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RADIATIVE AND LEPTONIC TRANSITIONS OF MESONS IN RELATIVISTIC QUARK MODEL

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Ստացված է ինչպես ռադիացիոն ու լեպտոնային տրոհումների վերաբերյալ եղած փորձարարական տվյալների, այնպես էլ պսևդոսկալյար, վեկտորային, առանցքային և թենզորային մեզոնների էլեկտրամագնիսական շառավիղների լավ նկարագրությունը:

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И.Г.АЗНАУРЯН, К.А.ОГАНЕСЯН

РАДИАЦИОННЫЕ И ЛЕПТОННЫЕ ПЕРЕХОДЫ МЕЗОНОВ В РЕЛЯТИВИСТСКОЙ МОДЕЛИ КВАРКОВ

Получено хорошее описание имеющихся экспериментальных данных по радиационным и лептонным распадам, а также электромагнитным радиусам для псевдоскалярных, векторных, аксиальных и тензорных мезонов.

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RADIATIVE AND LEPTONIC TRANSITIONS OF MESONS IN RELATIVISTIC QUARK MODEL

A good description of available experimental data on radiative and leptonic decays, and also on electromagnetic radii for pseudoscalar, vector, axial and tensor mesons, is obtained.

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1. Introduction.

Recently, owing to the creation of high-energy pion and K-meson beams, a great number of new experimental data on the light meson radiative decays is obtained. For the first time are measured widths of decays $b_1 \to \pi\gamma$ [1], $a_1 \to \pi\gamma$ [2], $K_2^*(1430) \to K\gamma$ [3], the charge radius of the K-meson [4]; more precise data on the decays $\rho \to \pi\gamma$ [5], $K^*(890) \to K\gamma$ [6,7], $a_2 \to \pi\gamma$ [8] as well as on the pion charge radius [9,10] are obtained. This enables one to study sufficiently complete-ly the properties of the mesons and the possibility of their self-consistent description within the quark model. In this work we shall consider the S- and P-wave mesons. Our description will be founded on the relativistic quark model which was constructed within the light-front dynamics in Refs. [11-13] and then formulated in the infinite momentum frame in [14,15]. This model was successfully applied to describe the low-energy characteris-tics of nucleons [14,15], baryons [16] and nucleon resonances [17], as well as the characteristics of pseudoscalar and vector mesons composed of nonstrange quarks [13,18,19].

In this work we'll show that all available experimental data on the radiative and leptonic transitions of the mesons consisting of quarks with $L = 0$ and 1 can be self-consistently described within the relativistic quark model. In this case, a minimum number of parameters will be introduced. These are masses of the strange and nonstrage quarks ($m_u = m_d \neq m_s$) and the parameters β_{uu} , β_{us} , β_{ss} characterizing the mean square momenta of the quarks which we consider to be dependent only on the quark content of mesons. The anomalous magnetic moments of the quarks, which were incorporated to the description of the magnetic moments of baryons in Refs. [14-16] and turned out to be small, will not be taken into account.

In Sect. 2, we briefly present the main formulae of the model which are necessary to obtain subsequent results. In Sect. 3, we describe the S-wave mesons. It is shown that in order to obtain the model parameters, it is enough to use the data on coupling constants of the decays $\pi \to \mu\nu$ and $K \to \mu\nu$ (f_π and f_K) and the VDM coupling constants f_ρ and f_φ . The other observables $\Gamma(\omega \to \pi\gamma)$, $\Gamma(K^{*+,0} \to K^{+,0}\gamma)$, τ_{π^-} , τ_{K^-} and τ_{K^0} are calculated with these values of parameters and are in good agreement with experiment. In Sect. 4, we consider the P-wave mesons. Using the values of parameters obtained in Sect. 3, we calculated widths of the $b_1 \to \pi\gamma$, $a_1 \to \pi\gamma$, $a_2 \to \pi\gamma$, $K_2^*(1430) \to K\gamma$ decays, which are in good agreement with experiment, as well as obtained predictions for the $K_1(1280) \to K\gamma$ and $K_1(1400) \to K\gamma$ decay widths.

