индекс 3624

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

Preprint ЕФИ-991(41)-87

ԵՐԵՎԱՆԻ ՖԻԶԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ
ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ
YEREVAN PHYSICS INSTITUTE

S.G.ARUTYUNIAN, H.M.BABUJIAN

ORTHOGONAL FIELDS ELECTRODYNAMICS AS DYNAMICS OF FORCE LINES

ЦНИИатоминформ
ЕРЕВАН — 1987

Նախնատիպ ЕФИ-99I(4I)-87

Ս.Գ.ՀԱՐՈՒԹՅՈՒՆՅԱՆ,Հ.Մ.ԲԱԲՈՒՋՅԱՆ

ՕՐԹՈԳՈՆԱԼ ԴԱՇՏԵՐԻ ԷԼԵԿՏՐԱԴԻՆԱՄԻԿԱՆ ՈՐՊԵՍ ՈՒԺԱԳԾԵՐԻ ԴԻՆԱՄԻԿԱ

Ստացված են Մաքսվելի հավասարումներին համարժեք համապարփակ կովարիանտ հավասարումներ՝ շարժվող ուժագծերի համար։

Երևանի Ֆիզիկայի ինստիտուտ

Երևան 1987

Препринт ЕФИ-99I(4I)-87

С.Г.АРУТЮНЯН,Г.М.БАБУДЖЯН

ЭЛЕКТРОДИНАМИКА ОРТОГОНАЛЬНЫХ ПОЛЕЙ КАК ДИНАМИКА СИЛОВЫХ ЛИНИЙ

Получены замкнутые ковариантные уравнения для движущихся силовых линий, эквивалентные уравнениям Максвелла

Ереванский физический институт

Ереван 1987

Preprint ЕФИ-99I(4I)-87

S.G. ARUTYUNIAN, H.M. BABUJIAN

ORTHOGONAL FIELDS ELECTRODYNAMICS AS DYNAMICS OF FORCE LINES

Closed covariant equations for moving force lines equivalent to Maxwell equations are obtained.

Yerevan Physics Institute
Yerevan 1987

The systematic introduction of moving covariant force lines describing the given field is tolerable for orthogonal electric and magnetic fields [1]. But one can reverse the problem and find the field by a system of lines which obey definite equations. The posssibility for such an approach was discussed by Dirac [2].

This reversal was done and the closed equations for force lines equivalent to Maxwell's equations were obtained in this work. Such electrodynamics formulation by means of force lines may be useful in the Rainich-Wheeler geometrodynamics [3]. The geometric approach turned out efficient, in particular, when solving applied problems, where the knowledge of the field fine spatial dependence is necessary, for instance, in the accelerator physics [4-5]. In this work the connection between the obtained equations and those of magnetohydrodynamics of liquid with infinite conductivity is pointed out [6].

At ultrahigh energies the particle field is constricted into a narrow extended γ-region [4] of transverse sizes $R\gamma^{-3}$ and $L\gamma^{-1}$, where R is the radius of the trajectory curvature, $\gamma = E/mc^2$ (E is the total energy), L is the path length of delayed signals which form this region. It means that at high energies interaction in the electrodynamics is of a localized in the space (string) character.

1. Let $F^{\mu\nu}$ be the electromagnetic field tensor. Then equations in total differentials [1]

$$F_{\mu\nu}\, dx^{\mu} = 0 \tag{1}$$

define the system of Lorentz-invariant magnetic force lines. In three-dimensional indications $dx^{\mu} = (c\,dt, d\vec{r})$ the equations (1) are written in the form

$$\begin{aligned} [d\vec{r} \times \vec{H}] + c\,dt\,\vec{E} &= 0, \\ (\vec{E}\, d\vec{r}) &= 0, \end{aligned} \tag{2}$$

where $\vec{E}$ and $\vec{H}$ are electric and magnetic fields, respectively. The system (1) has non-zero solutions if $\det F^{\mu\nu} = 0$, i.e. for orthogonal fields. The condition of the system integrability is written as

$$[\vec{H} \times (\mathrm{rot}\,\vec{E} + \frac{1}{c}\frac{\partial \vec{H}}{\partial t})] - \vec{E}\,\mathrm{div}\,\vec{H} = 0 \tag{3}$$

which is automatically satisfied owing to Maxwell equations.

Under $\vec{E}\vec{H} = 0$ the antisymmetric matrix $F^{\mu\nu}$ is of rank two and the system (1) determines the surface $x^{\mu}(\tau,\sigma)$ of dimension two, which is swept out by a unidimensional magnetic force line.

As seen from (1), on the surface $x^{\mu}(\tau,\sigma)$ lie the world lines of charged particles of masses tending to zero. Really, the expression $\frac{e}{c} F_{\mu\nu} \frac{dx^{\mu}}{ds}$ describes the Lorentz force which acts on a particle (s is the intrinsic time). Note, that the condition of the Lorentz force being equal to zero is applied to equations of the magnetohydrodynamics of liquid with infinite conductivity [6].

