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SECOND QUANTIZATION OF RELATIVISTIC PARTICLE WITH SPIN 1/2 AND QUANTUM SUPERSYMMETRY

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Р.П.МАНВЕЛЯН

ВТОРИЧНОЕ КВАНТОВАНИЕ РЕЛЯТИВИСТСКОЙ ЧАСТИЦЫ СО СПИНОМ 1/2 И КВАНТОВАЯ СУПЕРСИММЕТРИЯ

Используя процедуру первичного квантования Баталина-Фрадкина-Вилковиского, получено вторично-квантованное действие для спиновой частицы, обладающее одномерной линейной суперсимметрией.

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R.P. MANVELYAN

SECOND QUANTIZATION OF RELATIVISTIC PARTICLE WITH SPIN 1/2 AND QUANTUM SUPERSYMMETRY

The second quantized action for a spinning particle with one-dimensional linear supersymmetry is obtained through the first quantization of Batalin-Fradkin-Vilkovisky.

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Yerevan 1987

Introduction

The method of BRST quantization of Batalin-Fradkin-Vilkovisky (BFV) [1] permits a large choice of gauge fixing for the quantum action of the relativistic spinning particle [2]. It leads to a class of second-quantized field theory actions. We prove in this work that one can choose such a gauge fixing function Ψ, that the second quantized action of the particle with spin 1/2, as in the case of a spinless particle [4], will have a linear supersymmetry instead of a nonlinear BRST symmetry.

When adding an interaction in the supersymmetric manner, the obtained supersymmetric field theory coincides with that [3] obtained from stochastic quantization.

Such an approach to the second quantization can be useful for the construction of the string field interaction and the analysis of the string perturbation theory.

1. The First BRST Quantization

Let us consider the naive action for the particle with spin 1/2 and mass m [7]

$$S = \int d\tau \,(P_\mu \dot{x}_\mu + \xi_\mu \dot{\xi}_\mu + \xi_5 \dot{\xi}_5 - \lambda L_1 - \chi L_2), \qquad (1)$$

where x_μ , P_μ are the ordinary coordinates and momenta, respectively; ξ_μ and ξ_5 are real Grassman variables describing the spin; L_1 , L_2 are the first-class constraints

$$L_1 = \frac{1}{2}(p^2 - m^2), \quad L_2 = p_\mu \xi^\mu - m\xi_5$$

satisfying the following conditions:

$$\{L_1, L_1\} = 0, \quad \{L_2, L_2\} = 2L_1, \quad \{L_1, L_2\} = 0.$$

The action (1) has a local supersymmetry defined by two generators L_1 , L_2 (superreparametrization).

Let us apply the BRST quantization of BFV to this system. The phase space has the following additional variables:

1) canonical momenta π_λ and π_χ for Lagrange multipliers λ and χ ;

2) a pair of Grassman ghosts C and $\bar{C}$ and a pair of boson ghosts S and $\bar{S}$;

3) canonical conjugate momenta for ghosts and antighosts: $\mathcal{B}$, $\bar{\mathcal{B}}$ for C , $\bar{C}$ and P_S , $P_{\bar{S}}$ for S , $\bar{S}$. The BRST charge is defined as:

$$Q = CL_1 + SL_2 + \bar{\mathcal{B}}\pi_\lambda + P_{\bar{S}}\pi_\chi + S^2\mathcal{B}. \tag{2}$$

The quantum action is

$$S_q = \int d\tau \{ p_\mu \dot{x}_\mu + \xi_\mu \dot{\xi}_\mu + \xi_5 \dot{\xi}_5 + \pi_\lambda \dot{\lambda} + \pi_\chi \dot{\chi} + \mathcal{B}\dot{c} + \bar{\mathcal{B}}\dot{\bar{c}} + \\ + P_S \dot{S} + P_{\bar{S}} \dot{\bar{S}} - \{Q, \Psi\}, \tag{3}$$

where $\{Q, \Psi\} = H_Q$ is the quantum Hamiltonian, and Ψ is an arbitrary gauge fixing function with the ghost number -1,