2. Initial Formulae.

Below, we'll mainly consider transitions of the type of $A(P) \to B(P') + \gamma^*(K)$ and $A \to B + e\nu(K)$, where brackets contain the momenta of mesons, virtual photon and leptonic pair. If we proceed from the model formulation in the infinite momentum frame (IMF), then in a specially chosen IMF ($P \to \infty$) obtained by a "boost" along the z axis, in which $K_o = -K_z = (m_A^2 - m_B^2 - \vec{K}_\perp^2)/4P$ (m_A and m_B are masses of A and B mesons), the matrix elements of the longitudinal components of the electromagnetic and axial currents corresponding to the considered transitions have the following form [14,15] :

$$\frac{1}{2P} \langle B, P', \lambda' | \bar{J}_{0,3}^{(em,ax)} | A, P, \lambda \rangle |_{P \to \infty} = \tag{1}$$

$$= \sum_{i=a,\bar{b}} \int \Psi^{IMF}_{B,\lambda'} (x, \vec{P}_\perp + x\vec{K}_\perp) \hat{T}_i^{(em,ax)} \Psi^{IMF}_{A,\lambda} (x, \vec{P}_\perp) d\Gamma,$$

where we assume that the considered mesons are bound states $a\bar{b}$, $\hat{T}_i^{(em)} = Q_i$ (Q_i is charge of a and $\bar{b}$ quarks), $T_i^{(ax)} = \hat{\tau}(u \to s)\, \sigma_3^i$ ($\hat{\tau}$ is operator transforming the u -quark into the s -quark), λ and λ' are helicities of the initial and final mesons, the variables x , $\vec{P}_\perp$ are connected in IMF with the quark momenta in the initial meson by the relations

$$\vec{q}_a = x_a \vec{P} - \vec{P}_\perp , \quad \vec{q}_{\bar{b}} = x_b \vec{P} + \vec{P}_\perp , \quad x_a = 1 - x , \quad x_b = x. \tag{2}$$

Analogous variables for the quarks in final state are x and $\vec{P}_\perp + x\vec{K}_\perp$. In what follows we shall need also the variables $P_i = (\vec{P}_{i\perp} , P_{iz} , \varepsilon_i)$, having the meaning of 4-momentum of quarks in their c.m.s.:

$$P_{iz} + \varepsilon_i = M_o x_i\,,\ \varepsilon_i = (\vec{P}_i^{\,2} + m_i^2)^{1/2},\ M_o = \varepsilon_a + \varepsilon_b = \Big(\sum_{i=a,b} \frac{m_i^2 + \vec{P}_{i\perp}^{\,2}}{x_i}\Big)^{1/2}. \quad (3)$$

In terms of variables $x, \vec{P}_\perp$ and $\vec{P}(\vec{P}_\perp, P_{bz})$, the phase space $d\Gamma$ is:

$$(2\pi)^3 d\Gamma = \frac{dx\, d\vec{P}_\perp}{2x(1-x)} = \frac{M_o\, d\vec{P}}{2\varepsilon_a\, \varepsilon_b}. \quad (4)$$

In relation (1), $\Psi^{IMF}_{A,\lambda}$ and $\Psi^{IMF}_{B,\lambda'}$ are vertex functions of meson transitions to quarks in IMF. Ref. [15] has shown that these vertex functions are obtained from the wave functions of quarks in their c.m.s. by spin rotation given by the Melosh matrix. Concretely, for the initial meson we have:

$$\Psi^{IMF}_{A,\lambda}(x, \vec{P}_\perp) = U_a^+(P_a)\, U_b^+(P_b)\, \varphi^{c.m.s}_{A,\lambda}(\vec{P})\, \phi_A(M_o^2), \quad (5)$$

where $U_i(P_i)$ are the Melosh matrices defined by the relation:

$$U_i(P_i) = \frac{m_i + M_o x_i + i\varepsilon_{\ell m}\, \sigma_\ell (\vec{P}_i)_m}{[(m_i + M_o x_i)^2 + \vec{P}_{i\perp}^{\,2}]^{1/2}}, \quad i = a, b, \quad (6)$$

$\varphi^{c.m.s}$ are spin-orbital parts of the wave functions of quarks in their c.m.s., $\phi(M_o^2)$ are the radial parts of these wave functions which we'll treat as functions of one variable [13] M_o - invariant mass of quarks composing meson. To obtain numerical results, we'll make use of the following form of these functions:

$$\phi(M_o^2) = N \exp(-M_o^2/4\beta^2), \quad (7)$$

where β is the parameter characterizing the mean square momentum of quarks in the meson, N is the normalization parameter defined by the normalization condition:

$$\int \left| \varphi^{c.m.s}(\vec{P})\, \phi(M_0^2)\right|^2 d\Gamma = 1 . \tag{8}$$

In Sects. 3 and 4, we'll consider the S- and P-wave mesons. The normalization conditions for the radial parts of the wave functions for these mesons come out from (8) via replacement of $\left|\varphi^{c.m.s}\right|^2$ by 1 and $\frac{\vec{P}_\perp^2}{2}$, respectively.

