} - F_{\mu\nu}^d + H_{\mu\nu} + H_{\mu\nu}^d),$$

$$C_{\mu} = \frac{1}{2} (A_{\mu} + B_{\mu}) + \frac{i}{2} (A_{\mu} - B_{\mu}).$$

In this case

$$G_{\mu\nu}^- = \frac{1+i}{2\sqrt{2}} F_{\mu\nu} + \frac{1-i}{2\sqrt{2}} F_{\mu\nu}^d,$$

$$G_{\mu\nu}^+ = \frac{1+i}{2\sqrt{2}} H_{\mu\nu} - \frac{1-i}{2\sqrt{2}} H_{\mu\nu}^d,$$

where

$$G_{\mu\nu}^{d\pm} = \mp i G_{\mu\nu}^{\pm}$$

Thus, the real function $F_{\mu\nu}$ through $G_{\mu\nu}^-$ describes the field $j=-1$, while the $H_{\mu\nu}$ through $G_{\mu\nu}^+$ describes $j=+1$.

From the Lagrangian (2.1) there follows

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2\sqrt{2}} (a_1 F_{\mu\nu} + a_2 H_{\mu\nu}) (\partial^\mu A^\nu - \partial^\nu A^\mu) - \\ & -\frac{1}{2\sqrt{2}} (a_1 F_{\mu\nu}^d - a_2 H_{\mu\nu}^d) (\partial^\mu B^\nu - \partial^\nu B^\mu) + \\ & + \frac{1}{2} m_1 F_{\mu\nu} H^{\mu\nu} + \frac{1}{2} m_2 (A_\mu A^\mu + B_\mu B^\mu). \end{aligned} \quad (2.4)$$

The conserved energy-momentum tensor has the form:

$$\begin{aligned} T_{\mu\nu} = & -\frac{m_1}{2} \frac{a_1}{a_2} (F_{\mu\alpha} F_\nu^\alpha + F_{\mu\alpha}^d F_\nu^{d\alpha}) - \frac{m_1}{2} \frac{a_2}{a_1} (H_{\mu\alpha} H_\nu^\alpha + H_{\mu\alpha}^d H_\nu^{d\alpha}) + \\ & + m_2 (A_\mu A_\nu - \frac{1}{2} g_{\mu\nu} A_\alpha A^\alpha) + m_2 (B_\mu B_\nu - \frac{1}{2} g_{\mu\nu} B_\alpha B^\alpha). \end{aligned}$$

Introduce, at $a_1, a_2 \neq 0$, the transformations:

$$\begin{aligned} P_{\mu\nu} &= \frac{1}{\sqrt{2}} (a_1 F_{\mu\nu} + a_2 H_{\mu\nu}), \\ Q_{\mu\nu} &= \frac{1}{\sqrt{2}} (a_1 F_{\mu\nu}^d - a_2 H_{\mu\nu}^d), \end{aligned} \quad (2.5)$$

Here

$$P_{\mu\nu} P^{\mu\nu} + Q_{\mu\nu} Q^{\mu\nu} = 2a_1 a_2 F_{\mu\nu} H^{\mu\nu}$$

and the Lagrangian is decomposed:

$$\mathcal{L} = -\frac{1}{2} P_{\mu\nu} (\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2} Q_{\mu\nu} (\partial^\mu B^\nu - \partial^\nu B^\mu) + \\ + \frac{1}{4} \frac{m_1}{\alpha_1 \alpha_2} (P_{\mu\nu} P^{\mu\nu} + Q_{\mu\nu} Q^{\mu\nu}) + \frac{1}{2} m_2 (A_\mu A^\mu + B_\mu B^\mu).$$

