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Ed.Sh.EGORIAN

SECOND QUANTIZATION OF SPINLES RELATIVISTIC PARTICLE AND QUANTUM SUPERSYMMETRY

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Էդ.Շ. ԵԳՈՐՅԱՆ

ՍՊԻՆԱԶՈՒՐԿ ՌԵԼՅԱՏԻՎԻՍՏԱԿԱՆ ՄԱՍՆԻԿԻ ԵՐԿՐՈՐԴԱՅԻՆ ՔՎԱՆՏԱՑՈՒՄԸ ԵՎ ՔՎԱՆՏԱՅԻՆ ԳԵՐՀԱՄԱՉԱՓՈՒԹՅՈՒՆԸ

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Ed.Sh. EGORIAN

SECOND QUANTIZATION OF SPINLESS RELATIVISTIC PARTICLE AND QUANTUM SUPERSYMMETRY

A second-quantized theory of a relativistic particle with one-dimensional linear supersymmetry is obtained, starting from the Batalin-Frandkin-Vilkovisky (BFV) first quantization method.

Yerevan Physics Institute

Yerevan 1987

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Эд.Ш.ЕГОРЯН

ВТОРИЧНОЕ КВАНТОВАНИЕ БЕССПИНОВОЙ РЕЛЯТИВИСТСКОЙ ЧАСТИЦЫ И КВАНТОВАЯ СУПЕРСИММЕТРИЯ

Получена вторично-квантованная теория релятивистской частицы, обладающая одномерной линейной суперсимметрией.

Ереванский физический институт
Ереван 1987

1. Introduction.

There is a class of first-quantized actions of a relativistic particle, corresponding to gauge fixing functions χ in the BFV [1] approach. This brings to a class of second-quantized field theory actions. We prove in this work, that one can choose such a χ at first-quantized level, that the corresponding second-quantized theory should have a linear supersymmetry instead of a nonlinear BRST symmetry. Our approach with one-dimensional linear supersymmetry is an alternative to Neveu-West approach [2] with Parisi-Sourlas supersymmetry.

The interacting particle field theory is constructed with the requirement to preserve the linear supersymmetry. The resulting supersymmetry field theory coincides with one [3] obtained from stochastic quantization.

This quantum supersymmetry as applied to the strings can give a good instrument for constructing the string field interactions and analysing the string perturbation theory.

2. First Quantization.

The reparametrization-invariant action for a spinless relativistic particle

$$S_{cl} = \int_0^T d\tau \left(p_\mu \dot{x}_\mu - \frac{1}{2}\lambda(\tau)(p^2 + m^2)\right) \tag{1}$$

has the following infinitesimal local symmetry

$$\delta x^\mu = 2\varepsilon p^\mu \quad , \qquad \delta\lambda = 2\dot{\varepsilon} \quad , \qquad \delta p^\mu = 0 \tag{2}$$

where the local infinitesimal parameter $\varepsilon(\tau)$ is restricted at the end points [4]:

$$\varepsilon(0) = \varepsilon(T) = 0 \tag{3}$$

The quantum BFV-Hamiltonian for the arbitrary gauge function χ may be written in terms of the following Poisson bracket:

$$H = \{ Q, \lambda\bar{\mathcal{B}} - \bar{c}\chi \} , \tag{4}$$

where Q is the BRST charge:

$$Q = \pi\mathcal{B} + \frac{1}{2}c(p^2 + m^2) \tag{5}$$

$c, \bar{c}$ are the ghost and $\mathcal{B}, \bar{\mathcal{B}}$ the corresponding anti-ghost variables.

We take

$$\chi = \lambda\bar{\mathcal{B}}c \tag{6}$$

instead of $\chi = 0$ in Ref. [4]. The resulting Hamiltonian is

$$H = [\lambda \tfrac{1}{2}(p^2 + m^2) + \bar{\mathcal{B}}\mathcal{B} + \lambda\pi\bar{\mathcal{B}}c](1 + \bar{c}c) , \tag{7}$$

where π is the canonical momentum to λ .

3. Second Quantization.

We now postulate the second-quantized action, which leads to the stationary Schrodinger equation with Hamiltonian H (Eq. (7)), where the following operator replacements are made:

$$x^{\mu} \to x^{\mu} \quad c \to c, \quad \bar{c} \to \bar{c}, \quad \lambda \to \lambda$$
$$p^{\mu} \to -i\frac{\partial}{\partial x^{\mu}}, \quad \mathcal{B} \to i\frac{\partial}{\partial \bar{c}}, \quad \bar{\mathcal{B}} \to -i\frac{\partial}{\partial c}, \quad \pi \to -i\frac{\partial}{\partial \lambda} \tag{8}$$

We consider a phase space of wave functions $\Psi(Z)$ of the coordinates $Z=(x^{\mu}, \lambda, c, \bar{c})$. The action is

$$S=\int dZ\, \bar{\Psi} H \Psi \tag{9}$$

$\bar{\Psi}, \Psi$ are arbitrary functions.

Let us make a few remarks concerning our choice of the wave functions phase space and the action (9). Our phase space certainly contains the physical-state wave functions which depend on the coordinates x^{μ} only. Our space is the stationary subspace of the Neveu-West space of functions $\Psi(Z,\tau)$. The independence of physical amplitudes on the arbitrary parametrization is an argument for one to neglect the τ -dependence in the second-quantized theory from the beginning. But the basic argument for our choice of functional space $\Psi(Z)$ and the action (9) is our final result.

