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ARBITRARY SPIN MESONS IN QCD SUM RULES

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ԿԱՄԱՅԱԿԱՆ ՍՊԻՆՈՎ ՄԵԶՈՆՆԵՐԸ ՔՔԴ ԳՈՒՄԱՐՆԵՐԻ ԿԱՆՈՆՆԵՐՈՒՄ

ՔՔԴ գումարների կանոնների շրջանակներում, մեզոնների գրգռված վիճակների հատկությունների վերլուծության համար որպես հադրոնների աղբյուրներ դիտարկված են ոչ տեղային, տրամաչափորեն ինվարիանտ քվարկային հոսանքները: Հաշվարկված են ,,մերկ,, քվարկային օղակի և գլյուոնային շտկման ներդրումները հոսանքի կոռելյատորի մեջ՝ կամայական սպինով վիճակների համար: Ցույց է տրված, որ ռեջեյան հետագծերի վրա ընկած զանգվածների տիրույթում առկա է քվարկ-հադրոնային երկվություն. գլյուոնային խտուցքի հետ կապված աստիճանային շտկման ներդրումը, համեմատած խոտորումների տեսության ներդրման հետ՝ փոքր է: Ստացված բանաձևերը ճիշտ են նկարագրում այն ռեզոնանսների զանգվածները, որոնք ընկած են ռեջեյան α_ρ , α_ω , α_{A_2} , α_f հետագծերի վրա:

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А.А.ГРИГОРЯН, А.Э.ТЫУГУ

МЕЗОНЫ С ПРОИЗВОЛЬНЫМИ СПИНАМИ В ПРАВИЛАХ СУММ КХД

Для анализа свойств возбужденных состояний мезонов в рамках правил сумм КХД в качестве адронных источников рассмотрены нелокальные калибровочно-инвариантные кварковые токи. Вычислены вклады "голой" кварковой петли и глюонной поправки в токовый коррелятор для состояний с произвольным спином. Показано, что в области масс, лежащих на реджевских траекториях, имеет место кварк-адронная дуальность - вклад степенной поправки, связанный с глюонным конденсатом, мал по сравнению с вкладом теории возмущений. Полученные формулы правильно описывают массы резонансов, лежащих на реджевских траекториях α_ρ, α_ω, α_{A_2}, α_f.

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A.A. GRIGORYAN, A.E. TYUGU

ARBITRARY SPIN MESONS IN QCD SUM RULES

To analyze the meson excitations in the framework of QCD sum rules, we consider as hadron sources the non-local gauge-invariant quark currents. Constructing the given spin state correlator of currents, we calculate the contributions of "bare" quark loop and $\langle G^2 \rangle$ - power correction into this correlator. In the mass region on Regge-trajectories there takes place the quark-hadron-duality, i.e. the $\langle G^2 \rangle$ - power correction contribution is small as compared with that of the perturbation theory. Our formulae describe correctly the masses of resonances on Regge-trajectories α_ρ , α_ω , α_{A_2} and α_f .

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1. Introduction

At present, the dispersion method of QCD sum rules, developed in the works of Shifman, Vainstein and Zakharov [1] (SVZ-method) is widely and successfully used for the calculations of low-lying hadron states static characteristics.

On the other hand, an attempt to apply the QCD sum rules to the calculation of large spin meson masses, using the local currents, is doomed to failure [2] . For the particles lying on the Regge-trajectories (for which spin $n \sim M^2$), the so-called quark-hadron duality turns out violated. Namely, at large n the contribution of power correction, connected with gluon condensate, starts to dominate over the contribution of perturbation theory, making senseless the ideology of QCD sum rules.

The purpose of our work is to analyze the large spin mesons in the framework of the SVZ-method, considering as the hadron sources the gauge-invariant non-local currents (see Sect. 2).

Using the definite spin states extraction technique, developed in Ref. [3] , we calculated the one-loop perturbative contribution, as well as those of power corrections connected with condensates $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, $m \langle \bar{\Psi} \Psi \rangle$, $(\langle \bar{\Psi} \Psi \rangle)^2$, into the sum rule for the two-current correlator with given spin. The calculations show that as distinct from the case of local currents, the power correction connected with two-gluon condensate is indeed correction for all mesons on the trajectory $n = \alpha(0) + \alpha' M^2$ (the operator-expansion coefficients in front of quark condensates $m \langle \bar{\Psi} \Psi \rangle$ and $(\langle \bar{\Psi} \Psi \rangle)^2$ vanish at $n \geqslant 3$). This allows us to determine from the sum rules the slopes α' of meson Regge-trajectories which consist of light quarks: $\alpha' \approx 1$ - in good agreement with experiments.