Thus, at $\alpha_1, \alpha_2 \neq 0$ the Lagrangian is decomposed into two independent systems: $P_{\mu\nu}, A_\mu$ and $Q_{\mu\nu}, B_\mu$. The transformations (2.5) transform the two fields $j = \pm 1$ into the fields $F_{\mu\nu}$ and $H_{\mu\nu}$ with helicities $j = -1, +1$. In this case, from the energy-momentum tensor we have:

$$(P_{\mu\alpha} P_\nu^\alpha + P_{\mu\alpha}^d P_\nu^{d\alpha}) + (Q_{\mu\alpha} Q_\nu^\alpha + Q_{\mu\alpha}^d Q_\nu^{d\alpha}) = \\ = \alpha_1^2 (F_{\mu\alpha} F_\nu^\alpha + F_{\mu\alpha}^d F_\nu^{d\alpha}) + \alpha_2^2 (H_{\mu\alpha} H_\nu^\alpha + H_{\mu\alpha}^d H_\nu^{d\alpha}).$$

The transformations (2.5) are analogous to those of Bogolyubov's [5].

The Lagrangian (2.4) contains four free parameters bound by condition (2.2). We choose the parameters so that we could isolate the state $j=-1$. This can be attained at the following choice of parameters:

$$m_1 = \alpha_2 = \varepsilon, \quad \alpha_1 = 1, \quad m_2 = m^2 \quad (2.6)$$

At $m^2 = 0$ we have the state $j=\pm 1$. We shall come to $j=-1$ after the transition $\varepsilon \rightarrow 0$.

Owing to the singularity of the state $j=-1$, we'll be able to realize the limiting transition $\varepsilon \rightarrow 0$ only in physically observed quantities.

The Maxwell equations describe the field $j=\pm 1$. The electromagnetic field, to be more correct, is the set of the states $j = \pm 1, -1, +1$.

From (2.4) at $m_1 = \alpha_1 \alpha_2, m_2 = 0$ there follows the electromagnetic

field Lagrangian:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2\sqrt{2}} (\alpha_1 F_{\mu\nu} + \alpha_2 H_{\mu\nu}) (\partial^\mu A^\nu - \partial^\nu A^\mu) - \\ & -\frac{1}{2\sqrt{2}} (\alpha_1 F_{\mu\nu}^d - \alpha_2 H_{\mu\nu}^d) (\partial^\mu B^\nu - \partial^\nu B^\mu) + \frac{1}{2} \alpha_1 \alpha_2 F_{\mu\nu} H^{\mu\nu} \end{aligned} \quad (2.7)$$

and the energy-momentum tensor:

$$T_{\mu\nu} = -\frac{1}{2} \alpha_1^2 (F_{\mu\alpha} F_\nu^\alpha + F_{\mu\alpha}^d F_\nu^{d\alpha}) - \frac{1}{2} \alpha_2^2 (H_{\mu\alpha} H_\nu^\alpha + H_{\mu\alpha}^d H_\nu^{d\alpha}) \quad (2.8)$$

At $\alpha_1, \alpha_2 \neq 0$ after the transformation (2.5) we obtain the Maxwell equations for $F_{\mu\nu}, A_\mu$. If $\alpha_2 \rightarrow 0$ ($\alpha_1 \rightarrow 0$), then (2.8) describes the field $j = -1$ ($j = +1$).

In the Euclidean metric the Lagrangian describing the nonabelian fields with the spin $s=1$ and the limiting fields $j = \pm 1, 0, -1, +1$ has the form:

$$\begin{aligned} \mathcal{L} = & -\text{tr} (\alpha_1 R_{\mu\nu}^- + \alpha_2 R_{\mu\nu}^+) (\partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]) + \\ & + \frac{1}{2} m_1 \text{tr} (\alpha_1 R_{\mu\nu}^- + \alpha_2 R_{\mu\nu}^+)^2 + m_2 \text{tr} A_\mu^2, \end{aligned}$$

where

$$R_{\mu\nu}^\pm = \frac{1}{2} (R_{\mu\nu} \pm R_{\mu\nu}^d).$$

The energy-momentum tensor is

$$\begin{aligned} T_{\mu\nu} = & -m_1 \alpha_1 \alpha_2 \text{tr} (R_{\mu\alpha}^- R_{\nu\alpha}^+ + R_{\mu\alpha}^+ R_{\nu\alpha}^-) + \\ & + m_2 \text{tr} (A_\mu A_\nu - \frac{1}{2} g_{\mu\nu} A_\alpha^2). \end{aligned}$$