During the concrete calculations, the products of the Melosh matrices entering the relation (1) are convenient to be written as follows:

$$u_a(P_a)\, u_a^+(P_a) = \left(a_0 + i \sum_{K=1}^{3} a_K \sigma_K^{(a)}\right) \Big/ \left(R_a R_a'\right)^{1/2}, \tag{9}$$

where

$$
\begin{aligned}
a_0 &= (m_a + M_0 x_a)(m_a + M_0' x_a) + \vec{P}_\perp^2 + x\, p_x K_x \, , \\
a_2 &= (m_a + M_0' x_a)\, p_{ax} - (m_a + M_0 x_a)\, p_{ax}' \, , \\
a_1 &= (M_0 - M_0')\, x_a\, p_{ay} \, , \quad a_3 = x\, p_y K_x \, , \\
R_a &= (m_a + M_0 x_a)^2 + \vec{P}_\perp^2 , \quad R_a' = (m_a + M_0' x_a)^2 + \vec{P}_\perp^2 .
\end{aligned} \tag{10}
$$

Analogous expressions for the b-quark come out from (9) and (10) via the replacement $a \to b$. In (9) and (10), the photon momentum is assumed to direct along the x axis, and in this case $p_x' = p_x + x K_x$, $p_y' = p_y$. Results presented in Sects. 3 and 4 will contain coefficients of expansions of a_0 and a_i in powers of K_x , which can readily be found from (10).

and are denoted by us according to the formulae:

$$a_0 = a_0^{(0)} + a_0^{(1)} K_x + a_0^{(2)} K_x^2 ,$$
$$a_i = (a_i^{(0)} + a_i^{(1)} K_x) K_x , \quad i = 1,2,3 . \qquad (11)$$

3. Pseudoscalar and Vector Mesons.

Start our consideration with the mesons consisting of nonstrange quarks. To study a possibility of their self-consistent description within the model framework, it is enough to consider only quantities f_π , f_ρ , τ_π and $\Gamma(\omega \to \pi\gamma)$. This is accounted for by the fact that under the made assumption on the equality of the mean square momenta of quarks in the mesons with the same quark content the constant f_ω is connected with the f_ρ by the SU(3)-symmetry relation $f_\rho = 3 f_\omega$ which is in good agreement with experiment. The ratios $\Gamma(\rho \to \eta\gamma)/\Gamma(\omega \to \pi\gamma)$, $\Gamma(\omega \to \eta\gamma)/\Gamma(\rho \to \pi\gamma)$, $\Gamma(\eta' \to \rho\gamma)/\Gamma(\omega \to \pi\gamma)$ and $\Gamma(\eta' \to \omega\gamma)/\Gamma(\rho \to \pi\gamma)$ give only the admixtures of nonstrange quarks in η and η' , which under available experimental data on these ratios are determined with great errors and agree with the estimates obtained in other works. The data on $\Gamma(\rho \to \pi\gamma)/\Gamma(\omega \to \pi\gamma)$, $\Gamma(\eta' \to \omega\gamma)$ / $\Gamma(\eta' \to \rho\gamma)$ and $\Gamma(\omega \to \eta\gamma)/\Gamma(\rho \to \eta\gamma)$ determine anomalous magnetic moment of the u - and d -quarks not proportional to their charges $\tilde{æ}$ ($æ_{u,d} = Q_{u,d}\, æ + \tilde{æ}$) [20] . Under experimental data available, informative for determining the $\tilde{æ}$ are only those on the ratio of widths of decays $\rho \to \pi\gamma$ [5] and $\omega \to \pi\gamma$ [21,22] , from which we obtain:

$$\tilde{\ae} = -0.014 \pm 0.11 \qquad [5,21] \ ,$$

$$\tilde{\ae} = -0.009 \pm 0.012 \qquad [5,22] \ . \tag{12}$$

In the framework of the considered model the quantities f_π, f_ρ, τ_π and the amplitude of the decay $\omega \to \pi\gamma$ ($g_{\omega\pi}$) were calculated originally in Ref. [13] and have the following forms:

$$f_\pi = \frac{\sqrt{3}}{2\pi^2} \int m_u \frac{p^2}{\varepsilon_u^2} \phi_\pi (M_0^2) dp \ , \tag{13a}$$

$$f_\rho = \frac{\sqrt{3}}{2\pi^2} \int \left(\varepsilon_u - \frac{1}{3} \frac{p^2}{\varepsilon_u + m_u}\right) \frac{p^2}{\varepsilon_u^2} \phi_\rho (M_0^2) dp \ , \tag{13b}$$

$$g_{\omega\pi} = \frac{1}{4\pi^2} \int \left[1 + \frac{m_u}{2p\varepsilon_u}(2\varepsilon_u + m_u) \ln \frac{\varepsilon_u + p}{\varepsilon_u - p}\right] \frac{\phi_\pi(M_0^2)\phi_\omega(M_0^2)}{\varepsilon_u(\varepsilon_u + m_u)} p^2 dp, \tag{13c}$$