The electric force lines are given by

$$F^{*}_{\mu\nu}\, dx^{\mu} = 0 \tag{4}$$

where $F^{*}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$ is a tensor dual to $F_{\mu\nu}$ *. These equations too have nontrivial solutions at $\vec{E}\vec{H} = 0$. Nevertheless, the integrability condition is satisfied in a space with no charges and currents.

Later on we'll consider only magnetic force lines.

2. The problem stated in the preceding section can be reversed, i.e. the field $F^{\mu\nu}$ be determined by a system of force lines.

The substitution of $x^{\mu} = x^{\mu}(\tau,\sigma)$ into (1) yields

$$F^{*\mu\nu} = \lambda (\dot{x}^{\mu} x'^{\nu} - x'^{\mu} \dot{x}^{\nu}), \tag{5}$$

where λ is an arbitrary scalar function, and the dot and prime denote differentiation over τ and σ, respectively. The expression (5) determines the field only on the surface $x^{\mu}(\tau,\sigma)$: that is where it can be arbitrary. Equations on the function x^{μ} arise if a congruence of such surfaces is given which fills the 4-volume.

Let parameters C_1 and C_2 realize such a congruence. Now, considering x^{μ} as a function of four variables $x^{\mu} = x^{\mu}(\tau,\sigma,c_1,c_2)$, and expressing the derivatives $\partial/\partial\xi$ through derivatives $\partial/\partial\xi^{\ell}$, where $\xi^{\ell} = (\tau,\sigma,c_1,c_2)$,

---

*The eq.(4) can be also interpreted as one of motion of a massless particle, but with a magnetic charge other than zero. From this point of view the eq.(1) is more "real".

we'll obtain equations equivalent to those of Maxwell:

$$\frac{\partial}{\partial \xi^{\ell}} \left[\varepsilon^{ijk\ell} \lambda x^{\mu}_{,i} (\dot{x} x_{,j})(x' x_{,k}) \right] = 0 , \tag{6}$$

$$\dot{x}^{\mu} (\lambda G)' - x'^{\mu} (\lambda G)^{\cdot} = 0 , \tag{7}$$

Here $x^{\mu}_{,i} = \partial x^{\mu} / \partial \xi^{i}$, $G = \det (\partial x^{\mu} / \partial \xi^{i})$. For linearly independent $\dot{x}^{\mu}$, x'^{μ} the eq.(7) determines the function of λ :

$$\lambda = f(c_1, c_2) G^{-1} , \tag{8}$$

where $f(c_1, c_2)$ is an arbitrary function of c_1 and c_2. The field $F^{\mu\nu}$ must depend only on the functions x^{μ} and their derevatives with respect to ξ^{i}, hence one can choose $f(c_1, c_2) = 1$.

In a space without charges and currents the electric force line dynamics is determined again by the eqs.(6-7) (in (5) one is to substitute $F^{*\mu\nu} \longrightarrow F^{\mu\nu}$).

Equations for magnetic force lines are easily generalized into a case with charges and currents. Really, here only the eq.(6) will be changed:

$$\frac{\partial}{\partial \xi^{\ell}} \left[\varepsilon^{ijk\ell} G^{-1} x^{\mu}_{,i} (\dot{x} x_{,j})(x' x_{,k}) \right] = G \frac{4\pi}{c} j^{\mu} , \tag{9}$$

where the given current j^{μ} depends on the function $x^{\nu}(\xi)$.

For electric force lines the current part is added to the right side of eq.(7), thus depriving it of an obvious solution $\lambda = G^{-1}$.

3. The expression (5) at $\lambda = G^{-1}$ is invariant under arbitrary reparametrization $\tau = \tau(\bar{\tau}, \bar{\sigma})$, $\sigma = \sigma(\bar{\tau}, \bar{\sigma})$.

The eqs.(6-7) are also invariant under such reparametrization. In (5) the substitution $c_1 = c_1(\bar{c}_1, \bar{c}_2)$, $c_2 = c_2(\bar{c}_1, \bar{c}_2)$ under the condition $\det(\partial \bar{c}_a / \partial c_b) = 1$ is also admissible.

By reparametrization (τ , σ) one can obtain a conformal calibration of the lines $\dot{x} x' = 0$, $\dot{x}^2 + x'^2 = 0$ on one of the surfaces c_1 , c_2 , but on other surfaces such gauging, generally speaking, may not be fulfilled. In case of $\dot{x}^2 x'^2 - (\dot{x} x')^2 = 0$ one may impose conditions $\left(\frac{\partial x}{\partial \bar{\tau}} \frac{\partial x}{\partial \bar{\sigma}} \right) = 0$, $\left(\frac{\partial x}{\partial \bar{\sigma}} \right)^2 = 0$ (on one surface). To just such gauging (in the whole space) correspond the solutions (6) which describe the linearly polarized plane wave:

$$x^0 = \tau , \; x^1 = c_1 , \; x^2 = \sigma \cos c_2 , \; x^3 = \tau + c_2 \tag{10}$$