It is obvious that at $\alpha_1, \alpha_2 \neq 0$ the parity is conserved. The theory has no singularities, all the limiting fields $j = \pm 1, 0, -1, +1$ are regular. At $m_2 = 0$ ($m_1 = 0$) this field is $j = \pm 1$ ($j=0$). If $m_2 = \alpha_2 = 0$ ($m_2 = \alpha_1 = 0$), then we have the field $j = -1$ ($j = \pm 1$).

3. Magnetic Charge.

Consider now the model in which the fields Ψ, χ with the spin $s=1/2$ interact with the components of the vector field $s=1, j=\pm 1, -1$. In the Minkowski space-time the Lagrangian has the form:

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{2\sqrt{2}} (F_{\mu\nu} + \varepsilon H_{\mu\nu}) (\partial^\mu A^\nu - \partial^\nu A^\mu) - \\
 & -\frac{1}{2\sqrt{2}} (F_{\mu\nu}^d - \varepsilon H_{\mu\nu}^d) (\partial^\mu B^\nu - \partial^\nu B^\mu) + \frac{1}{2} \varepsilon F_{\mu\nu} H^{\mu\nu} + \frac{1}{2} m^2 (A_\mu A^\mu + B_\mu B^\mu) + \\
 & + \frac{i}{2} (\bar{\Psi} \gamma^\mu \partial_\mu \Psi - \partial_\mu \bar{\Psi} \gamma^\mu \Psi) - \mu_1 \bar{\Psi} \Psi + e \bar{\Psi} \gamma^\mu \Psi A_\mu + \\
 & + \frac{i}{2} (\bar{\chi} \gamma^\mu \partial_\mu \chi - \partial_\mu \bar{\chi} \gamma^\mu \chi) - \mu_2 \bar{\chi} \chi + g \bar{\chi} \gamma^\mu \chi B_\mu.
 \end{aligned} \tag{3.1}$$

The conserved energy-momentum tensor is

$$\begin{aligned}
 T_{\mu\nu} = & -\frac{1}{2} (F_{\mu\alpha} F_\nu^\alpha + F_{\mu\alpha}^d F_\nu^{d\alpha}) - \frac{\varepsilon^2}{2} (H_{\mu\alpha} H_\nu^\alpha + H_{\mu\alpha}^d H_\nu^{d\alpha}) + \\
 & + m^2 (A_\mu A_\nu - \frac{1}{2} g_{\mu\nu} A_\alpha A^\alpha) + m^2 (B_\mu B_\nu - \frac{1}{2} g_{\mu\nu} B_\alpha B^\alpha) + \\
 & + \frac{i}{4} (\bar{\Psi} \gamma_\mu \partial_\nu \Psi - \partial_\nu \bar{\Psi} \gamma_\mu \Psi + \bar{\Psi} \gamma_\nu \partial_\mu \Psi - \partial_\mu \bar{\Psi} \gamma_\nu \Psi) +
 \end{aligned} \tag{3.2}$$

$$\begin{aligned}
& + \frac{i}{4} (\bar{\chi} \gamma_{\mu} \partial_{\nu} \chi - \partial_{\nu} \bar{\chi} \gamma_{\mu} \chi + \bar{\chi} \gamma_{\nu} \partial_{\mu} \chi - \partial_{\mu} \bar{\chi} \gamma_{\nu} \chi) + \\
& + \frac{1}{2} (e \bar{\Psi} \gamma_{\mu} \Psi A_{\nu} + e \bar{\Psi} \gamma_{\nu} \Psi A_{\mu} + g \bar{\chi} \gamma_{\mu} \chi B_{\nu} + g \bar{\chi} \gamma_{\nu} \chi B_{\mu}).
\end{aligned}$$