$$\tau_\pi^2 = \frac{3}{8\pi^2} \int \left[\frac{1}{2\varepsilon_u^2} \ln \frac{\varepsilon_u + p}{\varepsilon_u - p} + \frac{1}{\beta_{uu}^2}\left(\ln \frac{\varepsilon_u + p}{\varepsilon_u - p} - \frac{p}{\varepsilon_u}\right) \phi_\pi^2 (M_0^2) p dp \right., \tag{13d}$$

where under the made assumptions all wave functions coincide. The amplitude $g_{\omega\pi}$ as well as the analogous amplitudes for the radiative decays of the vector, axial and tensor mesons with the pseudoscalar particle production are connected with the widths of these decays by the following relations:

$$\Gamma = \frac{\alpha}{2S+1} g^2 \left(\frac{m_i^2 - m_f^2}{2m_i}\right)^{2S+1} , \tag{14a}$$

where $\alpha = 1/137$, S is the initial meson spin, m_i and m_f are masses of the initial and final mesons.

The coupling constants f_π and f_ρ are connected with the widths of the $\pi \to \mu\nu$ and $\rho \to e^+e^-$ decays as follows:

$$\Gamma(\pi \to \mu\nu) = \frac{G_F^2 f_\pi^2 \cos^2\theta_c}{4\pi} m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2, \quad \Gamma(\rho \to e^+e^-) = \frac{4\pi}{3}\alpha^2 \frac{f_\rho^2}{m_\rho}, \tag{14b}$$

where G_F is Fermi constant, θ_c is Cabibbo angle.

When deriving the m_u and β_{uu} parameters from the experiment, we proceeded only from the data on the f_π and f_ρ coupling constants because they are measured well experimentally and their functional dependence on the m_u and β_{uu} is such (see also [13,18]) that allows one to find these parameters unambiguously. In this case the ratio f_π / f_ρ fixes up the ratio β_{uu} / m_u , while the quantities f_π and f_ρ themselves determine the nonstrange quark mass m_u . Finally we obtain:

$$m_u = 0.25 \text{ GeV}, \quad \beta_{uu} = 0.5 \text{ GeV}. \tag{15}$$

The values of τ_π and $\Gamma(\omega \to \pi\gamma)$ obtained with these parameters from formulae (13), (14) are listed in the Table and are in good agreement with experiment. Note that parameters (15) differ somewhat from those obtained in Ref. [19] , this being due to the difference in the normalization conditions of the wave functions. It is interesting also to note that after Ref. [19] was published the experimental value of τ_π was essentially improved. The coincidence of the new value of τ_π with the result obtained in the considered approach can be treated as a success of the model in the description of mesons.

Turn now to the strange mesons. In their description we use experimental data on f_K , τ_{K^-} , τ_{K^0} and on the widths of the $K \to \pi e \nu$,

$K^{*0} \to K^0\gamma$, $K^{*+} \to K^+\gamma$ decays. The f_K coupling constant and the amplitudes of the $K^* \to K\gamma$ decays are calculated within the considered model in Refs. [13,20] and have the following forms:

$$f_K = \frac{\sqrt{3}}{4\pi^2}\int\left[\sqrt{(\varepsilon_u+m_u)(\varepsilon_s+m_s)} - \sqrt{(\varepsilon_u-m_u)(\varepsilon_s-m_s)}\right]\frac{p^2}{\varepsilon_u\varepsilon_s}\phi_K(M_0^2)\,dp,$$

$$g_{K^{*+}K^+} = \frac{2}{3}g(u,s) - \frac{1}{3}g(s,u), \quad g_{K^{*0}K^0} = -\frac{1}{3}\left(g(d,s)+g(s,d)\right), \tag{16}$$

$$g(u,s) = \frac{1}{4\pi^2}\int\left[1 - \frac{\varepsilon_u(M_0+m_u+m_s)}{2M_0(\varepsilon_s+m_s)}\left(1-\frac{m_u^2}{2p\varepsilon_u}\ln\frac{\varepsilon_u+p}{\varepsilon_u-p}\right)+\right.$$

$$\left.+\frac{m_u}{2p}\ln\frac{\varepsilon_u+p}{\varepsilon_u-p}\right]\frac{M_0 p^2}{\varepsilon_u\varepsilon_s(\varepsilon_u+m_u)}\phi_K^2(M_0^2)\,dp\,.$$