2. Two-Current Correlator.

Consider the operator:

$$J^{(i)}(x,\xi) = \bar{\Psi}(x+\xi) A^{(i)} P\exp\left[ig\int_{x-\xi}^{x+\xi} A_\mu(x')dx'_\mu\right]\Psi(x-\xi) , \tag{1}$$

where

$$A^{(i)} = I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu} . \quad A_\mu(x) = \frac{\lambda^a}{2} A^a_\mu(x) .$$

In what follows, we'll neglect the masses of quarks.

The properties of this operator as a possible candidate on the role of QCD analogue of string were discussed by a number of authors in a series of works [3] . It was found that under some additional assumptions it satisfies the free relativistic string equation. Thus the conclusion was made that this operator is the wave functional of the open string. At the same time, the analysis of QCD lagrangian shows that such interpretation is somewhat hasty [4] .

However, the expression (1) can be interpreted as the source of hadrons with arbitrary spins, and the dispersion sum rules can be used for calculations of slopes of Regge trajectories. We will work in the straight-line contour approximation. As was shown in [5] , the straight-line (single-mode) string is of a special physical interest, because it can be self-consistently quantized in relativistically invariant manner in the 4-dimensional space-time.

The expression (1) can be written in the following form:

$$J^{(i)}(x,\xi) = \sum_n \frac{\xi^{\mu_1}\dots\xi^{\mu_n}}{n!}\bar{\Psi}(x) A^{(i)}\overleftrightarrow{\nabla}_{\mu_1}\dots\overleftrightarrow{\nabla}_{\mu_n}\Psi(x) = \sum_n \frac{\xi^{\mu_1}\dots\xi^{\mu_n}}{n!} J^{n(i)}_{\mu_1\dots\mu_n}(x), \qquad (2)$$

where

$$\overleftrightarrow{\nabla}_\mu = -\overleftarrow{\partial}_\mu + \overrightarrow{\partial}_\mu - 2igA_\mu(x).$$

We see that the nonlocal current with the straight-line contour is nothing else than the infinite sum of the local hadron sources with arbitrary spins.

For simplicity, we consider first the case of "scalar" current. The correlator, containing the contributions of states with given spin n , has the form:

$$\Pi^{(S)n}(q^2) = i\int d^4x e^{iqx} \sum_m \int d^4\xi \int d^4\eta \sqrt{\frac{P_m^2}{\xi^2}}\,\delta(P_m\cdot\xi) f(\sqrt{\xi^2}),$$

$$\sqrt{\frac{P_m^2}{\eta^2}}\,\delta(P_m\cdot\eta)\, f(\sqrt{\eta^2})\, Y^{n\lambda}(P_m,\xi)\, Y^{*n\lambda}(P_m,\eta)\times$$

$$\times\begin{cases} <0|J^{(S)}(0,\xi)|m><m|\bar{J}^{(S)}(0,\eta)|0>e^{-iP_m x} & t_x>0 \\ <0|\bar{J}^{(S)}(0,\eta)|m><m|J^{(S)}(0,\xi)|0>e^{iP_m x} & t_x<0 \end{cases} \tag{3}$$

In this expression the sum is taken over the complete set of hadronic states emitted by currents $J^{(S)}(0,\xi)$, $\bar{J}^{(S)}(0,\eta)$, and over all intrinsic quantum numbers of these states. P_m is the m -state 4-momentum, the $Y^{n\lambda}(P_m,\xi)$ are the usual spherical harmonics written in the 4-dimensional form (see [5, 6]), $f(x)$ is the weight-function (see Sect. 3).

3. Calculations of "Bare" Quark Loop and Gluon Power Correction.

It is handy to carry out the calculations in the $q_\mu=(q_0\,,\,0)$ frame, using the fixed-point gauge [7] . Then the correlator, subject to "bare" loop and two-gluon power correction, has the following form:

$$\Pi^{(S)n}(q_0^2)=i\int d^4x\, e^{iq_0 t}\int d^3\vec{\xi}\int d^3\vec{\eta}\ \ (\vec{\xi}^2\,\vec{\eta}^2)^{-1}\times$$

$$\times f(|\vec{\xi}|)\, f(|\vec{\eta}|)\, Y^{n\lambda}(\vec{n}_\xi)\, Y^{*n\lambda}(\vec{n}_\eta)\times \tag{4}$$

$$\times\left\{\frac{3}{\pi^4}\ \frac{(\alpha\beta)}{\alpha^4\beta^4}+\frac{<\frac{\alpha_s}{\pi}G^2>}{12\pi^2}\left[\frac{(\alpha\beta)((x(\xi+\eta))^2-x^2(\xi+\eta)^2)}{\alpha^4\beta^4}-\frac{3\ (\alpha\beta)}{8\,\alpha^2\beta^2}\right]\right\},$$

where

$$\alpha = x+\eta-\xi\,,\quad \beta = -x+\eta-\xi\,,\quad \langle \frac{\alpha_s}{\pi} G^2 \rangle = \langle 0 | \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a .$$