The transformation (2.5) in the given case, at $\varepsilon \neq 0$, will take the form:

$$\begin{aligned}
P_{\mu\nu} &= \frac{1}{\sqrt{2}} (F_{\mu\nu} + \varepsilon H_{\mu\nu}), \\
Q_{\mu\nu} &= \frac{1}{\sqrt{2}} (F_{\mu\nu}^d - \varepsilon H_{\mu\nu}^d).
\end{aligned} \tag{3.3}$$

In the path-dependent formulation of Mandelstam [6]

$$\Psi(x, P) = \Psi(x) \exp\left(-ie \int_{-\infty}^x A_{\mu}(\xi) d\xi^{\mu}\right),$$

the dependence on the path is defined by the relations:

$$\begin{aligned}
\delta_{\underline{z}} \Psi(x, P) &= -ie \Psi(x, P) P_{\mu\nu}(z) \delta^{\mu\nu}, \\
\partial^{\nu} P_{\mu\nu}^d &= 0.
\end{aligned}$$

These differential relations are valid in the consideration of the field $j = \pm 1$; in the transformation (3.3) this is true only at $\varepsilon \neq 0$, since the fields $F_{\mu\nu}$, $H_{\mu\nu}$ are singular. In the integral form we have

$$\Psi(x, P') = \Psi(x, P) \exp\left(-\frac{ie}{2} \int_{\mathcal{S}} P_{\mu\nu} d\sigma^{\mu\nu}\right), \tag{3.4}$$

$$\exp\left(-\frac{ie}{2} \oint P_{\mu\nu} d\sigma^{\mu\nu}\right) = 1. \tag{3.5}$$

where S is the surface between P' and P .

The relation (3.5) is written in the integral form, and after the transformation (3.3) we can perform the limiting transition $\varepsilon \rightarrow 0$.

From the Lagrangian (3.1), at $m^2 = 0$, we have

$$\begin{aligned} \partial^\nu F_{\mu\nu} &= e' \bar{\Psi} \gamma_\mu \Psi, \\ \partial^\nu F_{\mu\nu}^d &= g' \bar{\chi} \gamma_\mu \chi, \end{aligned} \quad (3.6)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - (\partial_\mu B_\nu - \partial_\nu B_\mu)^d.$$

The transformations (3.3) bring to the new charges:

$$e' = \frac{e}{\sqrt{2}}, \quad g' = \frac{g}{\sqrt{2}}$$

From the equality (3.5), after the substitution of the decomposition (3.3) and limiting transition $\varepsilon \rightarrow 0$ we obtain

$$\exp\left(-\frac{ie'}{2} \oint F_{\mu\nu} d\sigma^{\mu\nu}\right) = 1.$$

Taking integral along a closed surface, using (3.6) we'll come to the Dirac relation [1]:

$$e'g' = 2\pi n$$

where n - is integer.

Only the limiting transition $\varepsilon \rightarrow 0$ at $m^2 = 0$ introduces interactions of electric and magnetic charges into theory. So long as the state $j = -1$ is singular, then we can realize the limiting transition $\varepsilon \rightarrow 0$ only in the integral relations for $F_{\mu\nu}$, after excluding the potentials.

For the further consideration we choose the gauge introduced originally

by Fock [7] : $x^\mu A_\mu = x^\nu B_\nu = 0$; in this case

$$\begin{aligned} A_\mu(x) &= \int_0^1 \alpha x^\nu P_{\nu\mu}(\alpha x) d\alpha, \\ B_\mu(x) &= \int_0^1 \alpha x^\nu Q_{\nu\mu}(\alpha x) d\alpha. \end{aligned} \quad (3.7)$$

These relations are valid under the fulfilment of additional conditions written in the integral and gauge-invariant form:

$$\begin{aligned} \exp\left(-\frac{ie}{2} \oint P_{\mu\nu} d\sigma^{\mu\nu}\right) &= 1, \\ \exp\left(-\frac{ig}{2} \oint Q_{\mu\nu} d\sigma^{\mu\nu}\right) &= 1. \end{aligned}$$