The $K \to \pi e\nu$ transition amplitude, calculated according to formulae of Sect. 2, at $K^2 = 0$ is:

$$f(0) = \int\frac{(M_a M_a' + \vec{p}_\perp^{\,2})(M_b M_b' + \vec{p}_\perp^{\,2}) - \vec{p}_\perp^{\,2}(M_a' - M_a)(M_b' - M_b)}{\left[(M_a^2+\vec{p}_\perp^{\,2})(M_a'^2+\vec{p}_\perp^{\,2})(M_b^2+\vec{p}_\perp^{\,2})(M_b'^2+\vec{p}_\perp^{\,2})\right]^{1/2}}\,\cdot \tag{17}$$

$$\cdot\,\phi_K(M_0'^2)\,\phi_\pi(M_0^2)\,d\Gamma,$$

where

$$M_a = m_u + M_0 x,\quad M_b = m_u + M_0(1-x),\quad M_0^2 = \frac{m_u^2+\vec{p}_\perp^{\,2}}{x(1-x)},$$

$$M_a' = m_u + M_0' x,\quad M_b' = m_s + M_0'(1-x),\quad M_0'^2 = \frac{m_u^2+\vec{p}_\perp^{\,2}}{x} + \frac{m_s^2+\vec{p}_\perp^{\,2}}{1-x}\,.$$

The K-meson charge radii defined by the coefficients at $\vec{K}_\perp^2$ in the right-hand side of relation (1) are given by the formulae:

$$r_{K^-}^2 = \frac{2}{3}\mathcal{F}(m_a = m_u, m_b = m_s) + \frac{1}{3}\mathcal{F}(m_a = m_s, m_b = m_u),$$
$$r_{K^0}^2 = -\frac{1}{3}\mathcal{F}(m_a = m_d, m_b = m_s) + \frac{1}{3}\mathcal{F}(m_a = m_s, m_b = m_d), \tag{18}$$

where

$$\mathcal{F} = 6\int \gamma\, \phi_K^2 (M_0^2)\, d\Gamma,$$

$$\gamma = -\sum_{i=1}^{3} \frac{a_i^{(0)} b_i^{(0)}}{R_a R_b} + \frac{x}{8(1-x)\beta_{us}^2} + \tag{19}$$
$$+ \frac{x^2 + (1-x)^2}{2 R_a x^2}\left[x^2 + \frac{p_x^2}{M_0^2} - \frac{p_x^2}{R_a}\left(1 + \frac{m_a}{M_0}\right)^2\right].$$

The numerical calculations show that at the fixed value of the strange quark mass m_s the functional dependence of the considered quantities on β_{us} is such that with increasing β_{us} the f_K coupling constant grows, whereas the quantities $\Gamma(K^{*0,+} \to K^{0,+}\gamma)$, r_{K^-} and $|r_{K^0}^2|$ fall off. Here it turns out that for the values of m_s in the range 0.32 - 0.42 GeV, there is possible a good description of data on these quantities at the same value of β_{us} . Before giving final numerical results, pay attention at the following. The theoretical values of the widths of the $K \to \mu\nu$ and $K \to \pi e \nu$ decays depend on the Cabibbo angle which usually is found from the leptonic decays of the baryons. As an argument in favour of this, there is taken the fact that the SU(3)-symmetry breaking is not large in relative values of the baryon masses, contrary to meson ones. However, in the determination of θ_c from the leptonic decays of baryons, there is an essential uncertainty (see, e.g. [23,24]) connected with various assumptions on the SU(3)-symmetry breaking and with the absence of experimental information about the dependence of these decays formfactors upon the leptonic pair invariant mass. In this connection, we'd like to note that the use of the relativistic quark model enables one to calculate the amplitude $f(0)$; hence the value of $\sin\theta_c$ can be found from the data on the $K \to \pi e \nu$ decays whose formfactors are known well from experiment. The numerical estimates of the $f(0)$ amplitude show that despite the great difference in masses of strange and nonstrange quarks, its deviation from the value obtained in the limit of exact SU(3)-symmetry is insignificant, namely $f(0) = 0.98$. Using the data on both $K^+ \to \pi^0 e\nu$ and $K_L \to \pi^+ e\nu$ reactions, we obtain:

$$\sin\theta_c = 0.221 \pm 0.002 \,. \tag{20}$$

The experimental value of f_K listed in the Table corresponds to this value of $\sin\theta_c$. If we fix up the strange quark mass as 0.37 GeV, then this value of f_K comes out at the following values of the m_s and β_{us} parameters:

$$m_s = 0.37 \text{ GeV} , \qquad \beta_{us} = 0.6 \text{ GeV} . \tag{21}$$

The quantities $\Gamma(K^{*+,0} \to K^{+,0}\gamma)$, τ_{K^-} and $\tau^2_{K^0}$ obtained with these parameters are listed in the Table and agree well with experiment. As mentioned above, a good consistent description of the data on strange mesons is possible also at other values of m_s in the range 0.32 - 0.42 GeV. For example, the obtained at

$$m_s = 0.4 \text{ GeV}, \qquad \beta_{us} = 0.61 \text{ GeV} \tag{21'}$$

values of observables which are given in the Table in brackets are in good agreement with experiment.