For our purpose it is convenient, instead of ordinary relativistic gauge $\dot{\lambda} = \dot{x} = 0$ [2] corresponding to $\Psi = \lambda \bar{b} + \chi P_s$, to choose Ψ in the following way:

$$\Psi_\alpha = \lambda \bar{b}(1 - \bar{c}c) + \alpha \chi P_s ,$$

where α is an arbitrary numerical parameter. The value of the functional integral does not depend on the choice of Ψ and hence, on the value of α. The quantum action is:

$$S_Q^\alpha = \int d\tau \{ P_\mu \dot{x}_\mu + \xi_\mu \dot{\xi}_\mu + \xi_5 \dot{\xi}_5 + b\dot{c} + \bar{b}\dot{\bar{c}} + \pi_\lambda \dot{\lambda} + \pi_\chi \dot{\chi} + + P_s \dot{s} + P_{\bar{s}} \dot{\bar{s}} - H_\alpha^Q ,$$

where

$$H_\alpha^Q = \{ \lambda L_1 - \bar{b} b - \lambda \pi_\lambda b c \}(1 - \bar{c}c) + \alpha(\chi L_2 - P_s P_{\bar{s}} + + \frac{\lambda s^2}{\alpha} b\bar{c} + 2\chi s b \} . \quad (4)$$

Now, in the functional integral

$$Z^\alpha = \int \exp\{-S_Q^\alpha\} \mathcal{D}(x_\mu, P_\mu, c, \bar{c}, b, \bar{b}, \lambda, \chi, \dots)$$

we shall change the variables with trivial determinant

$$\begin{array}{ll} \pi_x \to \alpha \pi_x & b \to \lambda b \\ P_{\bar{s}} \to \alpha P_{\bar{s}} & c \to \lambda^{-1} c \\ s \to \alpha s & \pi_\lambda \to \pi_\lambda - \frac{bc}{\lambda} . \end{array} \quad (5)$$

Then in view of α -independence of Z^α, choose $\alpha = 0$. The quantum action will be rewritten in the form:

$$S_Q^{\alpha=0} = \int d\tau \{ P_\mu \dot{x}_\mu + \xi_\mu \dot{\xi}_\mu + \xi_5 \dot{\xi}_5 + b\dot{c} + \bar{b}\dot{\bar{c}} + \pi_\lambda \dot{\lambda} - H^{QS} \},$$

where

$$H^{QS} = \{ \lambda L_1 - \lambda \bar{b} b - \lambda \pi_\lambda \frac{1}{2}(bc - cb) \} (1 - \frac{\bar{c}c}{\lambda}) . \quad (6)$$

Important is the fact, that under quantization and are realized by the gamma-matrices γ_μ , γ_5 and though the constraint $L_2 = P_\mu \zeta_\mu - \zeta_5 m$ is dropped in the gauge in (...), the states of the system are described by Dirac spinors.

2. Second Quantization and Quantum Supersymmetry

Proceeding from ref. [4] , we'll define the quantum field theory by an action, the equation of motion of which coincides with the stationary Shrodinger equation for the Hamiltonian (6). Thus, our wave function depends on x_μ , c , $\bar{c}$, λ , and is a Dirac spinor. This choice of the wave function follows from the Hamiltonian (6), where, due to the choice of the gauge there is no dependence on χ , S , $\bar{S}$.

Suppose an ordinary realization of the first quantization, i.e. substitution of P_μ , $\mathcal{B}$, $\bar{\mathcal{B}}$, π_λ by $-i\frac{\partial}{\partial x_\mu}, -i\frac{\partial}{\partial c}, -i\frac{\partial}{\partial \bar{c}}$ $-i\frac{\partial}{\partial \lambda}$, and of ζ_μ , ζ_5 by $\gamma_\mu \gamma_5$, γ_5 . The Hamiltonian is replaced by the operator

$$H^{QS} = \left\{-\frac{\lambda}{2}(\Box + m^2) - \lambda\partial_{\bar{c}}\partial_c + \lambda c\partial_c\partial_\lambda - \frac{\lambda}{2}\partial_\lambda\right\}\left(1 - \frac{\bar{c}c}{\lambda}\right) \quad (7)$$

The second-quantized action is now written as follows:

$$S = \int d^4x\, d\lambda\, dc\, d\bar{c}\; \tilde{\Psi}'\left(\frac{\lambda}{2}(-\Box - m^2 + 2\partial_c\partial_{\bar{c}} + 2c\partial_c\partial_\lambda - \partial_\lambda)\left(1 - \frac{\bar{c}c}{\lambda}\right)\Psi'\right), \quad (8)$$

where $\tilde{\Psi}'$, Ψ' are arbitrary Dirac spinors.