The energy-momentum tensor (3.2) in the Fock gauge (3.7), at $m^2 = 0$, $\epsilon \rightarrow 0$, has the form:

$$\begin{aligned} T_{\mu\nu} &= -\frac{1}{2} (F_{\mu\alpha} F_\nu^\alpha + F_{\mu\alpha}^d F_\nu^{d\alpha}) + \\ &+ \frac{i}{4} (\bar{\Psi} \gamma_\mu \partial_\nu \Psi - \partial_\nu \bar{\Psi} \gamma_\mu \Psi + \bar{\Psi} \gamma_\nu \partial_\mu \Psi - \partial_\mu \bar{\Psi} \gamma_\nu \Psi) + \\ &+ \frac{i}{4} (\bar{\chi} \gamma_\mu \partial_\nu \chi - \partial_\nu \bar{\chi} \gamma_\mu \chi + \bar{\chi} \gamma_\nu \partial_\mu \chi - \partial_\mu \bar{\chi} \gamma_\nu \chi) + \\ &+ \frac{1}{2} (e \bar{\Psi} \gamma_\mu \Psi A'_\nu + e \bar{\Psi} \gamma_\nu \Psi A'_\mu + g \bar{\chi} \gamma_\mu \chi B'_\nu + g \bar{\chi} \gamma_\nu \chi B'_\mu), \end{aligned} \quad (3.8)$$

where

$$\begin{aligned} A'_\mu(x) &= \frac{1}{\sqrt{2}} \int_0^1 \alpha x^\nu F_{\nu\mu}(\alpha x) d\alpha, \\ B'_\mu(x) &= \frac{1}{\sqrt{2}} \int_0^1 \alpha x^\nu F_{\nu\mu}^d(\alpha x) d\alpha. \end{aligned}$$

The interaction Lagrangian on the basis of (3.1) after the limiting transition $m^2 = 0$, $\varepsilon \rightarrow 0$, in the Fock gauge, has the form:

$$\begin{aligned} \mathcal{L}' = e' \bar{\Psi} \gamma^\mu \Psi \cdot \int_0^1 \alpha x^\nu F_{\nu\mu}(\alpha x) d\alpha + \\ + g' \bar{\chi} \gamma^\mu \chi \cdot \int_0^1 \alpha x^\nu F_{\nu\mu}^d(\alpha x) d\alpha. \end{aligned} \quad (3.9')$$

The energy-momentum tensor (3.8) as well as the interaction Lagrangian (3.9) describe the interactions of $F_{\mu\nu}$ with the fields Ψ and χ possessing the electric charge e' and magnetic charge g' , respectively.

Consider the set of fields $\Psi_\alpha(x)$, ($\alpha = 1, 2, \dots, n$): each particle possesses the electric charge e_α and magnetic charge g_α . In this case the relations (3.4), (3.5) will take the form:

$$\Psi_\alpha(x, p') = \Psi_\alpha(x, p) \exp \left[-\frac{i}{2} \int_S (e_\alpha P_{\mu\nu} + g_\alpha Q_{\mu\nu}) d\sigma^{\mu\nu} \right], \quad (3.10)$$

$$\exp \left[-\frac{i}{2} \oint (e_\alpha P_{\mu\nu} + g_\alpha Q_{\mu\nu}) d\sigma^{\mu\nu} \right] = 1. \quad (3.11)$$

After the transformation (3.3) and the limiting transition $\varepsilon \rightarrow 0$, from (3.11) we obtain:

$$\exp \left[-\frac{i}{2} \oint (e'_\alpha F_{\mu\nu} + g'_\alpha F_{\mu\nu}^d) d\sigma^{\mu\nu} \right] = 1$$

Here

$$\begin{aligned} \partial^\nu F_{\mu\nu} &= \sum_\alpha e'_\alpha \bar{\Psi}_\alpha \gamma_\mu \Psi_\alpha, \\ \partial^\nu F_{\mu\nu}^d &= \sum_\alpha g'_\alpha \bar{\Psi}_\alpha \gamma_\mu \Psi_\alpha. \end{aligned}$$

Taking integral along the closed surface, we obtain [8] :

$$e'_a g'_\varepsilon - e'_\varepsilon g'_a = 2\pi n_{a\varepsilon} \quad (3.12)$$

where $n_{a\varepsilon}$ - is integer.