Consider now the mesons composed of strange quarks, experimental data on which are: f_φ = (77.3 ± 1.8) MeV and $\Gamma(\varphi\to\eta\gamma)$ = (54.9 ± 4.2) keV [21]. To describe these quantities, we use formulae (13 b,c) with the replacements $f_\rho \to \frac{3}{\sqrt{2}} f_\varphi$, $m_u \to m_s$, $\beta_{uu} \to \beta_{ss}$ and $g_{\omega\pi} \to g_{\varphi\eta} \frac{3}{2y_\eta}$, where y_η is the admixture of strange quarks in η. The experimental value of f_φ comes out at

$$\beta_{ss} = (0.52 - 0.55) \text{ GeV} \tag{22}$$

for both values of the strange quark mass m_s = 0.37 and 0.4 GeV. Here it turns out that in order to describe the experimental value of $\Gamma(\varphi\to\eta\gamma)$, the strange quark admixture in η must be y_η = 0.55 ± 0.03 for m_s = 0.37 GeV and y_η = 0.57 ± 0.03 for m_s = 0.4 GeV. These values of y_η agree with the values of the mixing angle $\theta_P = -19^\circ$ (y_η = 0.58) from Ref. [27] and $\theta_P = -23^\circ$ [21] (y_η = 0.53) obtained from the mass formulae when using the linear mass operator.

4. Axial and Tensor Mesons.

In this section we'll consider the radiative decays of the P-wave mesons. These are axial mesons 1^{+-} (b_1(1235), h_1(1190), h_1', K_{1B}) and 1^{++} (a_1(1270), f_1(1285), f_1(1420), K_{1A}) as well as tensor mesons 2^{++} (a_2(1230), f_2(1270), f_2(1525), K_2^*(1430)). So long as the radiative decays of mesons are most convenient to study experimentally in the π- and K-meson scattering on nuclei with the use of the Primakoff effect, we shall consider only the radiative decays with the π- and K-mesons production. For this reason, we do not consider the scalar mesons.

Write down the Lorentz-covariant expressions for the transition currents of interest in the form:

$$<0^-, P', \lambda'=0 \,|\, J_\mu^{em} \,|\, 1^+, P, \lambda> = e g_A (V_\mu^{(\lambda)} PK - P_\mu V^{(\lambda)} \cdot K) \varphi_P \,, \qquad (23)$$

$$<0^-, P', \lambda'=0 \,|\, J_\mu^{em} \,|\, 2^+, P, \lambda> = \sqrt{2}\, g_T \epsilon_{\mu\nu\sigma\rho} V_{\nu\nu'}^{(\lambda)} P_\sigma K_\rho K_{\nu'} \varphi_P \,, \qquad (24)$$

where $V_\mu^{(\lambda)}$, $V_{\nu\nu'}^{(\lambda)}$ and φ_P are the wave functions of the axial (A), tensor (T) and pseudoscalar (P) mesons, and coupling constants $g_{A,T}$ are connected with the widths of the corresponding decays by the relation (14a). Calculating these coupling constants within the relativistic quark model in accordance with (1), for the $q_1 \bar{q}_2$ bound states we obtain:

$$g_{1+-} = (Q_{q_1} + Q_{q_2}) \int \frac{\vec{P}_\perp^2}{2\beta_{q_1 q_2}^2 (1-x)} \Phi_P(M_0^2)\, \Phi_A(M_0^2)\, d\Gamma, \qquad (25)$$

$$g_{1++} = \sqrt{2} \left[Q_{q_1} g_1(a=q_1, b=q_2) - Q_{q_2} g_1(a=q_2, b=q_1) \right], \qquad (26)$$

$$g_1 = \int \left[P_z \left(\frac{b_2^{(0)}}{R_b} - \frac{a_2^{(0)}}{R_a} \right) + x \vec{P}_\perp^2 \frac{R_b - R_a}{2 R_a R_b} \right] d\Gamma. \qquad (26')$$

Note that while calculating coupling constants (25) and (26) we considered the relation (1) at $\lambda = 1$ and $\lambda' = 0$, the coupling constants