After the transformations

$$\begin{aligned} (1+\bar{c}c)\Psi &\to \Phi \\ c &\to \lambda^{-1} c \\ \lambda^{2}\bar{\Psi} &\to \bar{\Phi} \end{aligned} \tag{10}$$

the action (9) can be rewritten in the form:

$$S=\int dz\,\bar{\Phi}\left[\frac{1}{2}(m^2-\partial^2)+\frac{\partial}{\partial c}\frac{\partial}{\partial\bar{c}}-\partial_c c\,\partial_\lambda\right]\Phi\,. \tag{11}$$

To describe a scalar particle field theory, one must reduce two arbitrary functions $\bar{\Phi}$, Φ to one. There are two reasonable possibilities to do this: either the BRST or the linear supersymmetry invariant way.

The BRST invariant choice is $\bar{\Psi}(x,\lambda,c,\bar{c})=\Psi(x,-\lambda,-c,\bar{c})$ in Eq. (9).

We choose the linear supersymmetry invariant way:

$$\bar{\Phi}(x,\lambda,c,\bar{c})=\Phi(x,\lambda,c,\bar{c}) \tag{12}$$

In this case the supersymmetry replaces the BRST invariance.

Taking into account Eq. (12) and supersymmetrizing the kinetic operator in Eq. (11), we get:

$$S=\frac{1}{2}\int dz\,\Phi(m^2-\partial^2+B)\,\Phi\,, \tag{13}$$

where

$$B=2\partial_c\partial_{\bar{c}}+2c\,\partial_c\partial_\lambda-\partial_\lambda \tag{14}$$

and the superfield Φ has the following expansion:

$$\Phi(z)=\varphi(x,\lambda)+\bar{\eta}(x,\lambda)c+\bar{c}\eta(x,\lambda)+\bar{c}c\alpha(x,\lambda)\,. \tag{15}$$

The action (13) has the following supersymmetry:

$$\lambda'=\lambda+\bar{c}\varepsilon\,,\quad c'=c-\varepsilon\,,\quad \bar{c}'=\bar{c}-\bar{\varepsilon} \tag{16}$$

$$\Phi'(x,\lambda',c',\bar{c}')=\Phi(x,\lambda,c,\bar{c})$$

with the generators

$$Q = \frac{\partial}{\partial c} + \bar{c}\,\partial_\lambda \quad , \qquad \bar{Q} = \frac{\partial}{\partial \bar{c}} \, . \tag{17}$$

The operator B (Eq. (14)) can be rewritten in the explicit super-invariant form:

$$B = [D, \bar{D}] ,$$

$$D = \frac{\partial}{\partial c} \quad , \qquad \bar{D} = \frac{\partial}{\partial \bar{c}} - c\,\partial_\lambda \, . \tag{18}$$

The derivatives $D, \bar{D}$ correspond to a chiral representation of the supersymmetry transformations with the generators $Q, \bar{Q}$.

The action (13) coincides with one [3] obtained from stochastic quantization. The coordinate λ plays the role of additional time.

The physical-state (off shell) condition is:

$$Q\Phi = \bar{Q}\Phi = 0 \tag{19}$$

which has the solution:

$$\Phi(x, \lambda, c, \bar{c}) = \varphi(x) \tag{20}$$

The Eq. (20) tells us that just the first component of the superfield Φ is the physical field.

4. Interaction.

We construct the interacting particle field theory under the requirement that the linear supersymmetry (16) should be preserved at the interaction level. This supersymmetry guarantees the unitarity (the physical-state condition (19) is preserved after scattering) and the renormalizability of the five-dimensional supersymmetric Φ^4 theory [5,6]. The supersymmetric interaction term is:

$$S_{int} = \int V(\Phi) dz \ ,$$
$$V(\Phi) = a\Phi^3(z) + g\Phi^4(z) . \qquad (21)$$

In conclusion, we bring the formula from Ref. [3], which gives the connection between the five-dimensional supersymmetric and usual four-dimensional scalar field theories:

$$\int \exp \int dz \left[\frac{1}{2} \Phi (m^2 - \partial^2 + [illegible]) \Phi + V(\Phi) + H\Phi \right] \mathcal{D}\Phi(z) =$$
$$= \int \exp - \int d^4x \left[\frac{1}{2} \varphi(x)(m^2 - \partial^2)\varphi(x) + V(\varphi) + h(x)\varphi(x) \right] \mathcal{D}\varphi(x) . \qquad (22)$$

where

$$H = h(x [illegible]) [illegible]$$

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REFERENCES

1. Fradkin E.S., Vilkovisky G.A. Quantization of relativistic systems with constraints. - Phys. Lett., 1975, v. 55B, N. 2, p. 224-226.
2. Neveu A., West P. Unification of the Poincare group with BRST and Parisi-Sourlas supersymmetries. - Preprint CERN-TH. 4547/86, Geneva, 1986.
3. Egorian Ed.Sh., Kalitsin S. A superfield formulation of stochastic quantization with fictious time. - Phys. Lett., 1983, v. 129B, N. 5, p. 320-323.
4. Monaghan S. BRS Hamiltonian quantization of a spinless relativistic particle in relativistic gauges. - Phys. Lett., 1986, v. 178 B, N. 2, 3, p. 231-234.
5. Егорян Эд.Ш., Манвелян Р.П. Стохастическая регуляризация и перенормировка. ТМФ, 1986, т.68, № 2, с.187-197.
6. Zin-Justin. Stochastic quantization. - Nucl. Phys., 1986, v. B 275, N. 1, p. 134.

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ЧАСТИЦЫ И КВАНТОВАЯ СУПЕРСИММЕТРИЯ

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Ереван 36, Маркаряна 2

The address for requests:
Information Department
Yerevan Physics Institute
Markaryan St., 2
Yerevan, 375036
Armenia, USSR

индекс 3624

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