4. Nonabelian Fields.

The equations of the massive Yang-Mills field describe the fields $s=1$, $j=\pm 1$. Nonabelian fields, to be more correct, are the set of the states $s=1$, $j=\pm 1, 0, -1, +1$, to describe which, one should introduce four mass parameters $m_1, m_2, \alpha_1, \alpha_2$.

The Lagrangian describing the nonabelian fields $s=1$, $j=\pm 1, -1$ interacting with the Fermi fields Ψ, χ has the following form:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2\sqrt{2}} \text{tr} (F_{\mu\nu} + \varepsilon H_{\mu\nu}) (\partial^\mu A^\nu - \partial^\nu A^\mu + [A^\mu, A^\nu]) - \\ & -\frac{1}{2\sqrt{2}} \text{tr} (F_{\mu\nu}^d - \varepsilon H_{\mu\nu}^d) (\partial^\mu B^\nu - \partial^\nu B^\mu + [B^\mu, B^\nu]) + \\ & + \frac{1}{2} \varepsilon \text{tr} F_{\mu\nu} H^{\mu\nu} + \frac{1}{2} m^2 \text{tr} (A_\mu A^\mu + B_\mu B^\mu) + \\ & + \frac{i}{2} (\bar{\Psi} \gamma^\mu \partial_\mu \Psi - \partial_\mu \bar{\Psi} \gamma^\mu \Psi) - \mu_1 \bar{\Psi} \Psi + \\ & + \frac{i}{2} (\bar{\chi} \gamma^\mu \partial_\mu \chi - \partial_\mu \bar{\chi} \gamma^\mu \chi) - \mu_2 \bar{\chi} \chi + \\ & + \bar{\Psi} \gamma_\mu T A^\mu \Psi + \bar{\chi} \gamma_\mu T B^\mu \chi, \end{aligned} \quad (4.1)$$

where the T -matrices are taken in the corresponding representation. One can readily be convinced that in virtue of the identity

$$\begin{aligned} & \text{tr}(F_{\mu\nu} + \varepsilon H_{\mu\nu})(F^{\mu\nu} + \varepsilon H^{\mu\nu}) + \text{tr}(F_{\mu\nu}^d - \varepsilon H_{\mu\nu}^d)(F^{d\mu\nu} - \varepsilon H^{d\mu\nu}) = \\ & = 4\varepsilon \text{tr} F_{\mu\nu} H^{\mu\nu}. \end{aligned}$$

the parity, at $\varepsilon \neq 0$, is conserved.

The conserved energy-momentum tensor has the following form:

$$\begin{aligned} T_{\mu\nu} = & -\frac{1}{2} \text{tr}(F_{\mu\alpha} F_{\nu}^{\alpha} + F_{\mu\alpha}^d F_{\nu}^{d\alpha}) - \frac{\varepsilon^2}{2} \text{tr}(H_{\mu\alpha} H_{\nu}^{\alpha} + H_{\mu\alpha}^d H_{\nu}^{d\alpha}) + \\ & + m^2 \text{tr}(A_{\mu} A_{\nu} - \frac{1}{2} g_{\mu\nu} A_{\alpha} A^{\alpha}) + m^2 \text{tr}(B_{\mu} B_{\nu} - \frac{1}{2} g_{\mu\nu} B_{\alpha} B^{\alpha}) + \\ & + \frac{i}{4} (\bar{\Psi} \gamma_{\mu} \partial_{\nu} \Psi - \partial_{\nu} \bar{\Psi} \gamma_{\mu} \Psi + \bar{\Psi} \gamma_{\nu} \partial_{\mu} \Psi - \partial_{\mu} \bar{\Psi} \gamma_{\nu} \Psi) + \\ & + \frac{i}{4} (\bar{\chi} \gamma_{\mu} \partial_{\nu} \chi - \partial_{\nu} \bar{\chi} \gamma_{\mu} \chi + \bar{\chi} \gamma_{\nu} \partial_{\mu} \chi - \partial_{\mu} \bar{\chi} \gamma_{\nu} \chi) + \\ & + \frac{1}{2} (\bar{\Psi} \gamma_{\mu} T A_{\nu} \Psi + \bar{\Psi} \gamma_{\nu} T A_{\mu} \Psi + \bar{\chi} \gamma_{\mu} T B_{\nu} \chi + \bar{\chi} \gamma_{\nu} T B_{\mu} \chi). \end{aligned} \tag{4.2}$$

Introduce for the nonabelian fields the transformations:

$$\begin{aligned} P_{\mu\nu} &= \frac{1}{\sqrt{2}} (F_{\mu\nu} + \varepsilon H_{\mu\nu}), \\ G_{\mu\nu} &= \frac{1}{\sqrt{2}} (F_{\mu\nu}^d - \varepsilon H_{\mu\nu}^d). \end{aligned} \tag{4.3}$$

In this case we have

$$\begin{aligned} & \text{tr}(P_{\mu\alpha} P_{\nu}^{\alpha} + P_{\mu\alpha}^d P_{\nu}^{d\alpha}) + \text{tr}(Q_{\mu\alpha} Q_{\nu}^{\alpha} + Q_{\mu\alpha}^d Q_{\nu}^{d\alpha}) = \\ & = \text{tr}(F_{\mu\alpha} F_{\nu}^{\alpha} + F_{\mu\alpha}^d F_{\nu}^{d\alpha}) + \epsilon^2 \text{tr}(H_{\mu\alpha} H_{\nu}^{\alpha} + H_{\mu\alpha}^d H_{\nu}^{d\alpha}). \end{aligned}$$

Thus, owing to the transformation (4.3), the two nonabelian fields $j=\pm 1$ are transformed into the fields $j = -1, +1$.

Introduce for the nonabelian fields the Fock gauge

$$x^{\mu} A_{\mu} = x^{\mu} B_{\mu} = 0$$

In this case

$$\begin{aligned} A_{\mu}(x) &= \int_0^1 \alpha x^{\nu} P_{\nu\mu}(\alpha x) d\alpha, \\ B_{\mu}(x) &= \int_0^1 \alpha x^{\nu} Q_{\nu\mu}(\alpha x) d\alpha. \end{aligned} \tag{4.4}$$

These relations are valid under the fulfilment of the Bianchi identity written in the integral and gauge-invariant form:

$$\begin{aligned} P \exp\left(-\frac{i}{2} \oint P_{\mu\nu} d\sigma^{\mu\nu}\right) &= 1, \\ P \exp\left(-\frac{i}{2} \oint Q_{\mu\nu} d\sigma^{\mu\nu}\right) &= 1, \end{aligned} \tag{4.5}$$

where P - is the ordered exponent of the two-dimensional integral [9].

After the transformation (4.3) we can, at $m^2 = 0$, realize the limiting transition $\epsilon \rightarrow 0$ in (4.4), (4.5), and hence we'll obtain:

$$\begin{aligned}
A'_\mu(x) &= \frac{1}{\sqrt{2}} \int_0^1 \alpha x^\nu F_{\nu\mu}(\alpha x) d\alpha, \\
B'_\mu(x) &= \frac{1}{\sqrt{2}} \int_0^1 \alpha x^\nu F_{\nu\mu}^d(\alpha x) d\alpha.
\end{aligned} \tag{4.6}$$

and the generalized relation (3.12).