Integrating over x , we get:

$$\mathrm{Im}\,\Pi^{(s)n}(q_0^2) = \int d|\vec{\xi}| \int d|\vec{\eta}| f(|\vec{\xi}|) f(|\vec{\eta}|) \frac{2n+1}{\pi^2} 2^{-6} \times$$

$$\Big[6q_0^2 + \frac{\pi^2}{3} \langle \frac{\alpha_s}{\pi} G^2 \rangle \Big(2\vec{\xi}^2 \big(-5 - 5q_0 \frac{d}{dq_0} - q_0^2 \big(\frac{d}{dq_0}\big)^2 \big) -$$

$$-2|\vec{\xi}| \frac{d}{d|\vec{\xi}|} |\vec{\eta}| \frac{d}{d|\vec{\eta}|} \Big(\frac{1}{q_0^2} + \frac{1}{q_0} \frac{d}{dq_0} \Big) - \frac{1}{2} q_0^2 \Big(\frac{d}{dq_0}\Big)^4 - \tag{5}$$

$$-3q_0 \Big(\frac{d}{dq_0}\Big)^3 - 7\Big(\frac{d}{dq_0}\Big)^2 - \frac{8}{q_0} \frac{d}{dq_0} \Big) \Big] j_n(q_0|\vec{\xi}|)\, j_n(q_0|\vec{\eta}|),$$

Here $j_n(x)$ is the spherical Bessel's function.

To integrate over $|\vec{\xi}|$ and $|\vec{\eta}|$, it is necessary to choose the weight-function $f(x)$. The detailed analysis of sum rules dependence on $f(x)$ will be made in the subsequent publications. Here we consider the simple case $f(x) = x^{a}$. Then in the limit $n \gg (1, |a|)$ the imaginary part of correlator will be equal to:

$$\mathrm{Im}\,\Pi^{(s)n}(q^2) = B^{(s)}_{n,a} (q^2)^{-a} \Big(1 - \frac{\pi^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle}{9\,(q^2)^2} n^2 (4a^2 + 8a + 5) \Big), \tag{6}$$

where $B^{(s)}_{n,a}$ is the numerical factor. It can be shown, that in the case of $A^{(i)} = \gamma_\mu, \gamma_5, \gamma_\mu\gamma_5$ the results will be the same up to coefficient $B^{(i)}_{n,a}$. As to the contribution of quark condensates $\langle \bar{\Psi}\Psi \rangle$, $\alpha_s (\langle \bar{\Psi}\Psi \rangle)^2$, at $n \geq 5$ the coefficients of these operators are zero.

So, in our model the quark-hadron duality takes place at $q^2 \gg n$, i.e. the contribution of the main power correction, connected with the gluon condensate, is small against that of the perturbation theory, unlike the case with local currents [2] .

From expression (6) it follows, that the dispersion integral makes sense at $a < -1$. Taking the $a = -2$ and saturating the right - phenomenological - side of sum rules by resonance of mass M_n and spin n , and the quark continuum with threshold S_o , in the frames of standard SVZ-method we get the following result:

$$\alpha' \approx n/M_n^2 = (1 \pm 0.1)\ \mathrm{GeV}^{-2}\ , \quad S_o/n = (1.5 \pm 0.2)\ \mathrm{GeV}^2$$

which agrees well with experimentally established slopes α'_{VT} of the rightmost in j -plane trajectories of vector-tensor (VT) group (ρ , ω , A_2 , f - trajectories).

Conclusion.

We have shown that the suitable choice of nonlocal currents as hadron sources allows us to extend the technique of SVZ sum rules to the case of mesons of large spins and to get the correct values of Regge trajectories slopes in the case of light quarks. In the considered approximation (the account of $< G^2 >$, $m_q < \bar{\Psi} \Psi >$ and $\alpha_s (< \bar{\Psi} \Psi >)^2$ power corrections only) the slopes of trajectories with other quantum numbers of signature σ and parity P (the trajectories of axial, scalar and pseudoscalar groups) degenerate at large spins with α'_{VT} . Further, in this approximation the slopes of trajectories $\bar{q}q$, $\bar{q}s$, $\bar{s}s$ are the same. The case of trajectories, consisting of heavy, c , b -quarks, requires a special analysis. Evidently, the calculations of characteristics of low-lying states will be a good verification of consistency of this method (this requires rather complicated calculations of nonperturbative power corrections, whose contribution vanishes at large spins). It is interesting also to apply this method to the case of baryons and glueballs.

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МЕЗОНЫ С ПРОИЗВОЛЬНЫМИ СПИНАМИ В ПРАВИЛАХ СУММ КХД

(на английском языке, перевод З.Н.Асланян)

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