After the transformations

$$\left(1 - \frac{\bar{c}c}{\lambda}\right)\Psi' = \Psi, \quad \frac{\lambda}{2}\tilde{\Psi}' = \tilde{\Psi}$$

the action (8) can be rewritten in the form:

$$S=\int d^4x\, d\lambda\, dc\, d\bar{c}\, \tilde{\Psi}(-i\overleftarrow{\partial\!\!\!/}+m)\{i\partial\!\!\!/-m+(i\partial\!\!\!/+m)^{-1}(2\partial_c\partial_{\bar{c}}+2c\partial_c\partial_\lambda-\partial_\lambda)\}\Psi \quad (9)$$

Now let us connect $\tilde{\Psi}$ and Ψ in the following way:

$$\tilde{\Psi}(-i\overleftarrow{\partial\!\!\!/}+m)=\bar{\Psi}=\Psi^+\gamma_0\,. \qquad (10)$$

Note, that under the requirement of equal dimensions for the fields Ψ' and $\tilde{\Psi}'$ (equal to 3/2 for fermions), the equation (8) could be written in the form:

$$S=\int d^4x\, d\lambda\, dc\, d\bar{c}\, \tilde{\Psi}'[K]^{-1}H^{qs}\Psi',$$

where K is an operator with dimension 1. In this case the eq.(10) is equivalent to the following choice of :

$$K=i\partial\!\!\!/+m\,,\quad \Psi'(1-\frac{\bar{c}c}{\lambda})=\Psi\,,\qquad \tilde{\Psi}'=\bar{\Psi}=\Psi^+\gamma_0\,.$$

Finally, we get the action

$$S^{SS}=\int dz\,\bar{\Psi}(z)(i\partial\!\!\!/-m+(i\partial\!\!\!/+m)^{-1}[D,\bar{D}])\Psi(z)\,, \qquad (11)$$

where $z=(x_\mu,\lambda,c,\bar{c})$ and $D,\ \bar{D}$ are supercovariant derivatives in chiral representation of the one-dimensional linear supersymmetry:

$$D=\partial_c\,,\quad \bar{D}=\partial_{\bar{c}}-c\partial_\lambda\,,\qquad \{D,\bar{D}\}=-\partial_\lambda\,.$$

The corresponding supergenerators are:

$$Q=\partial_c+\bar{c}\partial_\lambda\,,\quad \bar{Q}=\partial_{\bar{c}}\,,\quad \{Q,\bar{Q}\}=\partial_\lambda\,.$$

Hence, the quantum action for fermions, as in the case with a spinless particle [4], has a one-dimensional supersymmetry

$$\delta\Psi = (\varepsilon Q + \bar{\varepsilon}\bar{Q})\Psi, \qquad \delta\bar{\Psi} = (\varepsilon Q + \bar{\varepsilon}\bar{Q})\bar{\Psi}$$

and, as in ref.[4], the physical sector is defined by the conditions

$$Q\Psi = \bar{Q}\Psi = 0,$$

This means, that the source in the functional integral is introduced only for the first component of the fields Ψ and $\bar{\Psi}$ taken in the point $\lambda = 0$.

The Lagrangian (11) coincides with the supersymmetric Lagrangian obtained in the stochastic quantization of fermions [3] . That is why it is evident that the functional integral

$$Z^{ss}(\eta(x_\mu), \bar{\eta}(x_\mu)) = \int \exp\left\{-S^{ss} + \int d^4x \left(\bar{\eta}(x)\Psi(x) + \bar{\Psi}(x)\eta(x)\right)\right\} \mathcal{D}\Psi(z)\,\mathcal{D}\bar{\Psi}(z)$$

will be equal to

$$Z(\eta(x), \bar{\eta}(x)) = \int \exp\left\{-S^{cl}(\Psi, \bar{\Psi}) + \int d^4x \left(\bar{\eta}\Psi + \bar{\Psi}\eta\right)\right\} \mathcal{D}\Psi(x)\,\mathcal{D}\bar{\Psi}(x)$$

where $\Psi(x) = \Psi(z)\big|_{\lambda = c = \bar{c} = 0}$ and $\bar{\Psi}(x) = \bar{\Psi}(z)\big|_{\lambda = c = \bar{c} = 0}$

In conclusion note, that such linear supersymmetry guarantees renormalization and unitarity of the 5-dimensional supertheory [5,6] .

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