(25), (26) being determined by the coefficients at K_x . In calculation of the $T \to P\gamma$ transition amplitude, we considered the relation (1) at $\lambda = 2$ and $\lambda' = 0$. Such a choice of λ is due to the fact that $V_\mu^{(\lambda)}$ and $V_{\nu\nu'}^{(\lambda)}$ should contain only transverse components ($\mu, \nu, \nu' = 1, 2$) which are independent of the meson mass. The longitudinal components V_μ and $V_{\nu\nu'}$ lead in relation (1) to ambiguity caused by the difference between the meson mass $m_{A,T}$ and M_0 , and therefore are not considered by us. Using the relation (1) at $\lambda = 2$ and $\lambda' = 0$ and calculating the coefficients at K_x^2 , we find:

$$g_T = a_{q_1} g_2 (a = q_1, b = q_2) - a_{q_2} g_2 (a = q_2, b = q_1), \tag{27}$$

where

$$g_2 = \int \frac{E_1 + E_2 + E_3}{\sqrt{2}\, R_a R_b} \Phi_P (M_0^2)\, \Phi_T (M_0^2)\, d\Gamma,$$

$$E_1 = G\left[P_x \left(a_0^{(0)} b_2^{(0)} - b_0^{(0)} a_2^{(0)}\right) + P_y \left(a_0^{(0)} b_1^{(0)} - b_0^{(0)} a_1^{(0)}\right)\right],$$

$$\begin{aligned} E_2 = {} & P_x \left(a_0^{(1)} b_2^{(0)} + a_0^{(0)} b_2^{(1)} - b_0^{(1)} a_2^{(0)} - b_0^{(0)} a_2^{(1)}\right) + \\ & + P_y \left(a_0^{(0)} b_1^{(1)} + a_0^{(1)} b_1^{(0)} - b_0^{(0)} a_1^{(1)} - b_0^{(1)} a_1^{(0)}\right), \end{aligned} \tag{27'}$$

$$E_3 = x P_y \left[P_x \left(b_1^{(0)} - a_1^{(0)}\right) + P_y \left(a_2^{(0)} - b_2^{(0)}\right)\right],$$

$$G = -\left[\frac{P_x}{2\beta^2_{q_1 q_2}(1-x)} + \frac{a_0}{R_a} + \frac{b_0}{R_b} \right].$$

The numerical calculations of coupling constants (25)-(27) show that if the parameters β_{uu} in the S- and P-wave mesons are treated as the same, then the experimental values of the $b_1 \to \pi\gamma$, $a_1 \to \pi\gamma$ and $a_2 \to \pi\gamma$ decay widths are described well. With increasing β_{uu} in the axial and tensor mesons, these widths drastically decrease and already at β_{uu} = 0.6 GeV noticeably disagree with the experiment. At a slight decrease in parameter β_{uu} in the P-wave mesons (β_{uu} = 0.4 GeV), the agreement with experiment is somewhat improved. At a larger decrease in β_{uu} , the widths of considered decays fall off again and at β_{uu} = 0.3 GeV come to disagreement with experiment. In order not to introduce a new parameter β_{uu} , we have imitated its slight decrease in the axial and tensor mesons by the replacement $\Phi_{A,T}(M_0^2) \to \Phi_{A,T}(M_0^2)/M_0$. and have given in the Table our results for the decay wisths of these mesons with the same β_{uu} and β_{us} as in the pseudoscalar and vector mesons. In the Table, we have listed also our predictions for $K_1(1280) \to K\gamma$ and $K_1(1400) \to K\gamma$ decay widths. Note that the physical states $K_1(1280)$ and $K_1(1400)$ are obtained by mixing the states K_{1A} and K_{1B} from the nonets 1^{++} and 1^{+-} : $K_1(1280) = K_{1A}\cos\varphi + K_{1B}\sin\varphi$; $K_1(1400) = -K_{1A}\sin\varphi + K_{1B}\cos\varphi$ For the mixing angle we have taken the value $\varphi = 33^0$ from Ref. [28] .

Conclusion.

The application of the quark model to the meson radiative decays usually reduces to the consideration of the magnetic-dipole transitions of the S-wave mesons in the framework of various schemes of the SU(3)-symmetry and Zweig rule breaking (see, e.g. Refs. [27,29,30]). This refers also to the bag models (see, e.g. [31]). The implication of the relativistic quark model to the description of mesons enables one to consider a wider range of observables within a unique approach. Within this model, we have considered a possibility of self-consistent description of charge radii, radiative and leptonic transitions of the S- and P-wave mesons. Here we obtained a good description of available experimental data under assumption that the SU(6)-symmetry is broken only in quark masses ($m_u = m_d \neq m_s$) and in their mean square momenta, and it turned out sufficient to treat the latters as dependent only on the quark content of mesons.