4. The eqs. (6) can be treated as Lagrange ones with a standard Lagrangian in the space x^{μ} , if one constructs the corresponding 4-potentials A^{μ} . For instance, in the gauging of $A^{\mu} x_{\mu} = 0$ [7]:

$$A^{\mu} = \int_0^{\lambda} d\lambda \cdot \lambda F^{\mu\nu}(\lambda x) x_{\nu} \tag{11}$$

But this expression is a non-local function of $x^{\mu}(\xi)$. That is why it is important to obtain a Lagrangian in the space of parameters ξ^{i} with changing variables $x^{\mu}(\xi)$.

5. The system of surfaces $x^{\mu}(\tau, \sigma, c_1, c_2)$ gives a certain separated curvilinear coordinate system. The electromagnetic field tensor has following form in these coordinates:

$$\tilde{F}^{*ik} = G^{-1}(\delta_0^i \delta_1^k - \delta_1^i \delta_0^k),$$
$$\tilde{F}^{ik} = -G^{-1} \varepsilon^{ikmn} \left(\frac{\partial x}{\partial \xi^0} \frac{\partial x}{\partial \xi^m}\right) \left(\frac{\partial x}{\partial \xi^1} \frac{\partial x}{\partial \xi^n}\right). \tag{12}$$

As it is seen, there remains the only non-zero electric field component for the tensor $\tilde{F}^{*ik}$ in coordinates corresponding to the force lines.

One pair of Maxwell equations is identically satisfied, but the second pair in the absence of charges and currents is reduced to equations of the first over the unknown functions g_{ik} order:

$$\frac{\partial}{\partial \xi^l} \left(E^{ikmn} g_{om}\, g_{1n} \right) = 0, \tag{13}$$

where g_{ik} is a metric tensor, $E^{ikmn} = (-g)^{-1/2} \varepsilon^{ikmn}$ is an antisymmetric single tensor of rank 4 in curvilinear coordinates.

The gauge field representation through one-dimensional objects was considered in refs.[8-9]. The connection between the electrodynamics and classic equations of motion of boson strings [10,12] was also studied, but it was necessary to introduce here nonphysical volume currents to satisfy the Maxwell equations.

The electrodynamics formulation through the force lines seems more natural for these purposes. First, the electromagnetic field is expressed by a system of surfaces locally swept by moving force lines. Second, the theory is constructed freely from the very beginning, i.e. without charges and currents.

The analysis of the field geometry of one charge [3,4]

allows one to assume that at ultrahigh energies the dependence on C_1 and C_2 in the force line equations becomes inessential and the field will mainly be a function of properties of only one surface swept by the γ-region. Here it is also important to obtain an expression for the action of electrodynamics with changing variables X^μ .

The authors are sincerely grateful to A.Ts. Amatuni, M.R. Mailian, S.G. Matinyan, G.A. Nagorsky and T.V. Shakhbazian for discussions and helpful comments.

References

1. Леонтович М.А. Эволюция представлений о магнитных и электрических силовых линиях. УФН,I964, т.84,вып.4, с.7I5.
2. Дирак П. Эволюция физической картины природы (элементарные частицы)/Под.ред. Б.В.Медведева. М.:Наука,I965,с.I23
3. Wheeler J.A. Neutrinos, Gravitation and Geometry, Bologna, 1960.
4. Арутюнян С.Г.,Нагорский Г.А. Эффекты интенсивности в ускорителях и накопителях,связанные с лиенар-вихертовскими полями частиц. Препринт ЕФИ-453(60)-80, Ереван, I980.
5. Арутюнян С.Г. Линии электромагнитного поля произвольно движущейся в вакууме точечной заряженной частицы. УФН,I984, т.I50, вы.3, с.445.
6. Ландау Л.Д.,Лифшиц Е.М. Электродинамика сплошных сред. М.: Наука, I982.
7. Ициксон К.,Зюбер Ж.Б. Квантовая теория поля, т.I, М.: Мир, I984.
8. Mandelstam S. Feynman Rules for Electromagnetic and Yang-Mills Fields from the Gauge-Independent Field-Theoretic Formalism. Phys.Rev., 1968, vol.175, p.1580.
9. Polyakov A.M. Gauge Fields as Rings of Glue. Nucl.Phys., 1979, vol.B164, p.171.
10. Nambu Y. Hamiltonian-Jacobi Formalism for Strings. Phys. Lett., 1980, vol.92B, p.327.
11. Kastrup H.A., Rinke M. Hamilton-Jacobi Theories for Strings. Phys.Lett., 1981, vol.165B, p.191.

12. Hosotani Y. Hamiltonian-Jacobi Formalism and Wave Equations for Strings. Phys.Rev.Lett., 1985, vol.55, p.1719.

The manuscript was received 30 April 1987