The energy-momentum tensor (4.2) and the interaction Lagrangian from (4.1), after the realization of the limiting transitions $m^2=0$, $\varepsilon \rightarrow 0$ are as follows:

$$\begin{aligned}
T_{\mu\nu} &= -\frac{1}{2} \text{tr} (F_{\mu\alpha} F_\nu^\alpha + F_{\mu\alpha}^d F_\nu^{d\alpha}) + \\
&+ \frac{i}{4} (\bar{\Psi} \gamma_\mu \partial_\nu \Psi - \partial_\nu \bar{\Psi} \gamma_\mu \Psi + \bar{\Psi} \gamma_\nu \partial_\mu \Psi - \partial_\mu \bar{\Psi} \gamma_\nu \Psi) + \\
&+ \frac{i}{4} (\bar{\chi} \gamma_\mu \partial_\nu \chi - \partial_\nu \bar{\chi} \gamma_\mu \chi + \bar{\chi} \gamma_\nu \partial_\mu \chi - \partial_\mu \bar{\chi} \gamma_\nu \chi) + \\
&+ \frac{1}{2} (\bar{\Psi} \gamma_\mu T A'_\nu \Psi + \bar{\Psi} \gamma_\nu T A'_\mu \Psi + \bar{\chi} \gamma_\mu T B'_\nu \chi + \bar{\chi} \gamma_\nu T B'_\mu \chi),
\end{aligned} \tag{4.7}$$

$$\begin{aligned}
\mathcal{L}' &= -\frac{1}{2\sqrt{2}} \text{tr} F^{\mu\nu} [A'_\mu, A'_\nu] - \frac{1}{2\sqrt{2}} \text{tr} F^{d\mu\nu} [B'_\mu, B'_\nu] + \\
&+ \bar{\Psi} \gamma^\mu T A'_\mu \Psi + \bar{\chi} \gamma^\mu T B'_\mu \chi,
\end{aligned} \tag{4.8}$$

where A'_μ and B'_μ are defined by relations (4.6).

The energy-momentum tensor (4.7) and the interaction Lagrangian (4.8) describe the nonabelian fields $F_{\mu\nu}$ with the helicity $j = -1$, interacting

with the spinors Ψ , χ which carry the electric- and magnetic-type charges.

From the Lagrangian (4.1), at $m^2 = 0$ and $\epsilon \neq 0$, there follow equations:

$$\partial^\nu F_{\mu\nu} + \frac{1}{2} [A^\nu + B^\nu, F_{\mu\nu}] + \frac{\epsilon}{2} [A^\nu - B^\nu, H_{\mu\nu}] = \frac{1}{\sqrt{2}} \bar{\Psi} \gamma_\mu \tau \Psi,$$

$$\partial^\nu F_{\mu\nu}^d + \frac{1}{2} [A^\nu + B^\nu, F_{\mu\nu}^d] + \frac{\epsilon}{2} [A^\nu - B^\nu, H_{\mu\nu}^d] = \frac{1}{\sqrt{2}} \bar{\chi} \gamma_\mu \tau \chi,$$

$$\epsilon \partial^\nu H_{\mu\nu} + \frac{\epsilon}{2} [A^\nu + B^\nu, H_{\mu\nu}] + \frac{1}{2} [A^\nu - B^\nu, F_{\mu\nu}] = \frac{1}{\sqrt{2}} \bar{\Psi} \gamma_\mu \tau \Psi,$$

$$\epsilon \partial^\nu H_{\mu\nu}^d + \frac{\epsilon}{2} [A^\nu + B^\nu, H_{\mu\nu}^d] + \frac{1}{2} [A^\nu - B^\nu, F_{\mu\nu}^d] = -\frac{1}{\sqrt{2}} \bar{\chi} \gamma_\mu \tau \chi.$$

The variables $F_{\mu\nu}$ and $H_{\mu\nu}$ are not divided. This means that the nonabelian fields describing the states $j = -1, +1$ are nonlocal [10].

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