Note, that the description of the considered observables was carried out within the quark model also in Ref. [32] . In that work the model parameters are found from the meson mass spectrum description performed with account of relativistic effects and with the use of one-gluon-exchange-plus-linear--confinement potential. To find the parameters, also the data on $\Gamma(\rho\to\pi\gamma)$, $\Gamma(a_2\to\pi\gamma)$ and τ_π are used. Further on, with these parameters there were predicted the meson decay widths, in calculation of which the methods close to those of the nonrelativistic quark model were applied. Here it turns out that the predicted values of f_π , f_K , $\Gamma(K^{*+}\to K^+\gamma)$ and $\Gamma(b_1\to\pi\gamma)$ exceed experimental data by 35-60 %, while the value of the $\omega\to\pi\gamma$ decay width is essentially less than experimental one. Predictions for the quantities f_ρ , $\Gamma(K^{*0}\to K^0\gamma)$, $\Gamma(K_2^*(1430)\to K\gamma)$, τ_K- and $\tau^2_{K^0}$ agree with experiment.

Thus, our obtained good description of experimental data on low-energy characteristics of mesons is based on the account of relativistic effects which is carried out within the constituent quark relativistic model formulated in Refs. [11-15] . The account of these effects plays an essential role in the results obtained and enables one to obtain a self-consistent description of considered quantities.

It is interesting to note that the use of the relativistic quark model allowed us to calculate the amplitude of the $K \to \pi e \nu$ transition, this being impossible in the nonrelativistic model, if taking into account the difference of masses of strange and nonstrange quarks and the π- and K-mesons. It turned out that irrespective of the great difference in masses of strange and nonstrange quarks, the SU(3)-symmetry prediction for this amplitude is practically not broken. The calculation of the amplitude of the $K \to \pi e \nu$ transition enabled us to find the Cabibbo angle for constituent quarks from the data on the $K^+ \to \pi^0 e \nu$ and $K_L \to \pi^+ e \nu$ decays, whose formfactors are measured experimentally with high precision.

Table

Quantity	Our results	Experiment
f_π , MeV	93^*	93
f_ρ , MeV	155^*	155 ± 4 [21]
r_π , F	0.67	0.663 ± 0.023 [9] 0.657 ± 0.012 [10]
$\Gamma(\omega \to \pi\gamma)$, keV	765	853 ± 75 [21] 789 ± 92 [22]
f_K , MeV	113^* (116)	113
$\Gamma(K^{*0} \to K^0\gamma)$, keV	125 (117)	116.5 ± 9.9 [7]
$\Gamma(K^{*+} \to K^+\gamma)$, keV	54 (57)	51 ± 5 [6]
r_{K^-} , F	0.58 (0.58)	0.53 ± 0.05 [4]
$r^2_{K^0}$, F^2	-0.05 (-0.05)	0.08 ± 0.05 [25] -0.054 ± 0.026 [26]
$\Gamma(b_1^\pm \to \pi^\pm\gamma)$, keV	275	230 ± 60 [1]
$\Gamma(a_1^\pm \to \pi^\pm\gamma)$, keV	323	640 ± 246 [2]
$\Gamma(a_2^\pm \to \pi^\pm\gamma)$, keV	324	295 ± 60 [8]
$\Gamma(K_2^{*+}(1430) \to K^+\gamma)$, keV	141	238 ± 50 [21]
$\Gamma(K_2^{*0}(1430) \to K^0\gamma)$, keV	2	
$\Gamma(K_1^+(1280) \to K^+\gamma)$, keV	2	
$\Gamma(K_1^0(1280) \to K^0\gamma)$, keV	175	
$\Gamma(K_1^+(1400) \to K^+\gamma)$, keV	495	
$\Gamma(K_1^0(1400) \to K^0\gamma)$, keV	538	

Note to Table. Quantities labelled by asterisks are taken by us as input ones to find the model parameters. The other quantities are calculated with these values of parameters. Predictions for the $K_1(1280) \to K\gamma$ and $K_1(1400) \to K\gamma$ decays are obtained at a 33° mixing angle. The results for f_ω and widths of the $\rho \to \eta\gamma$, $\omega \to \eta\gamma$, $\eta' \to \rho\gamma$, $\eta' \to \omega\gamma$, $h_1(1190) \to \pi\gamma$, $h_1 \to \pi\gamma$ decays are not listed in the Table, since they can be simply obtained from the given quantities with account of isotopic factors, phase spaces and admixture of nonstrange quarks in η, η', h_1, h_1'. The $f_{1,2} \to \pi\gamma$ decays violating C-parity conservation are forbidden.

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РАДИАЦИОННЫЕ И ЛЕПТОННЫЕ ПЕРЕХОДЫ МЕЗОНОВ В РЕЛЯТИВИСТСКОЙ МОДЕЛИ КВАРКОВ

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